

# e-Souvenir

## **Twenty-eighth International Workshop on Matrices and Statistics**

**December 13-15, 2021**



## **International Conference on Linear Algebra and its Applications**

**December 15-17, 2021**



## **International Workshop on Algorithms for Cooperative TU Games using Matrices and Graphs**

**December 20-31, 2021**



Centre for Advanced Research  
in  
Applied Mathematics and Statistics  
MAHE, Manipal



**MANIPAL**  
ACADEMY of HIGHER EDUCATION

(Deemed to be University under Section 3 of the UGC Act, 1956)

**Centre for Advanced Research in Applied Mathematics and Statistics  
(CARAMS, MAHE), Manipal Academy of Higher Education, Manipal, India**

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# Souvenir

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Centre for Advanced Research  
in  
Applied Mathematics and Statistics  
MAHE, Manipal

Centre for Advanced Research in  
Applied Mathematics and Statistics (CARAMS)  
Level VI, Health Sciences Library Building  
Manipal Academy of Higher Education, Manipal-576 104  
Karnataka, India



Centre for Advanced Research  
in  
Applied Mathematics and Statistics  
MAHE, Manipal

**Twenty Eighth  
International Workshop on Matrices  
and Statistics  
IWMS 2021**

**December 13-15, 2021**

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**International Conference on  
Linear Algebra and its Applications  
ICLAA 2021**

**December 15-17, 2021**

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**International Workshop on  
Algorithm for Cooperative TU Games  
using Matrices and Graphs  
ACTGMG 2021**

**December 20-31, 2021**

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**Welcome to CARAMS, MAHE**

## From the desk of Chairman, Local Organizing Committee

**Manipal Academy of Higher Education Manipal** is synonymous with excellence in Higher Education. 30,000+ students from 65+ different nations live, learn and play in the sprawling University town, nestled on a plateau in Karnataka's Udupi district. MAHE Manipal offers 28 disciplines, 12 Program levels, 350+ programs through 31 institutions / departments. MAHE also has campuses in Mangaluru, Bengaluru & Jamshedpur and off-shore campuses in Dubai (UAE) & Melaka (Malaysia).

**Centre for Advanced Research in Applied Mathematics & Statistics (CARAMS)** has been officially established in September 2018 at MAHE with objectives of fostering advanced research and training in the area of Mathematics, Statistics and their applications. It facilitates to bridge the gap between Mathematicians/Statisticians and the Applied Mathematicians working in the area of Health, Technology, and Social Sciences.

CARAMS consists of several chapters working in different dimension of pure and applied Mathematics. Among them are (i) Algebra & Analysis (ii) Matrix Analysis & Applications (iii) Network Science and its Applications (iv) Statistics & Operations Research. Adjunct faculties of MAHE are associated with CARAMS in promoting research and act as mentors for CARAMS. Along with the activities promoting the research in Mathematics and Statistics, the centre aims at offering advanced courses in the subjects of interest. CARAMS offers positions of JRF, PDF, and visiting scientists to work in the area of CARAMS' research interests. MAHE faculty, adjunct faculty associated with MAHE and the mentors associated with CARAMS are available as advisors for the conduct of research in the area of focus.



Narayana Sabhahit  
Registrar, MAHE

CARAMS continuously organizes national and international workshop and conferences in the focus area which help the promotion of Mathematics and Statistics in the university and also in the region. CARAMS emphasis on conducting series of Lectures in Pure and Applied Mathematics, which are popular in nature, to develop several teams with members from different branches of sciences and to develop multidisciplinary projects. Please visit <https://carams.in/> for more details on the past and upcoming academic events. Recently held national workshop matrix analysis and game theory was a great success and resource materials are available in <https://carams.in/courses/>. Publications of edited volumes, special issues dedicated to conferences organized describe the quality of academic activities at CARAMS. CARAMS thank several eminent mathematicians in its network, government and private agencies who supported the activities by awarding adequate financial grant. National Board of Higher Mathematics (NBHM), Science and Engineering Research Board (SERB), Council of Scientific and Industrial Research (CSIR), India National Academy of Sciences (INSA) are among several national agencies who continuously supporting the activities of CARAMS.

The following departments

- Department of Data Science, PSPH, MAHE, Manipal
- Department of Mathematics, MIT, MAHE Manipal and
- Department of Mathematics, SMIT, SMU, Sikkim

are continuously associated with different activities being held by CARAMS.

- Indian Statistical Institute Delhi

is also in the network and associated with CARAMS in organizing a few conferences.

**About present Conferences and Workshop.** In the sequel to the conferences CMTGIM 2012, ICLAA 2014, ICLAA 2017, we have planned to organize ICLAA 2020 in December 17-19, 2020. Beside the same, we have planned for IWMS 2020(International workshop in Matrices and Statistics), 28th in its sequence, was to be held in December 15-17, 2020, in which the common day was to celebrate and honor a great applied mathematician and living legend Professor Calyampudi Radhakrishna Rao who completed 100 years of his life in the year 2020. Unfortunately, Covid-19 situation didn't allow us to organize any of these conferences in the normal format. However, we have organized ALAPS 2020 to honor Professor Rao in the online format. ICLAA and IWMS are rescheduled to 2021 and planned to organize them in the hybrid format. Further, 'International Workshop on Algorithms for Cooperative TU Games using Matrices and Graphs (ACTGMG 2021)' will be held following the conferences in December 20-31, 2021 to cater young minds of India.

Recently held national workshop matrix analysis and game theory was a great success and resource materials used in the workshop are available in the form of video lecture available in <https://carams.in/courses/>. Selected talks to be delivered in the conferences and ACTGMG will be available in the same link.

Combining online and in-person participants, more than 200 participants (as per registration details in mid November) from more than 20 nations across the globe will be attending these wonderful academic events. Among Indian participants, almost every state has representation in these conferences. These information clearly spells the quality of the conferences and workshop organized by CARAMS in December.

I welcome and also thank all the resource personnel and other delegates participating and in particular, wish a pleasant and safe stay for all the participants attending the event in-person at MAHE, Manipal during the exciting activities organized by the CARAMS.

**Dr. Narayana Sabhahit**

Registrar, MAHE

(Chairman, Local Organizing Committee)

## Overview of the events

On behalf of entire organizing team, CARAMS and MAHE, we the members of SAC welcome all the speakers and participants of triple events for IWMS 2021, ICLAA 2021 and ACTGMG 2021 organized by CARAMS, MAHE. IWMS and ICLAA were scheduled in December 2020 which could not organize in a regular format admitting physical presence of delegates due to existing Covid-19 situation. In fact, the common day between IWMS and ICLAA was planned for honoring a great living legend Professor Rao who completed 100 years of his life in the September 2020. However, ALAPS 2020 was organized to celebrate 100 years of Rao in December 2020 and the conferences IWMS 2020 and ICLAA 2020 have been rescheduled to December 13-15, 2021 and December 15-17, 2021, respectively.

**IWMS 2020.** The proposed ‘International Workshop on Matrices and Statistics’ to be held in Manipal in December 2021 is 28th in its sequence, the first was held at the University of Tampere(Tampere University) in Tampere, Finland during 6–8 August 1990. The purpose of International Workshop on Matrices and Statistics is to foster, in an informal setting, the interaction of researchers in the interface between Matrix Theory and Statistics.

The theme of the workshop shall focus on the theory and applications of the following topics in different branches of science such as Biology, Computer and Information Science, Economics, Electronics, Genetics, Office Statistics, Social Statistics and Accountancy:

- Matrix Analysis
- Projectors in Linear Models & Multivariate Analysis
- Growth Curve Models
- Linear Regression Models
- Linear Statistical Inference
- Modelling Covariance Structures
- Multivariate and Mixed Linear Models
- Statistics in Big Data Analysis.

This series of Workshops has a long history, see <https://www.sis.uta.fi/tilasto/iwms/IWMS-history.pdf> and we welcome the opportunity to hold the workshop once again in India. The 9th Workshop was held at Hyderabad in 2000, celebrating 80th birthday of Calyampudi Radhakrishna Rao, popularly known as C. R. Rao. The present 28th IWMS will be held beside ICLAA 2021, the common day between IWMS and ICLAA, December 15, 2021, will have some common sessions in honor of Prof. Calyampudi Radhakrishna Rao. We are sure that 28th IWMS will be a great success with the participation of prominent researchers from across the globe working in Statistics, Matrix Theory and allied subjects.

The purpose of the Workshop is to stimulate research and, in an informal setting, to foster the interaction of researchers in the interface between statistics and matrix theory. The Workshop will provide a forum through which statisticians may be better informed of the latest developments and newest techniques in linear algebra and matrix theory and may exchange ideas with researchers from a wide variety of countries.

As well as range of plenary speakers we are to strengthening the interactions between participants by organizing a range of minisymposia in various specialist areas. Beside organizing invited talks from eminent speakers, we invite participants to submit their research articles in the sessions of contributory talks.

**ICLAA 2021** The present ICLAA scheduled in the December 2021 is fourth in its sequence at Manipal, the was called CMTGIM and held in January 2012.

ICLAA intended to provide a platform for leading Mathematicians and Statisticians, working around the globe in the theme area to discuss several research issues and to introduce new innovations. The main goal of the conference is to bring experts, young researchers, and students together to present recent developments in this dynamic and important field. The conference also aims to stimulate research and support the interaction between the scientists by creating an environment for the participants to exchange ideas and to initiate collaborations and professional partnerships.

The theme of the conference shall focus on

- Classical Matrix Theory and Linear Algebra
- Nonnegative Matrices and Special Matrices
- Matrices and Graphs
- Combinatorial Matrix Theory
- Linear Statistical Inference
- Matrix and Graph Methods in Statistics
- Matrix and Graph Methods in Biological Sciences
- Matrices in Error Analysis and its Applications.

Linear Algebra and Graph Theory are important branches of Mathematics having applications in each and every branch of Applied Sciences. The topic ‘Matrix Methods in Statistics’ is a branch of Linear Algebra and Matrix Theory containing a variety of challenging problems in Linear Statistical Models and Statistical Inference having applications in various branches of Applied Statistics such as Natural Sciences, Medicine, Economics, Electrical Engineering, Markov Chains, Digital Signal Processing, Pattern Recognition and Neural Network, to name a few. Advances in Combinatorial Matrix theory are motivated by a wide range of subjects such as Networks, Chemistry, Genetics, Bioinformatics, Computer Science, and Information Technology. The areas of Classical Matrix Theory and Combinatorial Matrix Theory interact with each other, which is evident from the interplay between Graphs and Matrices. Generalized Inverses of Matrices such as the Incidence Matrix and Laplacian Matrix are mathematically interesting and have great practical significance. Covariance Matrices play an important role in the study of uncertainty associated with data related to measurements, which is an important part of applied Mathematics and Statistics.

This conference is in sequel to the conferences CMTGIM 2012, ICLAA 2014, and ICLAA 2017 held in Manipal during January 2012, December 2014, and December 2017 respectively. ICLAA 2021 has been endorsed by ILAS as in the case of its predecessor and we than ILAS for the same.

A special session in memory of **Prof. Arbind Lal** will be held as a part of ICLAA 2021 for which Prof. Sukanta Pati is the organizer. Prof. Arbind K Lal was an young and dynamic mathematician who was always a part of all the activities organized by CARAMS.

**ACTGMG 2021.** CARAMS, MAHE will be organizing a two-week workshop, carrying a credit of 2 points, on the topic ‘Algorithms for Cooperative TU Games using Matrices and Graphs’ during December 20-31, 2021. This is in continuation of Prof. T E S Raghavan’s effort of organizing ‘Gurukulam’ for the last few years. The course will be organized in in-person/hybrid format depending on the prevailing COVID situation. For the same reason, the dates also remain to be flexible.

The objective of the workshop is to provide an advanced input on integrating the following:

- Matrices associated with graphs such as Incidence, Adjacency and Laplacian matrices, Distance matrix of a tree and its generalizations, Resistance distance, Proof of the sensitivity conjecture, and the Algorithmic aspects of Cooperative Game Theory which

involve many basic combinatorial developments like: Max-flow min-cut, Longest paths ending in a graph, Edmond's maximum matching in a general graph.

- Completely mixed games and Perron Frobenius Theorem, Matrix games with payoff  $M$  that guarantee the solvability of the associated LCP constructively via the Lemke Howson algorithm, Totally positive matrices with their combinatorial connections and the spectral properties, Jacobi matrices and their implications to continuous time stationary Markov processes in discrete state space, and Determinantal properties of Sylvester, Gantmacher and MG Krein.

The course will be delivered through lectures on the topic in-depth and tutorials. A limited number (up to 20) of seriously interested students (Doctoral/Post-doctoral) with a good background of Linear Algebra and Calculus will be selected for participation in the workshop. They having basic knowledge of Markov Chains and Continuous Time Discrete Stochastic Process is appreciated.

It may be noted that certificates will be issued only to the participants who complete the tutorial assignments.

A few students (up to 6) will be partially sponsored by CARAMS, MAHE under Scientific Social Responsibility of different projects. Selection depends on the Supervisor/HOD's recommendation describing the possible benefits to the candidate. Candidates aspiring the support may write to the Coordinator of CARAMS (kmpasad63@gmail.com) or to T. E. S. Raghavan (terctu@gmail.com) immediately after the registration, attaching the necessary recommendation.

We wish that IWMS 2021, ICLAA 2021 and ACTGMG 2021 will attain great successes with the participation of prominent researchers from across the globe working in Matrix Theory, Graph Theory, Network Science, Statistics, and allied subjects.

**Ravindra B Bapat**

Indian Statistical Institute, Delhi  
(Chair, Scientific Advisory Committee)

**Manjunatha Prasad Karantha**

Center for Advanced Research in Applied Mathematics and Statistics  
(Convener, Scientific Advisory Committee)

**Stephen J Kirkland**

University of Manitoba, Canada  
(Member, Scientific Advisory Committee)

**Simo Puntanen**

Tampere University, Finland  
(Member, Scientific Advisory Committee)

## Warm welcome: From the desk of Chairman, IOC, IWMS

On behalf of the International Organizing Committee, IOC, of the International Workshop on Matrices and Statistics, IWMS-2021, we are very pleased to extend a warm welcome to all the participants. We are most grateful to Professor K. Manjunatha Prasad, the local organizing committee secretary, and his team on putting together such an excellent programme: in particular, taking the demanding circumstances due to Covid-19 into account. Thanks go also to the Manipal Academy of Higher Education, MAHE, and its related departments.

Now, let's look a bit backwards, 31 years only, . . . , along the lines of *A Short History of the IWMS*.

