

McGill Matrix Wednesday: May 30, 2007

Otto Maass Chemistry Building, Room 217

Programme with abstracts

9:30 – 10:00 am Coffee/Tea

10:00 – 11:30 am Session 1, chair: George P. H. Styan

10:00 am Christopher C. PAIGE & Ivo PANAYOTOV (McGill University):
“Accuracy of Ritz values from a given subspace” [M-1]

10:30 am Götz TRENKLER (Universität Dortmund, Dortmund, Germany):
“Two subspaces—two projectors” [M-2]

10:50 am Oskar Maria BAKSALARY (Adam Mickiewicz University, Poznań, Poland):
“On the product of two orthogonal projectors” [M-3]

11:10 am Augustyn MARKIEWICZ (Agricultural University of Poznań, Poznań, Poland):
“Kiefer optimality in multivariate linear models” [M-4]

11:30 am – 2:00 pm Lunch: Le Taj, 2077 rue Stanley (514-845-9015), www.restaurantletaj.com

2:00 – 3:20 pm Session 2, chair: Oskar Maria Baksalary

2:00 pm Jarkko ISOTALO (University of Tampere, Tampere, Finland):
“On equivalent linear systems in the context of estimation in the linear statistical model” [M-5]

2:20 pm Simo PUNTANEN (University of Tampere, Tampere, Finland):
“A useful matrix decomposition and its statistical applications in linear regression” [M-6]

2:40 pm Kimmo VEHKALAHTI (University of Helsinki, Helsinki, Finland):
“Matrices as building blocks of the measurement framework” [M-7]

3:00 pm Jani A. VIRTANEN (University of Helsinki, Helsinki, Finland):
“Structured pseudospectra” [M-8]

3:20 – 4:00 pm Coffee/Tea

4:00 – 5:20 pm Session 3, chair: Simo Puntanen

4:00 pm Shuangzhe LIU (University of Canberra, Canberra, Australia):
“Matrix-trace Wielandt inequalities with statistical applications” [M-9]

4:20 pm Yongge TIAN (Shanghai University of Finance and Economics, Shanghai, China):
“On V -orthogonal projectors associated with a semi-norm” [M-10]

4:40 pm Xiaowen CHANG (McGill University):
“Backward error for scaled total least squares problems” [M-11]

5:00 pm George P. H. STYAN (McGill University):
“Some comments on a matrix associated with the paintings by
Johannes Vermeer (1632–1675) depicted on postage stamps” [M-12]

5:20 – 6:30 pm Refreshments

7:30 pm – ... Peking Duck Dinner: La Maison Kam Fung,
1111 rue Saint-Urbain (514-878-2888), www.lamaisonkamfung.com

Abstracts

10:00 am Christopher C. PAIGE & Ivo PANAYOTOV (McGill University):

[M-1] “Accuracy of Ritz values from a given subspace”

If $x, y \in \mathbb{C}^n$ are unit-length vectors ($x^H x = y^H y = 1$) where y is an approximation to an eigenvector x of $A = A^H \in \mathbb{C}^{n \times n}$ with $Ax = x\lambda$, $\lambda = x^H Ax \in \mathbb{R}$, then the Rayleigh quotient $y^H Ay$ can be shown to satisfy

$$|\lambda - y^H Ay| \leq \sin^2 \theta(x, y) \cdot \text{spread}(A). \quad (1)$$

Here if $\lambda_1(A) \geq \dots \geq \lambda_n(A)$ are the eigenvalues of A , then $\text{spread}(A) \equiv \lambda_1(A) - \lambda_n(A)$, and $\theta(x, y) \equiv \cos^{-1} |x^H y| \in [0, \pi/2]$ is the acute angle between x and y . Thus the Rayleigh quotient approximation $y^H Ay$ to the eigenvalue λ of A can be far more accurate than the approximation of $\text{range}(y)$ to the invariant subspace $\text{range}(x)$ of A . We generalize this result to a higher dimensional subspace \mathcal{Y} approximating an invariant subspace \mathcal{X} .

