Journal of Statistical Planning and Inference 88 (2000) 173–179

journal of statistical planning and inference

www.elsevier.com/locate/jspi

Two matrix-based proofs that the linear estimator Gy is the best linear unbiased estimator ☆

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Received 27 January 1998; received in revised form 8 June 1998

Abstract

We offer two matrix-based proofs for the well-known result that the two conditions $GX = X$ and $GVQ = 0$ are necessary and sufficient for Gy to be the traditional best linear unbiased estimator (BLUE) of $X\beta$ in the Gauss–Markov linear model $\{y, X\beta, V\}$, where y is an observable random vector with expectation vector $\mathscr{E}(y) = X\beta$ and dispersion matrix $\mathscr{D}(y) = V$; the matrix Q here is an arbitrary but fixed matrix whose range (column space) coincides with the null space of the transpose of X.

MSC: 15A09; 15A24; 15A45; 62J05

Keywords: Best linear unbiased estimator; Best unbiased estimators; BLUE; Consistency condition; Extended BLUE approach; Extended estimators; Gauss–Markov model; Idempotent matrices; Generalized matrix inverses; Linear models; Matrix methods for statistics; Natural restrictions; Quadratic unbiased estimators; Traditional BLUE approach

1. Introduction and preliminaries

Let $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$, and $\mathscr{P}^{n\times n}$ denote the set of n-dimensional real column vectors, the set of $n \times m$ real matrices, and the set of real $n \times n$ symmetric nonnegative-definite (nnd) matrices, respectively. Given $A \in \mathbb{R}^{n\times m}$, let A', $\mathscr{R}(A)$, and $\mathscr{N}(A)$ denote the transpose,

☆ This paper was presented at the Sixth International Workshop on Matrix Methods for Statistics, İstanbul, Turkey, August 1997. The research of G.P.H. Styan was supported in part by the Natural Sciences and Engineering Research Council of Canada. Financial support of H.J. Werner by the Deutsche Forschungsgemeinschaft, Sonderforschungsbereich 303, at the University of Bonn, is gratefully acknowledged.

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PII: S0378-3758(00)00076-8