The first workshop in the International Workshop on Matrices and Statistics (IWMS) series took place at the University of Tampere in Tampere, Finland, 6–8 August 1990. It was organized by a local committee from the Statistics Unit of the Department of Mathematical Sciences at the University of Tampere.

This first IWMS was actually called *The International Workshop on Linear Models, Experimental Designs, and Related Matrix Theory*. Since 1990 the name has been changed twice, and in 1998 the IWMS became the *International Workshop on Matrices and Statistics*, following a suggestion by Professor C. Radhakrishna Rao.

In 1990 in Tampere there were 98 participants from 18 different countries. The Keynote Address was given by C. Radhakrishna Rao. The invited speakers were

Jerzy K. Baksalary	Sujit Kumar Mitra	Friedrich Pukelsheim
R. Dennis Cook	Seppo Mustonen	Jagdish N. Srivastava
Yadolah Dodge	Heinz Neudecker	George P. H. Styan
Shanti S. Gupta	Ingram Olkin	

The organizers of the group meetings were

Jerzy K. Baksalary	Sanpei Kageyama	Kirti R. Shah
Tadeusz Caliński	Jürgen Kleffe	George P. H. Styan
R. Dennis Cook	Sujit Kumar Mitra	Götz Trenkler
R. William Farebrother	Seppo Mustonen	Song-Gui Wang
Yasunori Fujikoshi	Friedrich Pukelsheim	Haruo Yanai
T.P. Hettmansperger	Jorma Rissanen	

Many of these persons have also been active participants in later workshops.

The following is an up-to-date version of the aims of the IWMS:

The purpose of the IWMS is to stimulate research and, in an informal setting, to foster the interaction of researchers in the interface between statistics and matrix theory. The Workshop will provide a forum through which statisticians may be better informed of the latest developments and newest techniques in linear algebra and matrix theory and may exchange ideas with researchers from a wide variety of countries.

Quite soon after the 2nd IWMS in Auckland, New Zealand, in December 1992, the organizing system for the IWMS found its form as two committees: International and Local. The International Organizing Committee (IOC) for several years comprised Richard William Farebrother, Simo Puntanen, George P. H. Styan, and Hans Joachim Werner. Later on, the IOC was extended (new fresh blood, of course!) and George P. H. Styan was named Honorary Chair of the IOC of the IWMS. The current IOC comprises the following members:



S. Ejaz Ahmed (Canada)  
Francisco Carvalho (Portugal)  
Katarzyna Filipiak (Poland)  
Jeffrey J. Hunter (New Zealand)  
Daniel Klein (Slovakia)  
Augustyn Markiewicz (Poland)

Simo Puntanen (Finland)  
Martin Singull (Sweden)  
George P. H. Styan (Canada)  
Júlia Volaufová (USA)  
Dietrich von Rosen (Sweden)  
Hans Joachim Werner (Germany)

Jeffrey J. Hunter has announced his intention to resign from the IOC. He has several IOC Chairs on his belt and the bottomline is: how can we survive without Jeff!

The IWMS series has had four **ILAS Lecturers**:

1. Gene H. Golub (1999), Tampere, Finland,
2. Jerzy K. Baksalary (2003), Dortmund, Germany,
3. Ravindra B. Bapat (2008), Tomar, Portugal,
4. Karl Gustafson (2015), Haikou, China;

two **Nokia Lecturers**:

1. Ingram Olkin (2004), Będlewo/Poznań, Poland,
2. C. Radhakrishna Rao (2005), Auckland, New Zealand;

and one **SAS Lecturer**:

1. Chris Gotwalt (2015), Haikou, China.

As the **IWMS Birthday Boys**, the following have been celebrated:

- T.W. Anderson (80, 90), Fort Lauderdale 1998, Tomar 2008,
- Jerzy K. Baksalary (60), Będlewo/Poznań 2004,
- Kai-Tai Fang (75), Haikou 2010,
- Lynn Roy LaMotte (70), Toronto 2013,
- Jeffrey J. Hunter (75), Funchal, 2016,
- Ingram Olkin (80, 90), Będlewo/Poznań 2004, Ljubljana 2014,
- Tarmo Pukkila (60), Uppsala 2006,
- Simo Puntanen (70), Haikou 2015,
- C. Radhakrishna Rao (80), Hyderabad 2000,
- Muni S. Srivastava (75), Tartu 2011,
- George P. H. Styan (65, 70, 75, 80), Lyngby, Windsor, Będlewo, Montréal.

As mentioned earlier, Professor **C. Radhakrishna Rao** was the Keynote Speaker in Tampere in the first IWMS in 1990. He gave a talk on “Uncertainty, statistics and creation of new knowledge”, available in YouTube-1990. Before that, Professor Rao visited Tampere in 1983, 1985 (Honorary Doctorate; George received this in 2000), and twice in 1987. His conference talk in 1987 can be watched from YouTube-1987. Professor Rao has been an active attendant of the IWMS meetings: Tampere (1990), Montréal (1995), Istanbul (1997), Fort Lauderdale

(1998), Hyderabad (2000), and Auckland (2005). Professor Rao's contributions on building up the IWMS series have been instrumental.

In this context we may shortly tell an interesting saga related to **Jerzy K. Baksalary** (1944–2005). To begin with, it's worth mentioning that Jerzy spent the period of September 1989 to September 1990 in Tampere as a Visiting Finnish Academy Professor. It was partly due to his visit that the idea of the first IWMS became real. Anyway, it so happened that after returning back to Poland, Jerzy soon started a full-time administrative career for many years as a Rector and Dean of the Tadeusz Kotarbiński Pedagogical University in Zielona Góra. But then, in the IWMS-Hyderabad in December 2000, Jerzy was back in real old good business (with his son Oskar Maria)—first put accidentally into George's room in the Secunderabad Club! That was quite a reunion, after ten years. The career of Jerzy is described by Oskar Maria Baksalary and George P. H. Styan in their article in the *LAA* (2005).

The 9th IWMS was held in Hyderabad in 9–13 December 2000, in celebration for C. R. Rao's 80th birthday. The program started with a two-day course on recent advances in matrix theory with special reference to applications to statistics. The topics of Professor Rao's talks were "Statistical proofs of some matrix inequalities" and "Antieigen and singular values of a matrix and their applications to statistics".

Simo's first visit to India was in December 1990, attending C.G. Khatri (1931–1989) Memorial Conference. Most of the period December 1992 to February 1993 Simo was in Delhi with George (on sabbatical).

While in Delhi, we prepared an interesting interview with **Sujit Kumar Mitra** (1932–2004); see PDF. Mitra, who attended the IWMS in Tampere 1990 and Montréal 1995, did his Ph.D. studies in North Carolina at Chapel Hill 1954–1956, supervisor being S.N. Roy. Below is a piece of conversation.

*And then you wrote your thesis with Professor S.N. Roy, in USA, in 1954–56. Did you feel homesick in Chapel Hill?*

"I felt terribly homesick. I don't know if it was my homesickness which was responsible for even thinking that I may come back without the degree. So I wrote to Professor P.C. Mahalanobis asking whether he would take me from the USA back to the ISI, without a degree.

But in the middle of the arrangements, I got two letters, one from D. Basu and the other from C.R. Rao. Both of them told me that I was silly and that I must first complete my PhD degree and then come to the ISI."

Below is some info about three IWMS meetings: 1990, 2000 and 2019.

**1990/1:** International Workshop on Linear Models, Experimental Designs, and Related Matrix Theory Tampere, Finland, 6–8 August 1990,  $n = 98$ .

Chair of the Organizing Committee: Erkki Liski. Program.

Videos from the conferences in statistics in Tampere in 1987 and 1990, prepared by Jarmo Niemelä and Reijo Sund.

**2000/9:** 9th International Workshop on Matrices and Statistics, Hyderabad, India, 9–13 December 2000, in celebration of *C. Radhakrishna Rao's* 80th birthday,  $n = 100$ .

Local Chairs: S.B. Rao, P. Bhimasankaram, IOC Chair: Hans Joachim Werner.

Program. Report in *Image*.

**2019/27:** 27th International Workshop on Matrices and Statistics, Shanghai University of International Business and Economics, 6–9 June 2019,  $n = 70$ .

IWMS Chair: Jeffrey J. Hunter, Local Chair: Yonghui Liu

Website. Poster. Book of Abstracts & Program. Group photo. Report in *Image*.

Invited speakers: Oskar Maria Baksalary, Rajendra Bhatia, Kai-Tai Fang, Shuangzhe Liu, Jianxin Pan, K. Manjunatha Prasad, Yongge Tian, Fuzhen Zhang, Shurong Zheng, Lixing Zhu.

- As an after-workshop to the IWMS-2019, on 12 June 2019, some participants attended the *Workshop on Advanced Multivariate Analysis*, Centre for Statistical Sciences and College of Mathematics, Sichuan University, Chengdu, China. Organizers: Jianxin Pan & Jie Zhou. Program. Group photo.

## Acknowledgements

There is an open-access website for the IWMS at the Tampere University: [www](http://www.iwms.tampere.fi), where we intend to put a good subset of associated reports and photographs of the IWMS series from 1990 onwards, including those published in *IMAGE: The Bulletin of the International Linear Algebra Society*. Complete videos, prepared by Jarmo Niemelä and Reijo Sund, of the talks at two pre-IWMS Tampere conferences in statistics in 1987 and 1990 are on YouTube. Photographs in this article are taken by Simo Puntanen.

### **Simo Puntanen**

Tampere University, Finland

(Chair of the International Organizing Committee)

### **George P.H. Styan**

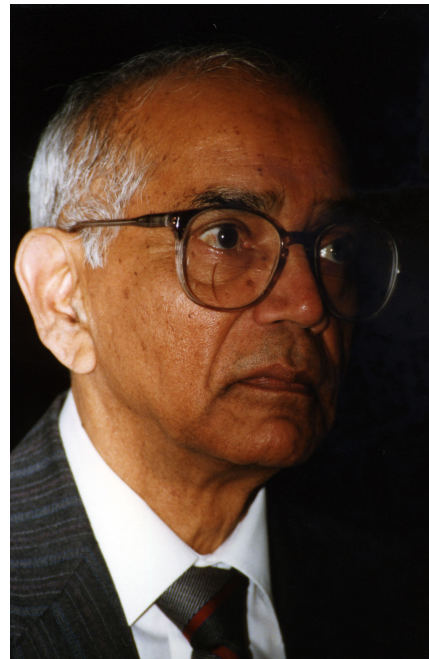
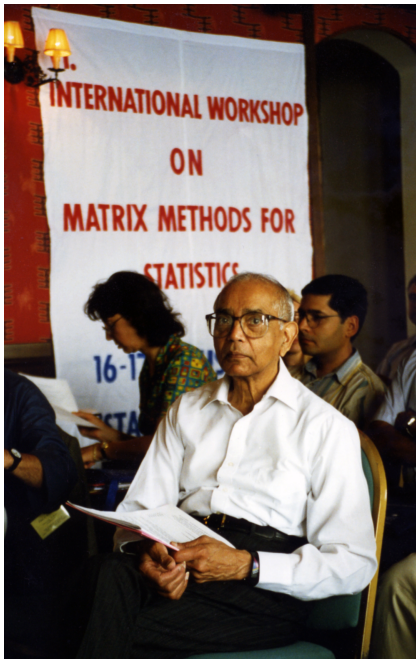
McGill University, Montréal (Québec), Canada

(Hon. Chair of the International Organizing Committee)

## Photographs



IWMS #4, Montréal, 15–16 July 1995: Geoffrey S. Watson, C.R. Rao (2nd row), Ingram Olkin. Program.



C.R. Rao in IWMS #6, Istanbul, 16–17 August 1997. Program.





IWMS #9, Hyderabad, India, 9–13 December 2000. To celebrate C.R. Rao's 80th birthday. Program.



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International Workshop on Linear Models, Experimental Designs & Related Matrix Theory  
6–8 August 1990, University of Tampere, Tampere, Finland

Monday, 6 August	Tuesday, 7 August	Wednesday, 8 August
<p>10:00–10:45 <i>Opening Session:</i> Tarmo PUKKILA Chair: E.P. Liski C. Radhakrishna RAO Chair: G.P.H. Styan</p> <p><i>Invited Talk:</i> 10:45–11:30 Ingram OLKIN Chair: J.K. Baksalary</p> <p>11:30–12:45 <b>Lunch</b></p> <p><i>Invited Talks:</i> 12:45–13:30 Sujit Kumar MITRA 13:30–14:15 R. Dennis COOK Chair: F. Pukelsheim</p> <p>14:15–14:45 <b>Coffee</b></p> <p><i>Group Meetings:</i> 14:45–15:45 Gustav Elfving: 1908–1984</p> <p>16:15–17:45 Mustonen Caliński Farebrother-1</p> <p>17:45–19:15 Rissanen Kageyama Farebrother-2</p> <p>20:00–... <b>Reception</b></p>	<p><i>Group Meetings:</i> 8:00–9:30 Trenkler-1 Wang Hettmansperger 9:30–11:30 Cook Baksalary</p> <p>11:30–12:45 <b>Lunch</b></p> <p><i>Group Meetings:</i> 12:45–14:15 Mitra Farebrother-3 14:15–15:45 Srivastava Yanai Farebrother-4</p> <p>15:45–16:15 <b>Coffee</b></p> <p><i>Group Meetings:</i> 16:15–17:45 Fujikoshi Pukelsheim Trenkler-2 17:45–19:15 Styan Kleffe Neudecker</p> <p>20:30–... <b>Sauna Party</b></p>	<p><i>Group Meetings:</i> 8:00–9:30 Shah Farebrother-5</p> <p>9:30–11:00 Editorial Policy in Statistical Journals</p> <p>11:00–11:30 <b>Coffee</b></p> <p>11:30–12:15 <i>Keynote Address:</i> C. Radhakrishna RAO Chair: T. Caliński</p> <p>12:15–13:30 <b>Lunch</b></p> <p><i>Invited Talks:</i> 13:30–14:00 Jagdish N. SRIVASTAVA 14:00–14:30 Shanti S. GUPTA 14:30–15:00 Yadolah DODGE Chair: T.P. Hettmansperger</p> <p>15:00–15:30 <b>Coffee</b></p> <p>15:30–16:00 Heinz NEUDECKER 16:00–16:30 Friedrich PUKELSHEIM Chair: N.R. Draper</p> <p>16:30–17:00 <b>Coffee</b></p> <p>17:00–17:30 George P. H. STYAN 17:30–18:00 Jerzy K. BAKSALARY Chair: T. Pukkila</p>

The first-mentioned groups meet in Room A I, the second in C IX (except on Monday in the Main Auditorium), and the third in A III.