Let $X, Y \in \mathbb{C}^{n \times k}$ be such that $X^H X = Y^H Y = I_k$, the $k \times k$ unit matrix, where $\mathcal{Y} \equiv \text{range}(Y)$ is an approximation to the invariant subspace $\mathcal{X} \equiv \text{range}(X)$ of A , so that $AX = X \cdot X^H AX$. Let $\lambda(X^H AX)$ and $\lambda(Y^H AY) \in \mathbb{R}^k$ be the vectors of eigenvalues in descending order of $X^H AX$ and $Y^H AY$, respectively. The elements of $\lambda(Y^H AY)$ are called Ritz values in the Rayleigh–Ritz method for approximating the eigenvalues $\lambda(X^H AX)$ of A . Let $\theta(\mathcal{X}, \mathcal{Y}) \in \mathbb{R}^k$ be the vector of angles (in descending order) between the subspaces \mathcal{X} and \mathcal{Y} , so that $\cos \theta(\mathcal{X}, \mathcal{Y}) = (\sigma_k, \dots, \sigma_1)^T$ where $\sigma_1 \geq \dots \geq \sigma_k$ are the singular values of $X^H Y$. Then if $\lambda(X^H AX)$ are the k largest eigenvalues of A , we show $\exists u$:

$$\lambda(X^H AX) - \lambda(Y^H AY) \prec u \leq \text{spread}(A) \cdot \sin^2 \theta(\mathcal{X}, \mathcal{Y}),$$

where “ $\prec u$ ” means “is majorized by u ”. Compare this with (1). This implies

$$|\lambda(X^H AX) - \lambda(Y^H AY)| \prec_w \sin^2 \theta(\mathcal{X}, \mathcal{Y}) \cdot \text{spread}(A), \quad (2)$$

where “ \prec_w ” denotes weak (sub-) majorization. It was shown in [1] that (2) holds for many practical cases. We suspect that (2) holds in general, but in [P1] we have only proven that a slightly weaker version *always* holds. [Joint work with Merico Argentati & Andrew Knyazev.]

[P1] M. E. Argentati, A. V. Knyazev, C. C. Paige, and I. Panayotov. Bounds on changes in Ritz values for a perturbed invariant subspace of a Hermitian matrix. Submitted to the *SIAM Journal on Matrix Analysis and Applications* in July 2006.

10:30 am Götz TRENKLER (Universität Dortmund, Dortmund, Germany):

[M-2] “Two subspaces—two projectors”

For two given subspaces of \mathbb{C}_n we investigate their distance and angle on the basis of a joint decomposition of the corresponding orthogonal projectors. Further attention is paid to the notions of inclinedness, orthogonal incidence and minimal angle. Some formulas for the spectral norm of orthogonal projectors or functions thereof are derived. Also the proofs of results known from Hilbert space theory, related to orthogonal projectors, are simplified considerably. [Joint work with Oskar Maria Baksalary.]

10:50 am Oskar Maria BAKSALARY (Adam Mickiewicz University, Poznań, Poland):

[M-3] “On the product of two orthogonal projectors”

It is known that a given square matrix is a projector (i.e., idempotent matrix) if and only if it is expressible as the Moore–Penrose inverse of the product of two orthogonal projectors (i.e., Hermitian idempotent matrices). Representing such an inverse as a partitioned matrix turns out to be very useful in studying the properties of the product of two orthogonal projectors and its functions. This fact is demonstrated by establishing several original results. [Joint work with Götz Trenkler.]

11:10 am Augustyn MARKIEWICZ (Agricultural University of Poznań, Poznań, Poland):

[M-4] “Kiefer optimality in multivariate linear models”

Kiefer optimality of designs, usually studied under univariate linear normal models, it is extended to multivariate linear normal models. This extension is straightforward in the case of known dispersion matrix by the use of univariate formulation of the multivariate linear model.

In the case of partially unknown dispersion matrix optimality is considered with respect to the precision in maximum likelihood estimation (MLE). The derivation of information matrices and precision matrices in MLE is presented. The relation between design optimality in a univariate model and in its multivariate extensions is studied.

