Revisiting the BLUE in a Linear Model via Proper Eigenvectors

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Abstract We consider two linear models, $\mathscr{M}_1 = \{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \mathbf{V}_1\}$ and $\mathscr{M}_2 = \{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \mathbf{V}_2\}$, having different covariance matrices. Our main interest lies in question whether a particular given BLUE under $\mathscr{M}_1$ continues to be a BLUE under $\mathscr{M}_2$. We give a thorough proof of a result originally due to Mitra and Moore (Sankhyā, Ser. A 35:139–152, 1973). While doing this, we will review some useful properties of the proper eigenvalues in the spirit of Rao and Mitra (Generalized Inverse of Matrices and Its Applications, 1971).

Keywords Best linear unbiased estimator · Gauss–Markov model · Linear model · Löwner ordering · Orthogonal projector · Proper eigenvalues

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1 Introduction

In this article, we consider the general linear model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, denoted as a triplet $\mathscr{M} = \{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \mathbf{V}\}$, where $\mathbf{X}$ is a known $n \times p$ model matrix, the vector $\mathbf{y}$ is an observable n-dimensional random vector, $\boldsymbol{\beta}$ is a $p \times 1$ vector of unknown parameters, and $\boldsymbol{\varepsilon}$ is an unobservable vector of random errors with expectation $\mathrm{E}(\boldsymbol{\varepsilon}) = \mathbf{0}$ and covariance matrix $\mathrm{cov}(\boldsymbol{\varepsilon}) = \mathbf{V}$. The nonnegative definite matrix $\mathbf{V}$ is known.

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