Above: S.K. Mitra; George & S.K. Mitra, New Delhi, January 1993. Below: schedule of the first IWMS; Tampere, 6–8 August 1990. Program.



## Arbind Kumar Lal

### A Tribute to a Gentleman



Professor Arbind Kumar Lal  
(1966 - 2021)

Indian Institute of Technology Kanpur

Known for his nature to help everyone (students, teaching and nonteaching staffs) in trouble, for his ability to resolve issues within the department, Professor Lal was popularly tagged as the 'permanent hod' by his colleagues.



Young Lal with brothers and sisters

Born at Uttara (Madhubani district, Bihar) to Smt. Sushila Devi and Shri Lekh Narayan Lal, his upbringing had made him a thorough gentleman and a person with a huge heart. An example that, I witnessed, when a few years back I was visiting IIT Kanpur. While driving his car to the department, I saw him waving his hands to the security persons at the gate nearby. When I looked at him with curious eyes, he replied, they work for us day and night, it is a nice gesture to say a hello or hi. He probably started caring others from his childhood. I don't think he ever changed.

He completed his schooling from Sainik School, Tilaiyaa in 1983 and obtained his BA with Mathematics, Honors in 1986 from Hansraj College, Delhi. He went to do his M Stat (1986-88) to ISI Delhi and this where his growth as a mathematician started. Because of his frank nature, being a sports loving person and being a thoughtful person he created many friends for life here. He completed his PhD (1988-93) from the same place ISI Delhi. He was a visiting fellow in TIFR Mumbai for a year

during which he worked on Combinatorics. After that he became a visiting fellow in HRI Allahabad for two years during which he worked on coding theory. After that he joined IIT Kanpur as a faculty in the Department of Mathematics (now it is called the Department of Mathematics & Statistics), in 1996. He got married in 1996 to the love of his life, Miss Niyati Padhi. They have two children, son Ayanesh and daughter Nilisha.



From the left: Daughter Nilish, Wife Niyati Padhi, Son Ayanesh, and Lal



Lal with Professor H.L. Janwa

He visited the University of Puerto Rico, Rio Piedras Campus, as a visiting professor from August 2000 to December 2001, to work on coding theory (his first field of research), with Professor H. L. Janwa. He has also visited Concordia University, Montreal from August 2010 to May 2011. This time he chose to work on Graphs and matrices, which is his second field of research. He immensely contributed to this area with numerous quality publications, collaborating with many researchers worldwide. He helped the area to grow in our country, being a part of organizing committees of many conferences, by helping many students working in the area. For example, for this same the present series of ICLAA conferences from the very first edition, he has been an organising member. His contribution 'A q-analogue of the Distance Matrix of a Tree', Linear

Algebra and its Applications, 416, 2-3, 2006, 799-814, has motivated many researchers to work on various aspects of the distance matrix. He was an editor of the journal 'Proceedings Mathematical Sciences' published by Springer and Indian Academy of Sciences.

It is a mystery to me how, he managed to get time, while doing many administrative works for the IIT (HOD, GATE chairman, Gymkhana, etc.), for the Department, and even while contributing in sports. Many will recall him as the 'net samrat' for his badminton play.





Lal with Bapat in the conference honoring R B Bapat in 2019

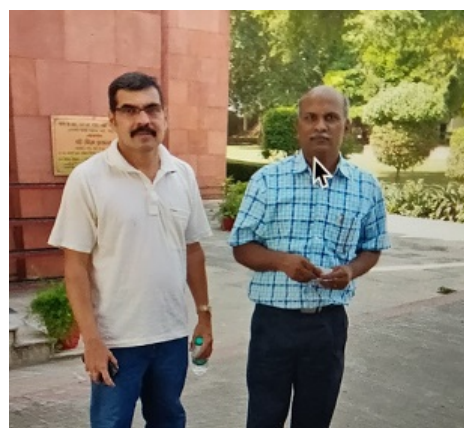
It is not that he would do any work half-heartedly. In fact, he did all those jobs so well, that everyone took it for granted, that if 'Arbind' is doing it, it shall be a job well done.

Many students would remember him as the 'final helper'. He was always ready to help every student who approached. The list of his PhD students and the varied subject areas will exhibit the same. Many of them joined him after facing initial hurdles in other subjects. He never discouraged anyone. All of his students (listed below) are well placed. One has become a colleague to me at IIT Guwahati.

- Dr. Chetan P. Nemade: On Irreducible Polynomials and Normal Bases over Finite Fields, 2001 (with Prof. M C Bhandari)
- Dr. Manish Kumar Gupta: On some Linear Codes over  $\mathbb{Z}_2^s$ , 2001 (with Prof. M C Bhandari)
- Dr. Tony Thomas: On Public-Key Cryptography using Hard Problems in Braid Groups, 2006
- Dr. Kamal L Patra: Laplacian Spectrum and Graph Structure, 2007
- Dr. Bikash Bhattacharjya: On Some Graph Labeling Problems, 2009
- Dr A S Reddy: Matrix Algebra and Graph Theory, 2012 (with S K Mehta)
- Dr S Mohanty: Analysis on Graphs, 2015 (with N Nilakantan).

As a teacher he constantly worked on making the material complete, more presentable year after year. His unpublished 'lecture notes on linear algebra' has been followed by many university and college teachers. Once he remarked 'there are many text books for the IIT (level) students, but there are very few text books for the college students'. Both his lecture notes (the other one is on discrete mathematics) were prepared keeping this philosophy. May be we will see them get published some day. Many students would recall his saying 'do not worry about the grades, just learn the subject'.

In his personal life, he loved visiting places. (I loved going with him, as I need not plan too much then.) Always cheerful. A loud hearty laugh and the words 'khush raho' (be happy) are his unique characteristics. They will echo in hearts of many, for years.



Lal with Rajaram Bhat

### **Sukanta Pati**

(Organizer, Sessions Dedicated to Arbind Lal)

Professor, IIT Guwahati

Assam, India.

**From Memories:**



PLEASE find Lal!!!!

# Acknowledgements

We acknowledge our sincere thanks to NMC and ILAS.



IWMS 2021 is partially supported by NMC



International Linear  
Algebra Society

ICLAA 2021 is an ILAS endorsed meeting

# **Call for Papers**

## Special Issue Dedicated to Conferences

We are in consultation with the journal 'AKCE International Journal of Graphs and Combinatorics' (An open access journal), for the possible publication of a special issue dedicated to the conference being held at CARAMS in this year. The details of journal is as under:

Journal : AKCE International Journal of Graphs and Combinatorics (An open access journal)

Web Link: <https://www.tandfonline.com/loi/uakc20>

Publisher: Taylor & Francis online.

We invite research articles in the following mentioned focus area for the possible publication in the special issue. Article need not be the one presented in any of the conferences held in CARAMS and similarly author need not be among the speakers or participants of conferences, though we encourage the our delegates to submit an article presented in the conference.

Keeping in view of focus area of the journal and conference, the articles submitted are expected to be in the following broad area:

- Linear Algebra & Matrix Theory
- Graphs & Matrices
- Applications of Matrices, Graphs, Network and Combinatorics in the area of Statistics & other branches of Sciences.

Guest Editors Team consists Scientific Advisory Committee (SAC) members:

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**How to Submit.** Corresponding author may submit an article with in the focus area mentioned above, directly to the convener of SAC, who is the coordinator for the special issue and the proceedings (km.prasad@manipal.edu; kmprasad63@gmail.com ) with the suggestion of a guest editor to handle the paper.

**Formatting.** Author may refer to instructions for authors on the journal site with the link : <https://www.tandfonline.com/action/authorSubmission?show=instructions&journalCode=uakc20>

**Submission fee & APC.** There is no Submission Fee or Article Professing Charges (APC) for the articles submitted for the special issue.

### Important Dates

- Receiving the articles: April 30, 2022

- Completing the review process including necessary revision: September 30, 2022
- Submitting the final manuscript in the necessary format: October 31, 2022
- Publication of special issue: January 31, 2023 (subject to convenience of AKCE Journal)

## Applied Linear Algebra, Probability and Statistics

This volume is to cherish the beautiful path laid by the living legend

Calyampudi Radhakrishna Rao

who completed 100 years of fruitful life in 1920 and in memories of young and dynamic

Arbind Kumar Lal

whom we missed in this year of 1921.

Research articles in detail and expository articles are invited for the possible publication in the volume of proceedings in the volume are in the focus area of ALAPS 2020, IWMS 2021, ICLAA 2021 and the work by Professor Rao. Though we appreciate the submission of work presented in any of these connected conferences, we also invite any good articles in the form of chapters which throw more light on the work of Professor Rao and help young scholars to understand Rao's work and further advancements. We welcome articles/chapters in memory of Arbind Lal too.

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6. Samir Kumar Neogy, Indian Statistical Institute, Delhi, India
7. Sukanta Pati, Indian Institute of Technology, Guwahati, India
8. Simo Puntanen, Tampere University, Finland

**Nature of article:** We welcome articles of the following nature:

1. Research or expository
2. It could be in the focus of area of any of the conferences ALAPS 2020, IWMS 2021, and ICLAA 2021 (Pure and applied linear algebra, Matrix Method in Statistics, Matrices and Graphs, Combinatorial Matrix Theory, and Applications of any of these in applied branches of science)
3. It could be in area of applied linear algebra and Statistics in which Rao worked

**Timeline:** Last Date for receiving the articles: April 30, 2022

Completion of Review, including necessary revision: October 31, 2022

Proof reading & Pre-production work by SPRINGER: December 31, 2022

Publication of edited volume: March 31, 2023.

**Number of pages:** We appreciate the number of pages of article restricted to 20–25, but in some exceptional cases it could be more.



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# **IWMS 2021**

Twenty eighth International  
Conference on Matrices and  
Statistics  
December 13-15, 2021

## IWMS: Plenary, Invited and Mini-Symposium Speakers

1. NARAYANASWAMY BALAKRISHNAN *McMaster University, Hamilton, Ontario, CANADA*
2. ADI BEN-ISRAEL *Rutgers Business School, Newark, New Jersey, UNITED STATES OF AMERICA*
3. ABRAHAM BERMAN *Technion-Israel Institute of Technology, ISRAEL*
4. ARUP BOSE *Indian Statistical Institute, Kolkata, INDIA*
5. GARRY KA LOK CHU *Dawson College, 3040 Sherbrooke Street West, CANADA*
6. SUSMITA DATTA *Clinical Translational Research Institute, University of Florida, UNITED STATES OF AMERICA*
7. KATARZYNA FILIPIAK *Poznan University of Technology, POLAND*
8. STEPHEN JOHN HASLETT *Massey University, Palmerston North, NEW ZEELAND*
9. ANDRZEJ JANUSZ *Institute of Informatics, University of Warsaw, POLAND*
10. SOMDEB LAHIRI *Pandit Deendayal Energy University, Gujarat, INDIA*
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20. GEORGE P H STYAN *McGill University, CANADA*

## IWMS 2021: Contributory Speakers

### IWMS 2021

1. STANISLAV ANATOLYEV *Center for Economic Research and Graduate Education - Economic Institute, Prague, Czech Republic*
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13. CHEPURI RAJITA *Kaktiya University, Warangal, India*

# **Abstracts: IWMS 2021**

## IWMS 2021: Invited Talks

### Some interesting feature of linear prediction

**Narayanaswamy Balakrishnan**

McMaster University, Hamilton, Canada  
bala@mcmaster.ca

#### Abstract

To be updated

**Keywords:** To be updated

**AMS subject classifications.** 62J05; 62J10

### On the Developments of Generalized Inverses

**Adi Ben-Israel**

Rutger University, New Jersey. adi.ben israel@gmail.com

#### Abstract

In the present talk, I shall review on the initial developments of generalized and its applications. .

---

### Completely Positive Matrices

**Abraham Berman**

Technion-Israel Institute of Technology, Israel  
berman@technion.ac.il

#### Abstract

Let  $S$  be a set of nonnegative numbers. A matrix  $A$  is *completely positive over  $S$*  if it can be decomposed as  $A = BB^T$ , where the entries of  $B$  are in  $S$ . The talk is a survey of results on completely positivity over  $S$  in the cases  $S = R$ ,  $S = Q$ ,  $S = Z$  and  $S = \{0, 1\}$

**Keywords:** Non negative matrix, Completely positive matrix

**AMS subject classifications.** 15A23; 15B48



## References

- [1] Abraham Berman and Changqing Xu.  $\{0, 1\}$  *Completely positive matrices*. Linear Algebra and its Applications, 399:35-51, 2005.
- [2] Abraham Berman, Naomi Shaked-Monderer *Completely Positive Matrices 2003*: World Scientific Publishing Company.
- [3] Man-Duen Choi. *Completely Positive Linear Maps on Complex Matrices*. Linear Algebra and its Applications, 10:285–290, 1975.
- [4] Naomi Shaked-Monderer, Abraham Berman *Copositive and Completely Positive Matrices 2021*: World Scientific Publishing Company.

## Spectral measures of empirical autocovariance matrices of high dimensional stationary processes

**Arup Bose**

Indian Statistical Institute, Kolkata, India  
bosearu@gmail.com

### Abstract

We begin with a brief glimpse at the bulk distribution results for some popular random matrices. Then we consider the empirical autocovariance matrix at a given non-zero time lag based on observations from a multivariate complex Gaussian stationary time series. We study the behavior of their spectral measures in the asymptotic regime where the time series dimension and the observation window length both grow to infinity, and at the same rate. Following a general framework in the field of the spectral analysis of large random non-Hermitian matrices, at first the probabilistic behavior of the small singular values of the shifted versions of the autocovariance matrix are obtained. This is then used to infer about the large sample behaviour of the empirical spectral measure of the autocovariance matrices at any lag. Matrix orthogonal polynomials on the unit circle play a crucial role in our study.

## Statistical Analysis of single cell RNA sequencing (ScRNA-seq) data

**Susmita Datta**

College of Public Health & Health Professions College of Medicine  
University of Florida, United States of America  
susmita.datta@ufl.edu

### Abstract

Transcriptomic studies such as in bulk RNA-sequencing, one can examine transcript abundance measurements averaged over bulk populations of thousands (or even millions) of cells. While these measurements have been valuable in countless studies, they often conceal cell-specific heterogeneity in expression signals that may be paramount to new biological findings.