2:00 pm Jarkko ISOTALO (University of Tampere, Tampere, Finland):

[M-5] “On equivalent linear systems in the context of estimation in the linear statistical model”

We consider the estimation of the linear parametric function $\mathbf{K}'\boldsymbol{\beta}$ in the linear statistical model $\{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \mathbf{V}\}$. It is well known that a linear statistic $\mathbf{G}\mathbf{y}$ is the best linear unbiased estimator (BLUE) of $\mathbf{K}'\boldsymbol{\beta}$ if and only if \mathbf{G} satisfies the fundamental equation of the BLUE:

$$\mathbf{G}(\mathbf{X} : \mathbf{V}\mathbf{X}^\perp) = (\mathbf{K}' : \mathbf{0}). \quad (3)$$

We introduce a linear system that is equivalent to the system of linear equations given in (3), and then present some properties of the BLUE based on the equivalent linear system.

2:20 pm Simo PUNTANEN (University of Tampere, Tampere, Finland):

[M-6] “A useful matrix decomposition and its statistical applications in linear regression”

It is well known that if \mathbf{V} is a symmetric positive definite $n \times n$ matrix, and $(\mathbf{X} : \mathbf{Z})$ is a partitioned orthogonal $n \times n$ matrix, then

$$(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1} = \mathbf{X}'\mathbf{V}\mathbf{X} - \mathbf{X}'\mathbf{V}\mathbf{Z}(\mathbf{Z}'\mathbf{V}\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{V}\mathbf{X}. \quad (*)$$

In this paper we show the usefulness of the formula (*), and in particular the version (**):

$$\mathbf{Z}(\mathbf{Z}'\mathbf{V}\mathbf{Z})^{-1}\mathbf{Z}' = \mathbf{V}^{-1} - \mathbf{V}^{-1}\mathbf{X}(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}^{-1} := \dot{\mathbf{M}}, \quad (**)$$

and present several related formulas, as well as some generalized versions. We also include several statistical applications. [Joint work with Jarkko Isotalo & George P.H. Styan.]

2:40 pm Kimmo VEHKALAHTI (University of Helsinki, Helsinki, Finland):

[M-7] “Matrices as building blocks of measurement framework”

The measurement framework approach allows estimating the variances of the measurement errors, and hence assessing the reliability and validity in multidimensional measurement settings. Matrices play a central role in all aspects of the framework. The measurement model is a multidimensional true score model, which is closely connected to the factor analysis model. The multivariate measurement scales in turn include factor scores, psychological test scales, or any other linear combinations of the observed variables, for example scales produced by regression analysis, discriminant analysis, or other multivariate statistical methods. Further research is proposed for a number of interesting topics.

3:00 pm Jani A. VIRTANEN (University of Helsinki, Helsinki, Finland):

[M-8] “Structured pseudospectra”

We recall some recent results on structured pseudospectra of finite Toeplitz matrices, and consider similar questions in the case of block matrices. Other structures, such as those of symmetric and Hankel matrices, will also be mentioned.

4:00 pm Shuangzhe LIU (University of Canberra, Canberra, Australia):

[M-9] “Matrix-trace Wielandt inequalities with statistical applications”

The vector correlation coefficient and other measures of association play a very important role in statistics and especially in multivariate analysis. In this paper a new measure of association is proposed and its upper bound is presented, using a matrix trace Wielandt inequality. Also given are relevant results involving Wishart matrices widely used in multivariate analysis, and especially a new alternative for the relative gain of the covariance adjusted estimator of a vector of parameters.

4:20 pm Yongge TIAN (Shanghai University of Finance and Economics, Shanghai, China):

[M-10] “On \mathbf{V} -orthogonal projectors associated with a semi-norm”

For any $n \times p$ matrix \mathbf{X} and $n \times n$ nonnegative definite matrix \mathbf{V} , the matrix $\mathbf{X}(\mathbf{X}'\mathbf{V}\mathbf{X})^+\mathbf{X}'\mathbf{V}$ is called a \mathbf{V} -orthogonal projector with respect to the semi-norm $\|\cdot\|_{\mathbf{V}}$, where $(\cdot)^+$ denotes the Moore–Penrose inverse of a matrix. Various new properties of the \mathbf{V} -orthogonal projector were derived under the condition that $\text{rank}(\mathbf{V}\mathbf{X}) = \text{rank}(\mathbf{X})$, including its rank, complement, equivalent expressions, conditions for additive decomposability, equivalence conditions between two (\mathbf{V}) -orthogonal projectors, etc. [Joint work with Yoshio Takane.]

