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The BLUE's covariance matrix revisited: A review

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Abstract

In this paper we comment on and review some unexpected but interesting features of the BLUE (best linear unbiased estimator) of the expectation vector in the general linear model and in particular, the BLUE's covariance matrix. Most of these features appear in the literature but are rather scattered or hidden.

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1. Introduction

Let us start by considering a very simple (almost the simplest) linear model $\mathscr{M} = \{\mathbf{y}, \mathbf{1}\beta, \mathbf{V}\}$, which means that

$$\mathbf{y} = \mathbf{1}\beta + \boldsymbol{\varepsilon}, \tag{1.1}$$

where

$$E(\mathbf{y}) = \mathbf{1}\beta, \quad E(\boldsymbol{\varepsilon}) = \mathbf{0}, \quad \operatorname{cov}(\mathbf{y}) = \operatorname{cov}(\boldsymbol{\varepsilon}) = \mathbf{V}. \tag{1.2}$$

By $E(\cdot)$ and $\operatorname{cov}(\cdot)$ we denote the expectation vector and covariance matrix of a random vector argument. The vector $\mathbf{y}$ is an $n \times 1$ observable random vector, $\boldsymbol{\varepsilon}$ is an $n \times 1$ random error vector, $\mathbf{1}$ (vector of 1's) is an $n \times 1$ model matrix (a really simple one), β is an unknown parameter, and $\mathbf{V}$ is a known $n \times n$ positive definite matrix. Let $\mathrm{OLSE}(\cdot)$ denote the ordinary least squares estimator, $\mathrm{BLUE}(\cdot)$, the best linear unbiased estimator, and $\operatorname{var}(\cdot)$ the variance. Then we have

$$\mathrm{OLSE}(\beta) = \hat{\beta} = (\mathbf{1}'\mathbf{1})^{-1}\mathbf{1}'\mathbf{y} = \bar{y}, \quad \operatorname{var}(\hat{\beta}) = \frac{1}{n^2}\mathbf{1}'\mathbf{V}\mathbf{1}, \tag{1.3a}$$

$$\mathrm{BLUE}(\beta) = \tilde{\beta} = (\mathbf{1}'\mathbf{V}^{-1}\mathbf{1})^{-1}\mathbf{1}'\mathbf{V}^{-1}\mathbf{y}, \quad \operatorname{var}(\tilde{\beta}) = (\mathbf{1}'\mathbf{V}^{-1}\mathbf{1})^{-1}. \tag{1.3b}$$

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