Fortunately, with single cell RNA-sequencing (scRNA-Seq), transcriptome data from individual cells are now accessible, providing opportunities to investigate functional states of cells, identify rare cell populations and uncover diverse gene expression patterns in cell populations that seemed homogeneous. Most importantly it provides an unprecedented resolution to the characterization of cellular clinical isolates. However, there are challenges analyzing such scRNA-Seq data. Amongst many challenges the most significant are the bimodal or multimodal distribution, sparsity and tremendous heterogeneity in the data. Consequently, we will describe potential ways of statistical modeling of such data, finding differentially expressed genes and methods for constructing gene-gene interaction network using this data.

**Acknowledgment.** The content of this talk is a part of my collaborative work with Dr. Jeremy Gaskins and Michael Sekula (University of Louisville).

## **Frobenius norm, entropy and quadratic loss functions as the measures of discrepancy from structured matrices**

**Katarzyna Filipiak**

Institute of Mathematics, Poznań University of Technology, Poznań, Poland  
katarzyna.filipiak@put.poznan.pl

### **Abstract**

The problem of measuring discrepancy of a given matrix from structured one often arises in statistics. In this talk the properties of the Frobenius norm, entropy and quadratic loss functions will be compared in the context of power of the test.

**Keywords:** Structured matrix, Frobenius norm, entropy loss function, quadratic loss function

## **The role of block diagonal matrices in conditions for equality of BLUEs in linear models**

**Stephen Haslett**

School of Fundamental Science and Centre for Public Health Research, Massey  
University, New Zealand  
Research School of Finance, Actuarial Studies and Statistics, The Australian  
National University, Australia  
NIASRA, Faculty of Engineering and Information Sciences, University of  
Wollongong, Australia.  
s.j.haslett@massey.ac.nz; stephen.haslett@anu.edu.au

### **Abstract**

The general condition for the BLUEs in a linear model being unaltered by a change in error covariance structure is due to C.R Rao (1971). This condition can be written so that the entire term that can be added to the original covariance is block diagonal in form. When the original full linear model is made smaller by reducing the number of regressors, block diagonal matrices for the quadratic form based on the matrix orthogonal to the design matrix also provide insight into conditions for full, small and intermediate models having the same

BLUEs for all the parameters they share. Haslett et al (2020) also consider properties of BLUEs in full versus small linear models with new observations where the supplementary quadratic form to the original covariance structure again involves block diagonal matrices, but based instead on the design matrix. Common themes will be explored using block diagonal matrices and the partitions they imply. Extension to linear mixed models and BLUPs is possible.

**Keywords:** linear model, BLUE, BLUP.

**AMS subject classifications.** 62J05; 62J10

**Acknowledgement.** This is joint work with Simo Puntanen and Augustyn Markiewicz.

## References

- [1] Haslett, S.J., Markiewicz, A., Puntanen, S. Properties of BLUEs and BLUPs in full vs. small linear models with new observations. *Recent Developments in Multivariate and Random Matrix Analysis*. (Holgersson, T. and Singull, M. eds.) 2020: Springer International Publishing, Cham. Chapter 8, pp. 123-146.
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# Learning from Video Game Logs: the Human-in-the-Loop Approach

**Andrzej Janusz**

Institute of Informatics, University of Warsaw, Poland  
andrzej.janusz@qed.pl

## Abstract

Active learning is a subfield of machine learning that considers interactive algorithms for training prediction models. In such a setup, the learner can iteratively query an oracle to obtain labels for a limited number of instances from a large volume of available data. This approach is particularly useful in practical applications when the label acquisition is time-consuming or expensive. During my talk, I will explain how we deal with such applications at QED Software. As an example, I will use one of our recent projects in which we are working on a model for the identification of critical turns (so-called, game-changers) in a multiplayer video game called Tactical Troops: Anthracite Shift. I will describe the main steps in the active learning cycle, and discuss the most common practical issues related to the deployment of successful active learning systems.

**Mini-Symposium:** Recent Methodologies in Big Data Analysis

## Quadratically optimal bi-matrix games

**Somdeb Lahiri**

School of Petroleum Management, Pt. Deendayal Energy University, India  
somdeb.lahiri@gmail.com

### Abstract

In this paper we introduce the class of quadratically optimal (bi-matrix) games, which are bi-matrix games whose set of equilibrium points contain all pairs of probability vectors which maximize the expected pay-off of some pay-off matrix. We call the equilibrium points obtained in this way, quadratically optimal equilibrium points. We prove the existence of quadratically optimal equilibrium points of identical bi-matrix games, i.e. bi-matrix games for which the two pay-off matrices are equal, from which it easily follows that weakly potential bi-matrix games (a generalization of potential bi-matrix games) are quadratically optimal. We also show that those weakly potential square bi-matrix games which have potential matrices that are two-way matrices are quadratically and symmetrically solvable games, i.e. there exists a square pay-off matrix whose expected pay-off maximizing probability vectors subject to the two probability vectors (row probability vector and column probability vector) being equal, are equilibrium points of the bi-matrix game. None of our results require using a fixed point theorem argument in the proofs. We show by means of an example of a  $2 \times 2$  identical symmetric potential bi-matrix game that for every potential matrix of the game, the set of pairs of probability distributions that maximizes the expected pay-off of the potential matrix is a strict subset of the set of equilibrium points of the potential game.

**Keywords:** Decision Maker, Game Theory

**AMS subject classifications.** 90B50, 91B06, 91A35

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# Covariance Matrix Estimation based on Matrix Projection

Augustyn Markiewicz

Poznań University of Life Sciences, Poland  
amark@up.poznan.pl; augustyn.markiewicz@up.poznan.pl

## Abstract

The need for estimation of covariance matrix with a given structure arises in various multivariate models. We are studying this problem for linear structures. The commonly used maximum likelihood estimation method is in general complex and time consuming when restricted to structured and positively definite matrices; cf. [1]. Therefore we consider some alternatives to the maximum likelihood estimation based on approximation of the unstructured sample covariance matrix by structured, positive definite matrices. The approximation via Frobenius as well entropy loss functions turn out to be numerically ineffective; cf. [2] and [3]. Instead, we propose a much more numerically efficient method of projecting the unstructured sample covariance matrix on a given linear structure (cf. [4]) and then, if necessary, restoring its definiteness by a specific shrinkage method. The statistical properties of these estimators and maximum likelihood estimators are compared via simulation study.

**Keywords:** covariance estimation, covariance structure, Frobenius loss, entropy loss, shrinkage method

**AMS subject classifications.** 62H20; 65F99

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# Learning from Video Game Logs: the Human-in-the-Loop Approach

**Samir K Neogy**

Indian Statistical Institute, Delhi Center, India  
sknisid@gmail.com

## Abstract

More and more data are being produced today by an increasing number of electronic devices physically surrounding us and on the internet. The large amount of data and the high frequency at which they are produced have resulted in the introduction of the term 'Big Data'. Today, in sectors like social development, healthcare, education, energy, governance etc have more data than they can handle, and recognize the potential for value, but the promise of big data still has not been realized, according to the leading academic and business media sources. Big data has caused the scientific community to re-examine its methodology of scientific research. Big data technologies and the corresponding fundamental research have become a research focus in academia. An emerging interdisciplinary discipline called data science has been gradually coming into place. This takes big data as its research object and aims at generalizing the extraction of knowledge from data. In fact we also need new approaches in statistics and computer science to analyse Big Data.

**Mini-Symposium:** Recent Methodologies in Big Data Analysis

# A review of the linear sufficiency

**Simo Puntanen**

Faculty of Information Technology and Communication Sciences, FI-33014  
Tampere University, Finland  
simo.puntanen@uni.fi

## Abstract

We consider the general linear model  $y = X\beta + \varepsilon$  supplemented with the new unobservable random vector  $y_*$ , coming from  $y_* = X_*\beta + \varepsilon_*$ , where the covariance matrix of  $y_*$  is known as well as the cross-covariance matrix between  $y_*$  and  $y$ . A linear statistic  $Fy$  is called linearly sufficient for  $X_*\beta$  if there exists a matrix  $A$  such that  $AFy$  is the best linear unbiased estimator, BLUE, for  $X_*\beta$ . The concept of linear sufficiency with respect to a predictable random vector is defined in the corresponding way but considering the best linear unbiased predictor, BLUP, instead of BLUE. In this paper, we consider the linear sufficiency of  $Fy$  with respect to  $y_*$ ,  $X_*\beta$ , and  $\varepsilon_*$ .

**Keywords:** BLUE, BLUP, linear sufficiency, linear model with new observations, Löwner ordering

**AMS subject classifications.** 62J05; 62J10

**Acknowledgement.** This is joint work with Stephen J. Haslett, Jarkko Isotalo, Radoslaw Kala and Augustyn Markiewicz

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## On Decompositions of Linear Spaces Connected to Linear and Bilinear Models

**Dietrich von Rosen**

Swedish University of Agricultural Sciences  
University in Uppsala, Sweden  
Dietrich.von.Rosen@slu.se

### Abstract

Some techniques to decompose linear and bilinear spaces are presented. The techniques are applied to the treatment of the Gauss-Markov model, linear equations, bilinear models, partial least squares (PLS) and reduced rank regression models. In particular for some of the models residuals will be considered.

## Exact Deep Learning Machines, Artificial Intelligence, and Deep Learning Methods

**Arni S.R. Srinivasa Rao**

Medical College of Georgia, Augusta University, USA  
arni.rao2020@gmail.com

### Abstract

Through this talk, a new method introduced by the author, called, exact deep learning machines (EDLM) will be introduced. Such methods will be compared with traditional artificial intelligence (AI) techniques. How the EDLMs provide better alternatives for AI-related experiments will be discussed. The difference of approaches of deep learning versus AI through two theoretical examples will be provided

**Keywords:** Deep Learning, Artificial Intelligence.

## Hack's Law in a Drainage Network Model: A Brownian Web Approach

**Anish Sarkar**

Indian Statistical Institute, Delhi Centre, India  
anish.sarkar@gmail.com

### Abstract

Hack (1957) while studying the drainage system in the Shenandoah valley and the adjacent mountains of Virginia, observed a power law relation  $l \sim a^{0.6}$  between the length  $l$  of a stream from its source to a divide and the area  $a$  of the basin that collects the precipitation contributing to the stream as tributaries. We study the tributary structure of Howard's drainage network model of headward growth and branching studied earlier by Gangopadhyay, Roy and Sarkar (2004). We show that the exponent of Hack's law is  $2/3$  for Howard's model. Our study is based on a scaling of the process whereby the limit of the watershed area of a stream is area of a Brownian excursion process. To obtain this, we define a dual of the model and show that under diffusive scaling, both the original network and its dual converge jointly to the standard Brownian web and its dual.

**Acknowledgment.** This is joint work with Rahul Roy and Kumarjit Saha.

## Trends in Big Data Technologies

**Santosh Singh**

Shiv Nadar University, UP, India  
santosh.singh@snu.edu.in

### Abstract

Now a days, very often we hear the common phrases in academic world, "Data is money", or "Data is like gold / diamond mine". In principle, will agree to the phrase "Data is like a gold or diamond mine". Diamond as a raw product don't have any value unless it's properly processed and presented. Similarly, data has no value unless it's processed, and the information is properly inferred using the statistical / mathematical tool and techniques. For the proper utilization of data, there are mainly two streams, one is data processing and mining, which mainly focuses on the development of different statistical and mathematical techniques and algorithms, whereas the other one is data technologies whose focus is handling of data. The talk will focus on the recent developments of the big data technologies and what are the possible future trends in the same domain.



# Residual Analysis for the GMANOVA-MANOVA Model

Beatrice Byukusenge<sup>1</sup>, Dietrich von Rosen<sup>2</sup>, and Martin Singull<sup>3</sup>

<sup>1</sup>Department of Mathematics, Linköping University, Sweden and  
Department of Mathematics University of Rwanda, Rwanda  
beatrice.byukusenge@liu.se

<sup>2</sup>Department of Energy and Technology, Swedish University of Agricultural  
Sciences, Uppsala, Sweden, and  
Department of Mathematics, Linköping University, Sweden  
dietrich.von.rosen@slu.se

<sup>3</sup>Department of Mathematics, Linköping University, Sweden  
martin.singull@liu.se

## Abstract

In both univariate and multivariate analysis, residuals are used for model diagnostics. In this presentation we consider the MANOVA and the GMANOVA-MANOVA models and different matrix residuals are established. The interpretation of the residuals is discussed and several properties are verified. A realistic, similar to some real studies, but artificial data will be analysed to illustrate how the residuals can be used in model validations.

**Keywords:** residual analysis, GMANOVA-MANOVA, parametric bootstrap,

**AMS subject classifications.** 62H10; 62H12; 62H15; 62F40

**Acknowledgement.** The research of Beatrice Byukusenge has been supported by the sub-program of Applied Mathematics and Statistics under the Sida-funded bilateral program, The University of Rwanda-Sweden Programme for Research, Higher Education and Institutional Advancement. Dietrich von Rosen is supported by the Swedish Research Council (2017-03003).

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# **Further Introduction to Philatelic Lattice Grids with Four-Sided Stamps (First international minisymposium on mathematical philately)**

**Garry Ka Lok Chu<sup>1</sup>, Simo Puntanen<sup>2</sup>, and George P.H. Styan<sup>3</sup>**

<sup>1</sup>Dawson College, Montreal, Quebec, Canada  
gchu@dawsoncollege.qc.ca

<sup>2</sup>Tampere University, Finland  
simo.puntanen@tuni.fi

<sup>3</sup>McGill University, Montreal, Quebec, Canada  
geostyan@gmail.com

## **Abstract**

SLIDES ARE APPENDED IN APPENDIX.

## IWMS: Contributory Talks

**Testing many restrictions under heteroskedasticity**Stanislav Anatolyev<sup>1</sup> and Mikkel Sølvsten<sup>2</sup><sup>1</sup>CERGE-EI, Czech Republic  
stanislav.anatolyev@cerge-ei.cz<sup>2</sup>University of Wisconsin, USA.**Abstract**

In many regression models, one is willing to test hundreds or thousands of restrictions on regression coefficients, for example, implied by the absence of a particular dimension of heterogeneity. The present paper provides a tool to conduct a test of such hypotheses.

We propose a test that allows for many tested restrictions in a heteroskedastic linear regression model. The test compares the conventional F statistic to a critical value that corrects for many restrictions and conditional heteroskedasticity. The correction utilizes leave-one-out estimation to correctly center the critical value, and a novel tool – leave-*three*-out estimation – to appropriately scale the recentered critical value. Large sample properties of the test are established in an asymptotic framework where the number of tested restrictions may be fixed or may grow with the sample size and can even be proportional to the number of observations.