4:40 pm Xiaowen CHANG (McGill University):

[M-11] “Backward error for scaled total least squares problems”

Given $A \in \mathbb{R}^{m \times n}$ (with $m \geq n$), $b \in \mathbb{R}^m$ and $\gamma \in (0, \infty)$, the scaled total least squares (STLS) problem is defined as follows: $\min_{E, f, x} \|[E, \gamma f]\|_F$, s.t. $(A + E)x = b + f$. It is known that the STLS problem is equivalent to

$$\min_x \frac{\|Ax - b\|_2^2}{\gamma^{-2} + \|x\|_2^2}.$$

When $\gamma = 1$, the STLS problem is the TLS problem. As $\gamma \rightarrow 0$ and $\gamma \rightarrow \infty$, the STLS problem reduces to the ordinary least squares (OLS) problem and data least squares (DLS) problem respectively.

Given a nonzero approximate solution $y \in \mathbb{R}^n$ to the STLS problem, we might want to verify whether it is a backward stable solution. So we would like to solve the following backward error (BE) problem:

$$\mu \equiv \min_{\Delta A, \Delta b} \|\Delta A, \theta \Delta b\|_F, \quad \text{s.t. } y = \arg \min_x \frac{\|(A + \Delta A)x - b\|_2^2}{\gamma^{-2} + \|x\|_2^2}, \quad (4)$$

where $\theta \in (0, \infty)$. In this talk we present a pseudo backward error (PBE) for the STLS problem. This PBE is a lower bound on the true BE μ , but they are equal if y is close enough to the true STLS solution. This PBE converges to the BE of the LS problem obtained in [C2] and the PBE of the DLS problem obtained in [C1], as $\gamma \rightarrow 0$ and $\gamma \rightarrow \infty$, respectively.

Since computing the PBE directly is expensive, we derive a lower bound and an asymptotic estimate, which can be computed or estimated more efficiently. [Joint work with C. C. Paige & D. Tittle-Peloquin.]

[C1] X.-W. Chang, G. H. Golub, C. C. Paige and D. Tittle-Peloquin, Towards a backward perturbation analysis for data least squares problems, submitted for publication, 2006.

[C2] B. Waldén, R. Karlson and J. Sun, Optimal backward perturbation bounds for the linear least squares problem, *Numerical Linear Algebra with Applications*, 2, 271–286 (1995).

5:00 pm George P. H. STYAN (McGill University):

[M-12] “Some comments on a matrix associated with the paintings
by Johannes Vermeer (1632–1675) depicted on postage stamps”

[This talk is dedicated to the memory of Jerzy K. Baksalary (1944–2005).]

We have found 72 postage stamps from 29 different “countries” that depict 20 of the 35 paintings by Johannes Vermeer (1632–1675). By “country” we mean a stamp-issuing region that issues or has issued its own postage stamps. Vermeer’s famous painting “The Lacemaker” is depicted on 8 stamps from 8 countries (8 is the most number of stamps and the most number of countries for a particular painting), while Manama-Ajman has 8 stamps depicting 5 paintings (Manama along with the rest of Ajman agreed to join the United Arab Emirates on December 2, 1971).



Suppose that v_{ij} stamps have been issued depicting painting number $i = 1, \dots, 20$ by country number $j = 1, \dots, 29$; then our Vermeer matrix $\mathbf{V} = \{v_{ij}\}$ is 20×29 , with $v_{ij} = 0, 1, 2, \dots$. Some properties of the Vermeer matrix \mathbf{V} will be presented. [Joint work with Oskar Maria Baksalary.]

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The purpose of this Workshop is to stimulate research, in an informal setting, and to foster the interaction of researchers in the interface between matrix theory and statistics. The Workshop will include both invited and contributed talks, and a special session with talks and posters by graduate students is planned. A special issue of the journal *Linear Algebra and its Applications* will be published devoted to selected papers presented at the conference.

This workshop is a satellite meeting of the 35th Annual Meeting of the Statistical Society of Canada, St. John's, Newfoundland, June 10-13, 2007.

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IN HONOUR OF PROF. T.W. ANDERSON'S
50TH BIRTHDAY

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