LINEAR ALGEBRA EDUCATION

Recent Linear Algebra Education Events

By Steven Leon

Among the highlights of the 2012 Joint Meetings of the AMS and MAA were the two sessions on Innovative and Effective Ways to Teach Linear Algebra that were organized by David Strong of Pepperdine University. The sessions featured nineteen talks offering a wealth of constructive ideas for improvements in the teaching of linear algebra. These sessions have become an popular feature at the annual meetings. A listing of all the talks given in these sessions during the last five years is available at the site: `http://faculty.pepperdine.edu/dstrong/LinearAlgebra/index.html`. This site features links to files containing the transparencies from many of the talks that were presented at the meetings.

Linear Algebra Education Events at Future Meetings

By Steven Leon

The 12$^{\text{th}}$ International Congress of Mathematics Education (ICME-12), July 8–15, 2012, Seoul, Korea, will feature a Discussion Group on Issues Surrounding the Teaching of Linear Algebra. The co-chairs of the discussion group are Avi Berman and Kazuyoshi Okubo. The 16$^{\text{th}}$ Haifa Matrix Theory Conference, November 12–15, 2012, will feature a session devoted to the teaching of linear algebra. The Annual ILAS Conference, June 3–7, 2013, Providence, Rhode Island will feature an invited minisymposium on Linear Algebra Education Issues. The organizers for the minisymposium are Avi Berman, Sang-Gu Lee, and Steven Leon

BOOK REVIEWS

Matrix Tricks for Linear Statistical Models: Our Personal Top Twenty By Simo Puntanen, George P.H. Styan and Jarkko Isotalo

Springer, Heidelberg, 2011, xvii+486 pages, ISBN 978-3-642-10472-5

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During the last decade, more than 50 monographs and textbooks have been published in the area of matrix algebra and statistical inference based on linear models. Among this collection, the book of Puntanen, Styan and Isotalo is exceptional. Of course, as it follows from the title, this new book is closely connected with matrices and linear models, but its organization is unique.

First of all, the authors extract from the matrix algebra and theory of linear models twenty key results or ideas which they call "tricks". The tricks are formulated as theorems and are placed at the beginning of each chapter, together with proofs, necessary references and specific remarks. At the end of each chapter, except the shortest ones, there is a list of exercises of various difficulty. The other sections of each chapter contain results connected with the leading trick of the chapter. Usually, they are formulated as propositions with formal proofs and frequently are supplemented by additional comments, citations, historical remarks and precise references from a collection of 661 items listed at the end of the book. The proofs of these propositions, however, contain cross-references to tricks of the other chapters. Consequently, the organization of the material is nonlinear. In addition, certain problems from the theory of linear models are recalled and revisited in many places of the book.

To help the reader, the authors begin the book with a rather long Introduction which contains preliminaries to linear algebra, statistics, random vectors, data matrices and linear models. Here the reader finds the definitions of the concepts, some examples and remarks, as well as basic methods which are discussed in detail in subsequent main chapters. The first main chapter, Chapter 1, contains the first trick, which is concerned with the column space of a matrix. The chapter begins with a collection of 13 simple and well-known facts; the rest of the chapter deals with applications derived from focusing on the column space. One section is devoted to the estimability of linear functions in a simple ANOVA model and the other to showing properties of the covariance matrix of a random vector.

Chapter 2 presents properties of orthogonal projectors. These properties are used to establish solutions to some minimization problems. The properties are extended to orthogonal projectors with respect to an arbitrary inner (or semi-inner) product. Chapter 3 is devoted to the sample correlation coefficient from a geometrical point of view. Moreover, the chapter develops the connections with the coefficient of determination and partial correlation coefficient in a simple regression model. Chapter 4 contains basic results on generalized inverses of a matrix. The chapter is supplemented with an exploration of their relations with the singular value decomposition, oblique projectors, the general solution and minimum norm solution to consistent linear equations, as well as with the least squares solution to inconsistent linear equations.