We show that the test is asymptotically valid and has non-trivial asymptotic power against the same local alternatives as the exact F test when the latter is valid. Simulations corroborate the relevance of these theoretical findings and suggest excellent size control in moderately small samples also under strong heteroskedasticity.

The leave-out machinery and derivations of the test's asymptotic properties make heavy use of properties of projection matrices some of which are novel in the literature.

**Keywords:** linear regression, hypothesis testing, leave-out estimation, many regressors, projection matrices

**AMS subject classifications.** 62J05, 62J20, 62F05

**Rank one summand and Star order**K Nayan Bhat<sup>1</sup>, and Manjunatha Prasad Karantha<sup>2</sup><sup>1</sup>Department of Mathematics, St. Joseph's College (Autonomous)  
Bangalore, India  
bhatnayan@gmail.com; nayan.bhatk@sjc.ac.in<sup>2</sup>Center for Advanced Research in Applied Mathematics and Statistics  
Manipal Academy of Higher Education, Manipal, India  
km.prasad@manipal.edu; kmprasad63@gmail.com**Abstract**

In this presentation, we define the rank one summand of a matrix. For a given real matrix  $A$  with rank greater than 1, we provide a characterization of vectors  $x$  in column space of  $A$  satisfying the property that there exists a rank one summand  $C$  of  $A$  with column space equal to  $\text{span}\{x\}$  and  $C$  is dominated by  $A$  under star order, in terms of separability.

**Keywords:** rank one summand, generalized inverse, star order.

**AMS subject classifications.** 15A09

**Acknowledgement.** First author acknowledge the support by St. Joseph's College (Autonomous), Bangalore. Second author acknowledge the support by Science and Engineering Research Board (DST, Govt. of India) under MATRICS (MTR/2018/000156) and CRG (CRG/2019/000238) schemes.

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## Multilevel Analysis of Self reported Asthma prevalence and its risk factors in India using NFHS-4

Pooja B S<sup>1</sup> and Vasudeva Guddattu<sup>2</sup>

<sup>1,2</sup>Department of Data Science, PSPH  
Manipal Academy of Higher Education, Manipal, India

<sup>1</sup> poojuindhu@gmail.com

<sup>2</sup>Vasudev.guddattu@manipal.edu

### Abstract

Globally, the number of people living with asthma is substantial. Asthma is a major chronic respiratory disease that not only results in death but also causes substantial suffering and reduced productivity. Asthma, though it is a major problem, it is remediable global health problem. With the proper measures and managements, we can reduce the burden of Asthma. Identifying the risk factors associated with Asthma is very important to take up the programs to manage the disease. This study has been done using the data of 696,211 women aged 15–49 years included in India's Largest Health survey, National Family Health Survey, 2015–2016 to identify the risk factors associated with Asthma. Multilevel logistic regression analysis was used to identify the risk factors associated with the disease and Local Indicator of Spatial Auto-correlation (LISA) analysis was then conducted using Local Moran's I to assess the degree of local clustering of asthma prevalence in the Indian districts.

**Keywords:** Asthma prevalence, Multilevel analysis, Spatial disparity

**AMS subject classifications.** 62M10; 62N02; 62H11

**Acknowledgement.** The authors thanks the support of Manipal Academy of Higher Education.

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# Vector Majorization based Stochastic Comparison of Order Statistics for Gompertz–Makeham Random Variables

Shovan Chowdhury<sup>1</sup> and Mikkel Sølvsten<sup>2</sup>

Indian Institute of Management Kozhikode, India

<sup>1</sup>meetshovan@gmail.com

## Abstract

The Gompertz–Makeham (GM) distribution, which is used commonly to represent lifetimes based on laws of mortality, is one of the most popular choices for mortality modelling in the field of actuarial science. This paper investigates ordering properties of the smallest and largest lifetimes (order statistics) arising from two sets of heterogeneous groups of insureds following respective GM distributions using vector majorization. Some sufficient conditions are provided for comparing the smallest and largest lifetimes from two sets of dependent variables in the sense of usual stochastic ordering. Comparison results on the smallest lifetimes, in the sense of hazard rate ordering are established for two groups of heterogeneous independent lifetimes. Under a similar set-up, no reversed hazard rate ordering is shown to exist between the largest lifetimes with the use of a counter-example. Finally, sufficient conditions are presented for comparing two sets of independent heterogeneous lifetimes under random shocks by means of usual stochastic ordering. Such comparisons for the smallest lifetimes are also carried out in terms of hazard rate ordering.

**Keywords:** Gompertz–Makeham distribution, Order Statistics, Vector Majorization, Usual stochastic ordering, Hazard rate ordering

# Artificial intelligence (AI) in marketing (customer service): Ease of use, usefulness, trust, problem-solving ability, and the intention to use / prefer artificial intelligence: Partial least squares (PLS) modelling analysis

**Bilal Mohammad Ahmad Eneizan**

Business School, Jadara University, Irbid, Jordan  
Bilalmomane@gmail.com / Bilal.e@jadara.edu.jo

## Abstract

In many regression models, one is willing to test hundreds or thousands of restrictions on regression coefficients, for example, implied by the absence of a particular dimension of heterogeneity. The present paper provides a tool to conduct a test of such hypotheses.

We propose a test that allows for many tested restrictions in a heteroskedastic linear regression model. The test compares the conventional F statistic to a critical value that corrects for many restrictions and conditional heteroskedasticity. The correction utilizes leave-one-out estimation to correctly center the critical value, and a novel tool – leave-*three*-out estimation – to appropriately scale the recentered critical value. Large sample properties of the test are established in an asymptotic framework where the number of tested restrictions may be fixed or may grow with the sample size and can even be proportional to the number of observations.

We show that the test is asymptotically valid and has non-trivial asymptotic power against the same local alternatives as the exact F test when the latter is valid. Simulations corroborate the relevance of these theoretical findings and suggest excellent size control in moderately small samples also under strong heteroskedasticity.

The leave-out machinery and derivations of the test's asymptotic properties make heavy use of properties of projection matrices some of which are novel in the literature.

**Keywords:** linear regression, hypothesis testing, leave-out estimation, many regressors, projection matrices

**AMS subject classifications.** 62J05, 62J20, 62F05

## Prediction intervals in Bi-additive Models

**Patrícia Antunes<sup>1</sup>, Sandra S. Ferreira<sup>2</sup>, Dário Ferreira<sup>3</sup>, and João T. Mexia<sup>4</sup>**

<sup>1,2,3</sup>Center of Mathematics and Applications, University of Beira Interior,  
Portugal

<sup>1</sup>alexandra.83.patricia@gmail.com

<sup>2</sup>sandraf@ubi.pt

<sup>3</sup>dario@ubi.pt

<sup>4</sup>Centre of Mathematics and its Applications, NOVA School of Science and  
Technology

<sup>4</sup>jtm@fct.unl.pt

## Abstract

We consider Bi-additive Models, which are linear models which are mixed linear models in which the random part may or may not be normally distributed. The aim is to obtain least squares estimators for the cumulants of the random vectors and generalized least squares estimators for the fixed effects vector. Moreover, using the cumulants' properties, in addition to obtaining prediction intervals for future observations, we also build confidence ellipsoids for these values. In order to illustrate the theory, we present an application divided in two parts. In the first part we consider that the random part of the model follows a normal distribution. In the second part, we consider that the random part of the model is distributed according to a gamma distribution. In fact, several cases are considered in each of the parts, comparing the values obtained with the real values.

**Keywords:** Cumulants, Mixed Linear Models, Prediction Intervals

**AMS subject classifications.** 62E20; 62F10; 62J10

**Acknowledgment.** This work was partially supported by the Portuguese Foundation for Science and Technology through the projects: UIDP/MAT/00212/2020 and UIDP/MAT/00297/2020.

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## Pseudoeigenvectors of bounded linear operators

**Krishna Kumar G.**

Department of Mathematics, University of Kerala  
Kariavattom Campus, Thiruvananthapuram, Kerala, India, 695581.  
krishna.math@keralauniversity.ac.in

### Abstract

The pseudospectrum is an essential generalization of the spectrum. We define the eigenvector corresponding to pseudospectral value of a bounded operator on a separable Hilbert space. The eigenvector corresponding to a pseudospectral value of an operator is called pseudoeigenvector. We develop various properties of pseudoeigenvectors of an operator. A sufficient condition using pseudoeigenvector is given, for an operator to possess an almost invariant subset. We find common pseudoeigenvectors for various classes of almost commuting pairs of operators.

**Keywords:** Pseudospectrum, almost commutative, almost invariant, pseudoeigenvector.

**AMS subject classifications.** Primary 47A15, 47A10; Secondary 47A55, 15A09, 15A18

**Acknowledgement.** The author thank University of Kerala for providing the startup research grant via order no. 1595/2021/UOK dated 22.03.2021.

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## Impulse controllability in rectangular descriptor systems

**Mahendra Kumar Gupta**

National Institute of Technology Jamshedpur, India  
mahendra14389@gmail.com

### Abstract

In this work, the following form of descriptor system is considered.

$$E\dot{x} = Ax + Bu,$$

where  $x \in \mathbb{R}^n$ ,  $u \in \mathbb{R}^k$ , are the state vector and the input vector, respectively.  $E \in \mathbb{R}^{m \times n}$ ,  $A \in \mathbb{R}^{m \times n}$ , and  $B \in \mathbb{R}^{m \times k}$  are known constant matrices. Descriptor systems are also known as Differential-Algebraic Equations (DAEs). In this work, the criteria of impulse controllability are established in terms of the original system matrices. It is shown that the condition for impulse controllability is necessary and sufficient condition for the existence of a derivative feedback control  $u = -K_d \dot{x} + v$  law such that the closed loop triple  $(E + BK_d, A, B)$  is impulse controllable.

**Keywords:** Differential-Algebraic Equations (DAEs), Impulse Controllability, Singular Value Decomposition

**AMS subject classifications.** 93B10; 93B05; 93B11; 93B52

**Acknowledgement.** The work is supported by Science and Engineering Research Board, New Delhi, under Grant No. SRG/2019/000451.



## A note on reverse order law for outer inverses

Umashankara Kelathaya<sup>1</sup> and Manjunatha Prasad Karantha<sup>2</sup>

Department of Data Science, Manipal Academy of Higher Education, Manipal,  
India.

<sup>1</sup> usksullia@gmail.com

<sup>2</sup> km.prasad@manipal.edu

### Abstract

It is well known that  $(AB)^{-1} = B^{-1}A^{-1}$  whenever the matrices  $A$  and  $B$  are invertible. This property is known as reverse order law. The reverse order law in the case of Moore-Penrose inverse, group inverse and Drazin inverse have been investigated by several researchers (See [1], [2], [3], and [4]).

We shall discuss reverse order law in the case of outer inverses when the elements are from associative rings. We note that  $\{(ab)^{-}\} \subseteq \{b^{-}a^{-}\}$  holds in general. In the present talk we shall try to explore the conditions for the other inclusion to hold.

**Keywords:** reverse order law, ginverse, associative rings

**AMS subject classifications.** 15A09, 16E50

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## A look on Hadamard Matrices

**Pankaj Kumar Manjhi**

Vinoba Bhawe University, Hazaribag, India  
19pankaj81@gmail.com

### Abstract

Some methods of construction of Hadamard matrices are discussed with their lacune

**Keywords:** Hadamard Matrix, Circulant matrix, Partial Hadamard Matrix.

**AMS subject classifications.** 05B15; 05B30

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## On 2–Stress Regular Graphs

**S. Arumugam<sup>1</sup>, Arathi Bhat<sup>2</sup>, Manjunatha Prasad Karantha<sup>3</sup>, and  
Raksha Poojary**

<sup>1</sup>Kalasalingam Academy of Research and Education, Tamilnadu, India  
s.arumugam.klu@gmail.com

<sup>2</sup>Manipal Institute of Technology, MAHE, Manipal, India  
arathi.bhat@manipal.edu

<sup>3</sup>CARAMS, Manipal Academy of Higher Education, Manipal, India  
km.prasad@manipal.edu

<sup>3,4</sup>Department of Data Science, PSPH, MAHE, Manipal, India  
<sup>4</sup>raksha.poojary@learner.manipal.edu

## Abstract

Stress is a centrality measure based on the shortest paths, which is used to study social and biological networks. In this paper, we make some basic observations on 2–stress regular graphs and we list all the 2–stress regular graphs.

**Keywords:** Centrality measure, stress, path

**AMS subject classifications.** 05C90; 94C15

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## Kurtosis-based projection pursuit for matrix-valued data

Una Radojicic<sup>1</sup>, Joni Virta<sup>2</sup>, and Klaus Nordhausen<sup>3</sup>

<sup>1</sup>Vienna University of Technology, Austria  
una.radojicic@tuwien.ac.at

<sup>2</sup>University of Turku, Finland  
joni.virta@utu.fi

<sup>3</sup>University of Jyväskylä, Finland  
klaus.k.nordhausen@jya.fi

### Abstract

We develop projection pursuit for data that admit a natural representation in matrix form. For projection indices we propose extensions of the classical kurtosis and Mardia's multivariate kurtosis. The first index estimates projections for both sides of the matrices simultaneously, while the second index finds the two projections separately. Both indices are shown to recover the optimally separating projection for two-group Gaussian mixtures in the full absence of any label information. We further establish the strong consistency of the corresponding sample estimators. Simulations and a real data example on hand-written postal code data are used to demonstrate the method.

**Keywords:** discriminant analysis, matrix-variate Gaussian mixture, rank-1 projection

## Fuzzy Assignment Problem with Random Triangular Fuzzy Numbers

Chepuri Rajita<sup>1</sup> and L P Raj Kumar<sup>2</sup>

<sup>1,2</sup>Department of Mathematics, Kakatiya University, Warangal, India

<sup>1</sup>rajithachepuri341@gmail.com

<sup>2</sup>ladalla.raj@gmail.com

### Abstract

Decision makers are responsible in gaining organizational commitment with the decision in their attitude and in their action. Decision making always need careful considerations and is a complex process that are based on emotions and intuition. In Classical Assignment Problems, the cost associated with assigning a job to a person is deterministic, which is not realistic in nature. If the cost in the assignment is with respect to its relative degree of similarity, is a Fuzzy Assignment Problem(FAP). In this paper, the cost associated are taken as Random Triangular Fuzzy Numbers and standard procedures are adopted in determining the fuzzy optimal solution.

