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Linear sufficiency and completeness in the context of estimating the parametric function in the general Gauss–Markov model

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ABSTRACT

In this paper we consider linear sufficiency and linear completeness in the context of estimating the estimable parametric function $\mathbf{K}'\boldsymbol{\beta}$ under the general Gauss–Markov model $\{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \sigma^2\mathbf{V}\}$. We give new characterizations for linear sufficiency, and define and characterize linear completeness in a case of estimation of $\mathbf{K}'\boldsymbol{\beta}$. Also, we consider a predictive approach for obtaining the best linear unbiased estimator of $\mathbf{K}'\boldsymbol{\beta}$, and subsequently, we give the linear analogues of the Rao–Blackwell and Lehmann–Scheffé Theorems in the context of estimating $\mathbf{K}'\boldsymbol{\beta}$.

1. Introduction

Consider the general Gauss–Markov model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon},$$

where $\mathbf{y}$ is an $n \times 1$ observable random vector, $\mathbf{X}$ is a known $n \times p$ model matrix, $\boldsymbol{\beta}$ is a $p \times 1$ vector of unknown parameters, and $\boldsymbol{\epsilon}$ is an $n \times 1$ random error vector. The expectation and the covariance matrix of random vector $\mathbf{y}$ are

$$\mathrm{E}(\mathbf{y}) = \mathbf{X}\boldsymbol{\beta} \quad \text{and} \quad \mathrm{cov}(\mathbf{y}) = \sigma^2\mathbf{V},$$

respectively, where $\sigma^2 > 0$ is an unknown scalar and $\mathbf{V}$ is a known nonnegative definite matrix. In short, we use the notation

$$\mathscr{M}_{\boldsymbol{\beta}} = \{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \sigma^2\mathbf{V}\}$$

to describe the general Gauss–Markov model.

Furthermore, let $\mathbb{R}_{m,n}$ denote the set of $m \times n$ real matrices and $\mathbb{R}_m = \mathbb{R}_{m,1}$. The symbols $\mathbf{A}'$, $\mathbf{A}^-$, $\mathbf{A}^+$, $\mathscr{C}(\mathbf{A})$, $\mathscr{C}(\mathbf{A})^\perp$, $\mathscr{N}(\mathbf{A})$, and $\mathrm{r}(\mathbf{A})$ will stand for the transpose, a generalized inverse, the Moore–Penrose inverse, the column space, the orthogonal complement of the column space, the null space, and the rank, respectively, of $\mathbf{A} \in \mathbb{R}_{m,n}$. By $\mathbf{A}^\perp$ we denote any matrix satisfying $\mathscr{C}(\mathbf{A}^\perp) = \mathscr{N}(\mathbf{A}') = \mathscr{C}(\mathbf{A})^\perp$. Further we will write $\mathbf{P}_\mathbf{A} = \mathbf{A}\mathbf{A}^+ = \mathbf{A}(\mathbf{A}'\mathbf{A})^-\mathbf{A}'$ to denote the orthogonal projector (with respect to the

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