A collection of results concerning the rank of a partitioned matrix and the matrix product constitute the trick of Chapter 5. Among other results, the reader finds here a nonstandard column space decomposition of a partitioned matrix into disjoint subspaces, a condition for equality of column subspaces of two matrices, and an explicit expression for the intersection of two column spaces.

Chapters 6 and 7 are the shortest. The first contains the rank cancellation rule which leads to the simplification of some matrix equations. In turn, Chapter 7 contains the conditions under which the sum (or difference) of two orthogonal projectors is an orthogonal projector.

Chapter 8 recalls the orthogonal decomposition of the column space of a partitioned matrix and then presents a decomposition of the orthogonal projector. This result is then applied to particular problems connected with estimation of parameters in linear models of various forms, related with testing of linear hypotheses or with expressing the coefficient of determination in different but equivalent forms. Chapter 9 considers the problem of finding the minimum – in Löwner sense – for the covariance matrix of a special linear transform of the partitioned vector random variable. This trick is presented in many statistical applications. Other sections in this chapter show its usefulness in establishing the best linear predictor (BLP) and its relations with linear regression and principal component analysis.

Chapter 10 is the longest one. It deals with the best linear unbiased estimation (BLUE) of linear functions of unknown parameters. This fundamental concept of linear models is characterized by a solution to an appropriate matrix equation. The main problems considered here concern the equality between the BLUE and the ordinary least squares estimator (OLSE), the efficiency of OLSE and its relation with canonical correlations, as well as the equality between the best linear unbiased predictor (BLUP) and the OLSE.

The trick of Chapter 11 is concerned with the basic matrix equation. Its general solution, when consistent, is utilized in discussion of connections between the BLUEs following from two fixed models with the same expectation but with different covariance structures. Similar problems concerning links between BLUEs and BLUPs under mixed models are also considered.

Chapter 12 presents the necessary and sufficient condition for invariance of matrix products involving generalized inverses with respect to the choice of the inverses. The results establishing invariance of the column space and rank of such products are also discussed.

Chapter 13 deals with the block partitioned symmetric nonnegative definite matrices. The main trick here concerns the block-diagonalization of such matrices in which the Schur complements of diagonal blocks appear. Formulas for generalized inverses, determinants and ranks of such matrices are also exhibited. Contrary to the trick of the previous chapter, Chapter 14 contains the necessary and sufficient condition under which a block partitioned symmetric matrix appears to be nonnegative definite. In this context, some consequences related with correlation and covariance matrices and also with the Löwner order in a set of nonnegative definite matrices are also presented. Properties of three specific nonnegative definite matrices form the trick of Chapter 15. Successive subsections show their usefulness in presenting solutions to many problems related with the BLUE and OLSE in linear models of various structures. The trick of Chapter 16 is composed of fourteen equivalent conditions for the disjointness of the column spaces of two matrices. These results are used to express the estimability condition in a partitioned linear model and in two problems concerning the BLUE and OLSE.

Chapters 17, 18 and 19 are devoted to three decompositions: full rank decomposition, the eigenvalue decomposition of a symmetric matrix, and the singular value decomposition of any rectangular matrix. The usefulness of these techniques in matrix algebra is well-known, but the authors focus their attention mainly on various statistical implications. Chapter 20 presents the last trick, the Cauchy–Schwarz inequality. This trick is supplemented by its specific versions, some statistical consequences, and their relations with inequalities of Kantorovich and of Wielandt.

The whole material is enriched with 30 figures illustrating the geometry of some statistical concepts and methods, with 31 photographs of statisticians, mainly from a collection of the first author, and with several images of very interesting philatelic items from a collection of the second author.

This exceptional book can be recommended to all interested students who wish to improve their skills in linear models and matrix manipulations. It can be recommended also to professors teaching statistics who may utilize this book as a rich source of various exercises, references and interesting historical notes.