**Keywords:** Optimal Decision, Fuzzy Assignment Problem, Triangular fuzzy numbers, Fuzzy Optimal Solution.

## Restricted Fuzzy Assignment Problem with Trapezoidal Fuzzy Numbers

**K Ravinder Reddy<sup>1</sup> and L P Raj Kumar<sup>2</sup>**

<sup>1,2</sup>Department of Mathematics, Kakatiya University, Warangal, India

<sup>1</sup> ravindhar.iitm18@gmail.com

<sup>2</sup> ladalla.raj@gmail.com

### Abstract

Decision makers take challenging decisions which impact the company operations and they know how to find the solutions to the problems effectively with critical thinking skills that help find solutions to problems. The cost associated with assigning a job to a person in classical assignment problems, is deterministic, which in general is not realistic in nature. If the associated cost of the assignment is in the form of composite relative to its degree of similarity, is a Fuzzy Assignment Problem(FAP). In this article, the costs associated are taken as Random Trapezoidal Fuzzy Numbers with the restricted cost assigned to one of the cell and the proposed ranking procedures are applied in determining the fuzzy optimal solution.

**Keywords:** Decision Makers, Restricted Fuzzy Assignment Problem, Trapezoidal Fuzzy Numbers, Fuzzy Optimal Solution.

# **Program: IWMS 2021**

**December 13, 2021 (Monday, Day 01)****09:15 - 09:45 - Inauguration**

- Opening Remarks & Welcome Address: K. Manjunatha Prasad, Coordinator, CARAMS, MAHE
- **Lighting the lamps**
- Overview of the events : Ravindra B Bapat, Chairman, SAC
- About IWMS & Welcome: Simo Puntanen, President IWMS
- Address by Chair, LOC: Narayana Sabhahit, Registrar, MAHE
- Message from Director, SMIT, SMU
- **Release of Souvenir**
- Inaugural Address: Helmut Brand, Director, PSPH, MAHE, Manipal
- Vote of Thanks

**09:45 - 10:15 Tea Break & Group Photo****SESSION 1 (10:15 - 11:05)**

*Chair Person: Steve Haslett, Massey University, New Zealand*

10:15 - 11:05 Arni Srinivasa Rao, Medical College of Georgia, Augusta University, USA  
*Exact Deep Learning Machines, Artificial Intelligence, and Deep Learning Methods*

**SESSION 2 (11:10 - 12:00)**

*Chair Person: Arup Bose, Indian Statistical Institute, Kolkata, India*

11:10 - 12:00 Susmita Datta, University of Florida, Gainesville, USA  
*Statistical Analysis of single cell RNA sequencing (ScRNA-seq) data*

**SESSION 3 (12:00 - 13:00)**

*Chair Person: Katarzyna Filipiak, Poznan University of Technology, Poland*

**12:00 - 13:00 Contributory Talks**

- Una Radojčić, Venna University of Technology, Vienna, Austria  
*Kurtosis-based projection pursuit for matrix-valued data*
- Sandra S. Ferreira, Center of Mathematics and Applications, University of Beira Interior, Portugal  
*Prediction intervals in Bi-additive Models*
- Mahendra Kumar Gupta, National Institute of Technology Jamshedpur (CT 7)  
*Impulse controllability in rectangular descriptor systems*

**13:00 - 14:00 Lunch Break**

**SESSION 4 (14:00 - 15:30)***Chair Person: Simo Puntanen, Tampere University, Finland*

14:00 - 15:30 Dietrich von Rosen, Swedish University of Agricultural University, Sweden  
*Matrix Decompositions in Linear / bilinear Methods*

**15:30 - 15:50 Tea Break****SESSION 5 (15:50 - 17:05)***Chair Person: Dietrich von Rosen, Swedish University of Agricultural University, Sweden*

15:50 – 16:25 Augustyn Marckiewicz, Poznan University of Life Science, Poland  
*Covariance matrix estimation based on matrix projection*

16:30 – 17:05 Martin Singull, Linkoping University, Sweden  
*Residual Analysis for the GMANOVA-MANOVA Model*

**December 14, 2021 (Tuesday, Day 02)****SESSION 6 (09:10 – 10:40)***Chair Person: Asha Kamath, DDS, PSPH, Manipal Academy of Higher Education, Manipal*

09:10 – 09:50 Steve Haslett, Massey University, New Zealand  
 The role of block diagonal matrices in conditions for equality of BLUEs in linear models

09:50 – 10:40 N Balakrishnan, McMaster University, Canada  
 Some interesting feature of linear prediction

**10:40 – 11:00 Tea Break****SESSION 7 (11:00 – 12:00)***Chair Person: Shreemathi Mayya, Manipal Academy of Higher Education, Manipal, India***11:00 – 12:00 Contributory Talks**

- Pooja B. S., Department of Data Science, PSPH, MAHE, Manipal, India  
*Multilevel Analysis of Self-reported Asthma prevalence and its risk factors in India using NFHS-4*
- Raksha Poojary, Manipal Academy of Higher Education, Manipal, India  
*On 2-Stress Regular Graphs*
- Umashankara Kelathaya, Manipal Academy of Higher Education, Manipal, India  
*A note on reverse order law for outer inverses*

**SESSION 8 (12:00 – 13:10)***Chair Person: Martin Singull, Linkoping University, Sweden*

12:00 – 12:35 Simo Puntanen, Tampere University, Finland  
*A Review of the Linear Sufficiency*

12:35 – 13:10 Katarzyna Filipiak, Poznan University of Technology, Poland  
*Frobenius norm, entropy and quadratic loss functions as the measures of discrepancy from structured matrices*

13:10 - 14:10 **Lunch Break**

**SESSION 9 (14:10 - 16:10), Mini-Symposium**

**Organizer: S. K. Neogy, Indian Statistical Institute, Delhi**

Topic : Recent Methodologies in Big Data Analysis

Speakers: Santosh Singh, Shiv Nadar University, UP, India

*Trends in Big Data Technologies* S.K. Neogy, Indian Statistical Institute, Delhi Centre

*Statistics & Big data: A Research Paradigm Shift* Andrzej Janusz, Q.E.D. Software, Poland

*Learning from Video Game Logs: the Human-in-the-Loop Approach*

16:10 - 16:30 **Tea Break**

**SESSION 10 (16:30 – 17:05)**

*Chair Person: Vasudeva Guddattu, DDS, PSPH, Manipal Academy of Higher Education, Manipal*

16:30 – 17:05 Somdeb Lahiri, Pandit Deendayal Energy University, Gujarat, India

*Game theory for decision making under ambiguity*

**December 15, 2021 (Wednesday)**

**SESSION 11 (09:30 - 10:30)**

*Chair Person: Vani Lakshmi R, Department of Data Science, PSPH, MAHE, Manipal, India*

**09:30 - 10:30 Contributory Talks**

- K Nayan Bhat, St. Joseph's College (Autonomous), Bangalore, India  
*Rank one summand and Star order*
- Pankaj Manjhi, Vinoba Bhave University, Hazaribag, India  
*A look on Hadamard Matrices*
- Shovan Chowdhury, Indian Institute of Management Kozhikode, India  
*Vector Majorization based Stochastic Comparison of Order Statistics for Gompertz–Makeham Random Variables*

10:30 - 11:00 **Tea Break**

**SESSION 12 (11:00 - 12:00)**

*Chair Person: Anisha P, Department of Data Science, PSPH, MAHE, Manipal, India*

**11:00 - 12:00 Contributory Talks**



- K Ravinder Reddy, Kakatiya University, Warangal, India  
*Restricted Fuzzy Assignment Problem with Trapezoidal Fuzzy Numbers*
- Chepuri Rajita, Kakatiya University, Warangal, India  
*Fuzzy Assignment Problem with Random Triangular Fuzzy Numbers*
- Krishna Kumar G, BJM Government College, Chavara, Kollam, India  
*Pseudoeigenvectors of bounded linear operators*
- Stanislav Anatolyev, CERGE - Economic Institute, Prague, Czech Republic  
*Testing many restrictions under heteroskedasticity*
- Bilal Mohammad Ahmad Eneizan, Business School, Jadara University, Irbid, Jordan  
*Artificial intelligence (AI) in marketing (customer service): Ease of use, usefulness, trust, problem-solving ability*

### SESSION 13 (12:00 – 12:50)

*Chair Person: Sharad S Sane, Chennai Mathematical Institute, Chennai*

12:00 – 12:50 Anish Sarkar, Indian Statistical Institute, Delhi  
*Hack's Law in a Drainage Network Model: A Brownian Web Approach*

13:00 - 14:00 **Lunch Break**

### SESSION 14 (14:00 – 14:50)

*Chair Person: Simo Puntanen, Tampere University of Technology, Finland*

14:00 – 14:50 Arup Bose, Indian Statistical Institute, Kolkata  
*Random Matrices*

### SESSION 15 (15:00 - 15:50)

*Balaji Ramamurthy, Indian Institute of Technology Madras, India*

15:00 - 15:50 Abraham Berman, Technion-Israel Institute of Technology, Israel  
*Completely Positive Matrices*

16:00 – 17: 20 **Tea Break & Break**

### SESSION 16 (17:20 – 18:00)

*Chair Person: Sudhakara G, Manipal Institute of Technology, MAHE, Manipal, India*

17:20 – 18:00 Rafikul Alam, Indian Institute of Technology Guwahati, India  
*A trace-moment based method for solving holomorphic eigenvalue problems*

### Mini Symposium ( SESSION 17; 18:00 – 19:30)

**Organizer: George PH Styan, McGill University, Canada**

18:00 – 19:30

Speakers: Garry Ka Lok Chu, Dawson College, Canada  
Simo Puntanen, Tampere University, Finland  
George PH Styan, McGill University, Canada

Title: A further introduction to Philatelic Lattice Grids (PLGs) with four-sided stamps  
(First International Mini-Symposium on Mathematical Philately)

**19:30 - Valedictory Function**

Remarks: Participants

Closing Remarks: Simo Puntanen, IWMS President  
GPH Styan, IWMS Hon. President

Vote of Thanks: K. Manjunatha Prasad, Convener, SAC

19:45 **Dinner**

# **Participants: IWMS 2021**

1. STANISLAV ANATOLYEV , Center for Economic Research and Graduate Education - Economic Institute, Prague, Czech Republic  
stanislav.anatolyev@cerge-ei.cz
2. KAZEEM ADELEKE , Obafemi Awolowo University, Nigeria  
aadeleke@oauife.edu.ng
3. S. EJAJ AHMED , Affiliation  
sahmed5@brocku.ca
4. S. ARUMUGAM , Kalasalingam University, India  
s.arumugam.klu@gmail.com
5. AMULYA AWASTHI , Banasthali Vidyapith, India  
amulyaawasthi13@gmail.com
6. PALLVI B , Indian Institute of Science Education and Research, Thiruvananthapuram, India  
pallavipoorna20@iisertvm.ac.in
7. POOJA B. S. , Manipal Academy of Higher Education, Manipal, India  
PoojaBS@gmail.com
8. BAYOWA BABALOLA , Kampala International University, Uganda  
teniola.babalola@eksu.edu.ng
9. ANAND BAJARANGI , Indian Institute of Information Technology Dharwad, India  
apb4maths@gmail.com
10. NARAYANASWAMY BALAKRISHNAN , McMaster University, Hamilton, Ontario, Canada  
bala@mcmaster.ca
11. RAVINDRA B. BAPAT , Indian Statistical Institute, Delhi, India  
rbb@isid.ac.in
12. KESANG BASI , Sikkim Manipal Institute Technology, Sikkim, India  
kessang\_20211041@smiit.smu.edu.in
13. KUNZANG CHUNI BASI , Government College Rhenock, Sikkim, India  
kunzang.basi.stats@gmail.com
14. ADI BEN-ISRAEL , Rutgers Business School, Newark, New Jersey, USA  
adi.benisrael@gmail.com
15. ANJALI BENIWAL , Indian institute of technology Kharagpur  
anjalibeniwal1811@gmail.com
16. ABRAHAM BERMAN , Technion-Israel Institute of Technology, Israel  
bermanabraham@gmail.com
17. ALOK BHARGAVA , Manipal University Jaipur, India  
alok.bhargava@jaipur.manipal.edu
18. K NAYAN BHAT , St. Joseph's College (Autonomous), Bangalore, India  
bhatnayan@gmail.com
19. S. PARAMESHWARA BHATTA , Mangalore University, Mangala Gangotri, India  
s\_p\_bhatta@yahoo.co.in
20. JOHN BIBBY , University of York, United Kingdom  
jb43@york.ac.uk
21. PARITOSH BISWAS , von Karman Society, India  
p.biswas1943@gmail.com
22. NIRANJAN BORA , Dibrugarh University Institute of Engineering & Technology, India  
niranjanbora11@gmail.com
23. ARUP BOSE , Indian Statistical Institute, Kolkata, India  
bosearu@gmail.com
24. FRANCISCO CAVALHO , Affiliation  
fpcarvalho@ipt.pt

25. ROBERT CASTELLANOS , CUN, Colombia  
rscastellanosr@unal.edu.co
26. RAMESHA C. , Manipal Institute of Technology, MAHE, Manipal  
ramesha.c@manipal.edu
27. ANKUR CHAKRABORTY , Indian Institute of Technology Dhanbad, West Bengal, India  
ankur.18dr0033@am.iitism.ac.in
28. PREYANKA CHANDRAMOHAN , Surigao del Sur State University, Singapore  
sangcoresa18@gmail.com
29. SRUTI CHOPPARAPU , Christ(Deemed to be University), Bangalore, India  
chopparapu.sruti@gmail.com
30. SHOVAN CHOWDHURY , Indian Institute of Management Kozhikode, India  
meetshovan@gmail.com
31. GARRY KA LOK CHU , Dawson College, 3040 Sherbrooke Street West, Canada  
gchu@dawsoncollege.qc.ca
32. PURAN DANGAL , Sikkim Manipal Institute Technology, Sikkim, India  
dangalpuran7@gmail.com
33. KOUSIK DAS , National Institute of Technology Raipur, India  
kousikdas1993@gmail.com
34. SUSMITA DATTA , Clinical Translational Research Institute, University of Florida, United States  
susmita.datta@ufl.edu
35. ARAVIND DHARMARAJ , cmc  
aravindradsat@gmail.com
36. SIBASISH DHIBAR , Indian Institute Technology Roorkee, India  
sibasishdhibar372@gmail.com
37. N. EAGAMBARAM , Former DDG, MOSPI, Govt. of India, India  
ekambaramn@gmail.com
38. BILAL MOHAMMAD AHMAD ENEIZAN , Business School, Jadara University, Irbid, Jordan  
Bilal.e@jadara.edu.jo
39. SANDRA MARIA BARGÃO FERREIRA , Universidade da Beira Interior, Portugal  
sandrabargao321@gmail.com
40. KATARZYNA FILIPIAK , Poznan University of Technology, Poland  
katarzyna.filipiak@put.poznan.pl
41. S. GANESAN , Bhabha Atomic Research Centre, Mumbai, India  
ganesan555@gmail.com
42. SANDEEP GARG , Mahatma Gandhi Medical College and Hospital, Jaipur, India  
gargsandeep42@yahoo.com
43. JESSICA GEORGE , Department of Data Science, PSPH, MAHE, Manipal, India  
georgejessica24@gmail.com
44. SUBRATA GHANTY , Santal Pargana College, Dumka, India  
subrataghanty.sg.2001@gmail.com
45. SUDHAKARA G. , Manipal Institute of Technology, MAHE, Manipal, India  
sudhakara.g@manipal.edu
46. VASUDEVA GUDDATTU , Manipal Academy of Higher Education, Manipal, India  
vasudev.guddattu@manipal.edu
47. MAHENDRA KUMAR GUPTA , National Institute of Technology Jamshedpur  
mahendra14389@gmail.com
48. SHRINATH HADIMANI , Indian Institute of Science Education and Research Thiruvananthapuram, India

- srinathsh33@gmail.com
49. STEPHEN JOHN HASLETT , Massey University, Palmerston North, New Zealand  
stephen.haslett@anu.edu.au
  50. JEFFREY HUNTER , Auckland University of Technology, Auckland, New Zealand  
jeffrey.hunter@aut.ac.nz
  51. AISHA JABEEN , Jamia Millia Islamia  
ajabeen329@gmail.com
  52. ANDRZEJ JANUSZ , Institute of Informatics, University of Warsaw, Poland  
andrzej.janusz@qed.pl
  53. NITIN JOHN , All India Institute of Medical Sciences, Hyderabad  
drnitinjohn@yahoo.co.in
  54. NEERAJ JOSHI , Department of Statistics, University of Delhi, India  
stats.joshi@gmail.com
  55. MOHANA K. S. , Mangalore University, Mangalore, India  
krish305@gmail.com
  56. AWOPEJU KABIRU , Nnamdi Azikiwe University, Nigeria  
ak.awopeju@unizik.edu.ng
  57. AKSHATA KALE , Department of Data Science, PSPH, MAHE, Manipal, India  
kale.madhura@learner.manipal.edu
  58. KAMARAJ K. , Anna University, Arni, India  
krajkj@yahoo.com
  59. ASHA KAMATH , Manipal Academy of Higher Education, Manipal  
asha.kamath@manipal.edu
  60. RAJESH KANNAN M , Indian Institute of Technology Karaghpur, India  
rajeshkannan1.m@gmail.com
  61. DEBAMALYA KAR , Maharani Kasiswari College, Kolkata, India  
kdebamalya15@gmail.com
  62. PRAJNA K. , Manipal Institute of Technology, MAHE, Manipal, India  
kprajna99@gmail.com
  63. KARPAGAM ARCHITECTURE , Anna University, Coimbatore, India  
karpagamarchcbe@gmail.com
  64. KANNADASAN KARUPPAIAH , Dr.M.G.R. Tamilnadu Medical University, India  
statkannadasan@gmail.com
  65. YUKTA KATARIA , Department of Data Science, PSPH, MAHE, Manipal, India  
yukta.kataria03@gmail.com
  66. NEERAJ KAUSHIK , National Institute of Technology Kurukshetra  
kaushikneeraj@gmail.com
  67. UMASHANKARA KELATHAYA , Manipal Academy of Higher Education, Manipal, India  
usksullia@gmail.com
  68. HARSHA NARASINHA KELKAR , Hope Foundation's Finolex Academy of Management and  
Technology, Ratnagiri, India  
harsha.kelkar@famt.ac.in
  69. HABIBA KHATUN , National Institute of Technology Rourkela  
habibakhatun7860@gmail.com
  70. STEPHEN JAMES KIRKLAND , University of Manitoba, Canada  
Stephen.Kirkland@umanitoba.ca
  71. DANIEL KLEIN , Pavol Jozef Safarik University in Košice, Slovakia  
daniel.klein@upjs.sk
  72. PUSHKAL KUMAR , National Institute of Technology Rourkela

- pushkalkumar0812@gmail.com
73. ANSHUL KUMAR , Indian Institute of Technology Roorkee, India  
akumar@ma.iitr.ac.in
74. ANANTH KUMAR , Department of Data Science, PSPH, MAHE, Manipal, India  
ananthkumarg169@gmail.com
75. KRISHNA KUMAR G , BJM Government College, Chavara, Kollam, India  
krishna.math@gmail.com
76. KALPANARANI KUMAR , VIT University, Kamarajapuram, Chennai, India  
kalpana91085@yahoo.com
77. POOJA KUMARI , Central University of Gujarat  
kumaripoojasingh10@gmail.com
78. SHIVA KUMAR KURVA , Pondicherry University, Puducherry, India  
shivakumar.sk137@pondiuni.ac.in
79. SOMDEB LAHIRI , Pandit Deendayal Energy University, Gujarat, India  
somdeb.lahiri@gmail.com
80. C. LALNGAIHAWMA , Govt. Kolasib College, Mizoram, India  
angaiha.anomaly@gmail.com
81. ANDRÉ LEROY , Université d'Artois, France  
andreleroy55@gmail.com
82. CHEDLY GÉLIN LOUZAYADIO , Université Marien NGOUABI, Brazzaville Republic of Congo  
gelinlouz@gmail.com
83. GNANASEKAR M , Rathinam college of Arts and Science, Coimbatore, India  
gnanasekar.kalam@gmail.com
84. VINAY MADHUSUDANAN , Manipal Institute of Technology, MAHE, Manipal, India  
vinay.m2000@gmail.com
85. SUMAN MAITI , National Institute of Technology Rourkela  
smaiti2711@gmail.com
86. AIJAZ AHMAD MALLA , University of Kashmir, Srinagar  
ahaijaz99@gmail.com
87. DEBJIT MANDAL , Utkal University, Bhubaneswar, India  
debjitmandal.burnpur@gmail.com
88. SANTANU MANDAL , National Institute of Technology Rourkela, India  
santanu.vumath@gmail.com
89. PANKAJ MANJHI , Vinoba Bhave University, Hazaribag, India  
19pankaj81@gmail.com
90. AUGUSTYN MARKIEWICZ , Poznan University of Life Sciences, Poznan, Poland  
amark@up.poznan.pl; augustyn.markiewicz@up.poznan.pl
91. ADILSON DE JESUS MARTINS DA SILVA , University of Cape Verde, Cape Verde  
adilson.dasilva@docente.unicv.edu.cv
92. MIKA MATTILA , Tampere University, Tampere, Finland  
mika.mattila@tuni.fi
93. SHREEMATHI MAYYA , Manipal Academy of Higher Education, Manipal, India  
shreemathi.mayya@manipal.edu
94. RAVI MEEL , Department of Mathematics, IISER Bhopal, India  
ravi17@iiserb.ac.in
95. SUNIL MEENA , Government College, Sirohi, India  
meenasunil461@gmail.com
96. VIDHYA MEENAKSHI V , Department of Statistics, Bharathidasan university, Trichy, India  
vidhyameena5799@gmail.com

97. RUCHIKA MEHTA , Manipal University Jaipur, India  
ruchika.mehta@jaipur.manipal.edu
98. PARVATHI G MENON , Sahridaya College Of Advanced Studies, Kodakara, Thrissur  
pgmenon174@gmail.com
99. DAVID RAJ MICHEAL , Sikkim Manipal Institute of Technology, Majitar, India  
davidddl2013@gmail.com
100. SURYANSHI MISHRA , Sam Higginbottom University of Agriculture, Technology and Sciences,  
Prayagraj, INDIA  
suryanshimishra2496@gmail.com
101. HEMANT MISHRA , Indian Statistical Institute, Delhi Center, India  
hemantmishra1124@gmail.com
102. VASANT KUMAR MISHRA , Banaras Hindu University, India  
vasantmishra99@gmail.com
103. ARCHANA MURUGANANTHAM , E.G.S. Pillay Engineering College, Nagapattinam, India  
archu288m@gmail.com
104. SANGEETA NAHAR , Krypt India, India  
sg30@iitbbs.ac.in
105. SANJANA NAIR , Department of Data Science, PSPH, MAHE, Manipal, India  
sanjana.nair27@gmail.com
106. NUPUR NANDINI , Manipal Academy of Higher Education, Manipal, India  
n.nandini048@gmail.com
107. DHANASEKARAN NATARAJAN , The Central Law College, Salem, India  
sekarraj2000@gmail.com
108. NISHA NAYAK , Department of Data Science, PSPH, MAHE, Manipal, India  
nisha.muma@learner.manipal.edu
109. S. K. NEOGY , Indian Statistical Institute, Delhi Centre, India  
skn@isid.ac.in
110. WINNIE ONSONGO , University of Ghana, Ghana  
onsongowinnie@gmail.com
111. HARIOM PACHORI , Central Institute of Psychiatry, Ranchi, India  
hariom.iips@gmail.com
112. SARVESH PANDEY , Banaras Hindu University, India  
pandeysarvesh798@gmail.com
113. KARUNA PANWAR , Indian Institute of Technology, Roorkee, India  
karunapanwar1@gmail.com
114. KARUNA PANWAR , Indian Institute of Technology, Roorkee  
karunapanwar1@gmail.com
115. SAMIRON PARUI , Indian Institute of Science Education and Research, Kolkata, India  
samironparui@gmail.com
116. SUKANTA PATI , Indian Institute of Technology Guwahati, India  
pati@iitg.ernet.in; pati@iitg.ac.in
117. C PERIYASAMY , Presidency College, Chennai, India  
periyasamyc1995@gmail.com
118. RAKSHA POOJARY , Manipal Academy of Higher Education, Manipal, India  
Rakshayogeesh@gmail.com
119. SUMATHI K. PRABHU , Manipal Institute of Technology, MAHE, Manipal, India  
k.sumathi@manipal.edu
120. BARSHA PRADHAN , Sikkim Manipal Institute Technology, Sikkim, India  
barshapradhan09@gmail.com



121. PUNAM KUMARI PRASAD , Sikkim Manipal Institute Technology, Sikkim, India  
psikkim@ymail.com
122. KUNCHAM SYAM PRASAD , Manipal Institute of Technology, MAHE, Manipal, India  
kunchamsyamprasad@gmail.com
123. PREETI , Indian Institute of Technology, Roorkee  
pnpreeti2@gmail.com
124. SIMO PUNTANEN , Tampere University, Finland  
simo.puntanen@tuni.fi
125. HARSHITH R A , Department of Data Science, PSPH, MAHE, Manipal, India  
harshith.a@learners.manipal.edu
126. UNA RADOJICIC , Venna University of Technology, Vienna, Austria  
una.radojicic@tuwien.ac.at
127. T. E. S. RAGHAVAN , University of Illinois at Chicago, United States  
terctu@gmail.com
128. CHEPURI RAJITA , Kaktiya University, Warangal, India  
rajithachepuri341@gmail.com
129. SRINIDHI RAMACHANDRAN , Department of Data Science, PSPH, MAHE, Manipal, India  
srinidhi.r28@gmail.com
130. BALAJI RAMAMURTHY , Indian Institute of Technology Madras, Chennai, India  
balaji5@iitm.ac.in
131. ARNI S R SRINIVASA RAO , Medical College of Georgia, Augusta University, USA  
arni.rao2020@gmail.com
132. T S S R K RAO , Manipal Academy of Higher Education, Manipal; Ashoka University, Sonipat, Haryana, India  
tssrkr Rao@gmail.com
133. JITA RASTOGI , Not available  
Jitirastogi@gmail.com
134. BILAL AHMAD RATHER , University of Kashmir, Srinagar, India  
bilalahmadrr@gmail.com
135. ANITHA RAVIKUMAR , Higher College of Technology, Muscat, Oman  
anitharaveekumar@gmail.com
136. SIDDHARTH RAWAT , Indian Institute of Management Bangalore, India  
siddharth.rawat19@iimb.ac.in
137. DHANANJAYA REDDY , Government Degree College, Puttur, India  
djreddy65@gmail.com
138. KONDAM RAVINDER REDDY , Kakatiya University, Warangal, India  
ravindhar.iitm18@gmail.com
139. DIETRICH VON ROSEN , Swedish University of Agricultural Sciences, Sweden  
Dietrich.von.Rosen@slu.se
140. NIRMAL CHANDRA ROUT , National Institute of Technology Raipur, India  
nirmal.rout1989@gmail.com
141. ARITRA ROY , Banaras Hindu University, India  
97aritra@gmail.com
142. MEDHA S , Department of Data Science, PSPH, MAHE, Manipal, India  
medha.s1@learner.manipal.edu
143. SACHIN SACHDEVA , School of Mathematics & Statistics, University of Hyderabad, India  
sachinstats007@gmail.com
144. TAPATEE SAHOO , Manipal Institute of Technology, MAHE, Manipal, India  
tapateesahoo96@gmail.com

145. DEEPAK SAINI , CCS University, Meerut, India  
deepaksaini1168@gmail.com
146. ANIRUDDHA SAMANTA , Indian Institute of Technology Kharagpur, India  
aniruddha.sam@gmail.com
147. RESA MAE SANGCO , Surigao del Sur State University, Philippines  
sangcoresa18@gmail.com
148. MOJAMMEL HAQUE SARKAR , National Institute of Technology Rourkela  
mojammelmhs3@gmail.com
149. ANISH SARKAR , Indian Statistical Institute, Delhi Centre, India  
anish.sarkar@gmail.com
150. PREETI SENGAR , Shri Govindram Seksaria Institute of Technology and Science, Indore, India  
preeti.sengar89@gmail.com
151. RAMJAN SHAIK , Al-Ameen College of Pharmacy, Bangalore, India  
ramjanshaik@gmail.com
152. NIDHI SHARMA , Indian Institute of Technology Roorkee, India  
6nidhisharma@gmail.com
153. SONALI SHARMA , Malaviya National Institute of Technology , Jaipur, India  
2019rma9072@mnit.ac.in
154. SHREENAVYA , Department of Data Science, PSPH, MAHE, Manipal, India  
shreenavyadm@gmail.com
155. PRIYANKA SINGH , Chhattisgarh Swami Vivekanand Technical University Bhilai  
ps183241@gmail.com
156. CHINGANGBAM SURAJ SINGH , Manipur University, Imphal, India  
suraj.ch@manipuruniv.ac.in
157. RAVI PRATAP SINGH , Banaras Hindu University, India  
ravibhu002@gmail.com
158. SANTOSH SINGH , Shiv Nadar University, Noida, Noida  
santosh.singh@snu.edu.in
159. MARTIN SINGULL , Linköping University, Linköping, Sweden  
martin.singull@liu.se
160. BIKAS KUMAR SINHA , Indian Statistical Institute, Kolkata, India  
bksinha@isical.ac.in
161. K. C. SIVAKUMAR , Indian Institutes of Technology Madras, Chennai, India  
kcskumar@iitm.ac.in
162. SIVARAMAKRISHNAN SIVASUBRAMANIAN , Indian Institute of Technology Bombay, India  
siva.krishnan@gmail.com
163. RACHANA SONI , Bennett University, Utter Pradesh, India  
rachanasoni007@gmail.com
164. PURNIMA SONKER , Punjabi University Patiala, India  
pani.sonkar@gmail.com
165. APOORVA SOOD , Department of Data Science, PSPH, MAHE, Manipal, India  
apoorva.sood@learner.manipal.edu
166. PRAJNYA SRINIVAS , Department of Data Science, PSPH, MAHE, Manipal, India  
prajnya.srinivas@learner.manipal.edu
167. GEORGE P H STYAN , McGill University, Canada  
geostyan@gmail.com
168. SUMATHI SUBBARAYAN , Vellur Institute of Technology, Chennai, India  
sumathi.manivannan04@gmail.com
169. LAVANYA SURIYAMOORTHY , Sri Ramachandra Institute for Higher Education and Research,

- Chennai India  
mathlavi@gmail.com
170. JYOTI TAK , Tania university Ganganagar  
jyotitak4985@gmail.com
171. LORENZO TEDESCO , Katholieke Universiteit Leuven, Belgium  
lorenzo.tedesco@kuleuven.be
172. VENKATESWARARAO TEEGA , Indian Institute of Technology Hyderabad, India  
ma19resch11001@iith.ac.in
173. MANJU BHASHINI THIYAGARAJAN , Vivekananda college of technology for women, Tamil-  
nadu, India  
manjutmpv13@gmail.com
174. KRISHNAN VARADHARAJAN , Jamal Mohamed College, Srirangam, Trichy, India  
krishnan@jmc.edu
175. SAVITHA V , Manipal Academy of Higher Education, Manipal, India  
savithavarkady@yahoo.co.in
176. PRIYA VERMA , C.S.J.M.University, Kanpur, India  
FANPRIYA49@gmail.com
177. PIYUSH VERMA , Indian Institute of technology, Bhubaneswar, India  
pv18@iitbbs.ac.in
178. PRATEEK KUMAR VISHWAKARMA , Indian Institute of Science, Bangalore, India  
prateekv@iisc.ac.in
179. JULIA VOLAUFOVA , Louisiana State University Health Sciences Center, New Orleans (USA)  
jvolau@lsuhsc.edu
180. XIKUI WANG , Warren Center for Acturial Research, University of Manitoba, Canada  
xikui.wang@umanitoba.ca
181. HITESH WANKHEDE , Department of Mathematics, IISER Pune, India  
hitesh.wankhede@students.iiserpune.ac.in
182. HANS JOACHIM WERNER , Affiliation  
hjwt.de@uni-bonn.de
183. YIMIN WEI , Fudan University, Shanghai, China  
yimin.wei@gmail.com; ymwei@fudan.edu.cn
184. Y. SANTHI SHEELA , National Institute of Technology Karnataka, India  
krishsanthi76@gmail.com

# **ICLAA 2021**

International Conference on Linear  
Algebra and its Applications  
December 15-17, 2021

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17. SUMIT MOHANTY *Gandhi Institute of Technology and Managemen, Vishakapatnam, India*
18. PALLAVI P *Manipal Institute of Technology, MAHE, Manipal, India*
19. KAMAL LOCHAN PATRA *National Institute of Science Education and Research, Bhubaneswar, India*
20. YOGESH KUMAR PRAJAPATY *Indian Institute of Science and Education, Thiruvananthapuram*
21. TAPATEE SAHOO *Manipal Institute of Technology, MAHE, Manipal, India*
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23. KULDEEP SARMA *Tezpur University, Assam, India*
24. SONALI SHARMA *Malaviya National Institute of Technology , Jaipur, India*
25. VAIBHAV SHEKHAR *National Institute of Technology Raipur, India*
26. TONY THOMAS *Kerala University of Digital Sciences, Innovation and Technology, Kerala, India*
27. SAVITHA V *Manipal Academy of Higher Education, Manipal, India*
28. PRATEEK KUMAR VISHWAKARMA *Indian Institute of Science, Bangalore, India*
29. PUNIT KUMAR YADAV *Malaviya National Institute of Technology Jaipur, India*

# **Abstracts: ICLAA 2021**

## ICLAA 2021: Invited Talks

### A trace-moment based method for solving holomorphic eigenvalue problems

Rafikul Alam<sup>1</sup> and Jibrail Ali

Indian Institute of Technology Guwahati, India

<sup>1</sup>rafik@iitg.ac.in

#### Abstract

Let  $U \subset \mathbb{C}$  be open and  $T : U \rightarrow \mathbb{C}^{n \times n}$  be holomorphic. Consider the nonlinear eigenvalue problem (NEP):

Find  $\lambda \in U$  and a nonzero  $v \in \mathbb{C}^n$  such that  $T(\lambda)v = 0$ .

NEPs arise in many applications in science and engineering. Computing solution of an NEP is a challenging task and various methods for solving NEPs have been proposed in recent years. We present a trace-moment based method for computing eigenvalues of  $T(\lambda)$ . If a region in  $U$  contains  $\ell$  distinct eigenvalue  $\lambda_1, \dots, \lambda_\ell$  of  $T(\lambda)$  then by utilizing trace-moments of  $T(\lambda)$  we construct an  $\ell \times \ell$  Hankel pencil  $L(\lambda) := \hat{H} - \lambda H$  such that  $\lambda_1, \dots, \lambda_\ell$  are simple eigenvalues of  $L(\lambda)$ . Hence by solving the linear eigenvalue problem  $L(\lambda)v = 0$  we obtain the eigenvalues  $\lambda_1, \dots, \lambda_\ell$  of  $T(\lambda)$ .

**Keywords:** Eigenvalue, eigenvector, matrix-valued function, matrix pencil

**AMS subject classifications.** 65F15; 15A18

### Nullity of a Graph

**S. Arumugam**

Kalasalingam Academy of Research and Education, Krishnankoil, Tamilnadu,  
India

s.arumugam.klu@gmail.com

#### Abstract

Let  $G$  be a simple graph of order  $n$ . The nullity  $\eta(G)$  of  $G$  is the algebraic multiplicity of 0 as an eigen value of the adjacency matrix of  $G$ . A graph  $G$  with  $\eta(G) > 0$  is called a singular graph. For a bipartite graph  $G$  (corresponding to an alterant hydrocarbon) if  $\eta(G) > 0$ , then it indicates that the molecule which such a graph represents is unstable. A characterization of graphs for which  $\eta(G) > 0$  still remains open. In this talk we present a survey of results on  $\eta(G)$  and indicate open problems and directions for further research.



# On the Developments of Generalized Inverses

**Adi Ben-Israel**

Rutger University, New Jersey. adi.benisrael@gmail.com

## Abstract

In the present talk, I shall review on the initial developments of generalized and its applications. .

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# Completely Positive Matrices

**Abraham Berman**

Technion-Israel Institute of Technology, Israel  
berman@technion.ac.il

## Abstract

Let  $S$  be a set of nonnegative numbers. A matrix  $A$  is *completely positive over  $S$*  if it can be decomposed as  $A = BB^T$ , where the entries of  $B$  are in  $S$ . The talk is a survey of results on completely positivity over  $S$  in the cases  $S = R$ ,  $S = Q$ ,  $S = Z$  and  $S = \{0, 1\}$

**Keywords:** Non negative matrix, Completely positive matrix

**AMS subject classifications.** 15A23; 15B48

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# On commuting isometries

**Tirthankar Bhattacharyya**

Indian Institute of Science, Bengaluru  
tirtha@iisc.ac.in

## Abstract

The study of a pair  $(V_1, V_2)$  of commuting isometries is a classical theme. In this talk, we shall try to shine new light on it by using the defect operator and the Berger-Coburn-Lebow Theorem. Especially interesting is the case when the defect operator is negative. The aim is to compute the joint spectrum and decide on the stage of the Koszul complex where the exactness breaks. A pair of operator valued functions  $(\varphi_1, \varphi_2)$  is canonically associated by Berger, Coburn and Lebow with  $(V_1, V_2)$ . If  $(V_1, V_2)$  is a pure pair, then in many cases,  $\sigma(V_1, V_2) = \overline{\cup_{z \in D} \sigma(\varphi_1(z), \varphi_2(z))}$ . It has been known that the fundamental pair of commuting isometries with positive defect is the pair of multiplication operators by the coordinate functions on the Hardy space of the bidisc. In the negative defect case, the modified bi-shift studied in the literature forms a canonical example. A functional model as well as the Berger-Coburn-Lebow triple are found in this case.

# Spectral measures of empirical autocovariance matrices of high dimensional stationary processes

**Arup Bose**

Indian Statistical Institute, Kolkata, India  
bosearu@gmail.com

## Abstract

We begin with a brief glimpse at the bulk distribution results for some popular random matrices. Then we consider the empirical autocovariance matrix at a given non-zero time lag based on observations from a multivariate complex Gaussian stationary time series. We study the behavior of their spectral measures in the asymptotic regime where the time series dimension and the observation window length both grow to infinity, and at the same rate. Following a general framework in the field of the spectral analysis of large random non-Hermitian matrices, at first the probabilistic behavior of the small singular values of the shifted versions of the autocovariance matrix are obtained. This is then used to infer about the large sample behaviour of the empirical spectral measure of the autocovariance matrices at any lag. Matrix orthogonal polynomials on the unit circle play a crucial role in our study.

# On the minimum number of distinct eigenvalues of a graph

Shaun M. Fallat

University of Regina, Canada  
Shaun.Fallat@uregina.ca

## Abstract

The inverse eigenvalue problem for graphs has become a central research enterprise for the past 30 years with many exciting and interesting advances and related applications. One aspect of this problem is studying the fewest possible eigenvalues associated with a given graph.

This presentation will provide a broad survey of current results involving this curious and important graph parameter, which is typically labeled as  $q(G)$  for a given graph  $G$ . In particular, I will discuss the cases when  $q(G) \geq |G| - 1$  and when  $q(G) = 2$ , along with other intriguing results on  $q$  for various families of graphs. I will also touch upon recent variants of the parameter  $q$  connected with certain ‘strong’ properties of matrices.

**Keywords:** eigenvalues of graphs, inverse eigenvalue problem, graphs, orthogonal matrices, threshold graphs

**AMS subject classifications.** 05C50, 15A18

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# Algebraic connectivity of graphs constructed with given Blocks

Debajit Kalita<sup>1</sup> and Kuldeep Sarma<sup>2</sup>

<sup>1,2</sup>Tezpur University, Napam, Tezpur, Assam, India

<sup>1</sup>k.debajit@gmail.com

<sup>2</sup>kuldeep.sarma65@gmail.com

## Abstract

A block is said to be pendant if it has exactly one point of articulation. Suppose that we are given a collection blocks and we construct all possible connected graphs with these blocks keeping the number of pendant blocks fixed. In this talk, we describe the structure of the graphs that minimize the algebraic connectivity among all such graphs. As an application, we conclude that over all such graphs made with the given blocks, the algebraic connectivity is minimum for a graph whose block structure is a path.

**Keywords:** Algebraic Connectivity, Block, Laplacian Matrix, Fiedler Vector

**AMS subject classifications.** 05C50, 15A18, 15A09, 05C20, 05C22.

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## Some generalizations of $P$ -property for operators on the space of real symmetric matrices

**K.C. Sivakumar**

Department of Mathematics, Indian Institute of Technology Madras, India  
kcskumar@iitm.ac.in>

### Abstract

Let  $\mathcal{S}^n$  denote the space of real symmetric matrices of order  $n \times n$ . A linear operator  $T$  on  $\mathcal{S}^n$  is said to be a  $P$ -operator, if the following implication holds for every  $X \in \mathcal{S}^n$ :

$$XT(X) = T(X)X \leq 0 \implies X = 0,$$

where, for  $Y \in \mathcal{S}^n$  we use  $Y \geq 0$  to denote that  $Y$  is positive semidefinite, i.e.,  $u^t Y u \geq 0$  for all  $u \in \mathbb{R}^n$ . Also,  $-Y \geq 0$  is denoted by  $Y \leq 0$ .

Let  $A \in \mathbb{R}^{n \times n}$  be fixed. The following three maps have been well studied, in the context of the semidefinite linear complementarity problems (SDLCP).

The Lyapunov transformation  $L_A : \mathcal{S}^n \rightarrow \mathcal{S}^n$ , is defined by

$$L_A(X) = AX + XA^T,$$

the Stein transformation  $S_A : \mathcal{S}^n \rightarrow \mathcal{S}^n$ , by

$$S_A(X) = X - AXA^T$$

and the multiplicative transformation  $M_A : \mathcal{S}^n \rightarrow \mathcal{S}^n$ , by

$$M_A(X) = AXA^T.$$

In [2, Theorem 5], it is proved that  $L_A$  has the  $P$ -property iff  $A$  is positive stable (meaning, all the eigenvalues of  $A$  have positive real part). Also, in [3, Theorem 6], it is shown that  $S_A$  has  $P$ -property iff  $A$  is Schur stable (meaning, all the eigenvalues of  $A$  lie in the open unit disk). Finally, it is shown in [1, Theorem 17] that  $M_A$  has  $P$ -property iff  $A$  is either positive definite or negative definite. It is important to observe that these results have some interesting and nontrivial implications to SDLCP.

A linear operator  $T$  on  $\mathcal{S}^n$  is a  $P_\#$ -operator (introduced in [5], Definition 1.3), if for every  $X \in \mathcal{S}^n$  we have:

$$X \in R(T), XT(X) = T(X)X \leq 0 \implies X = 0.$$

For  $x, y \in \mathcal{S}^n$ , define  $X \circ Y := \frac{1}{2}(XY + YX)$ . We say that the operator  $T$  has the Jordan- $P$ -property if

$$X \circ L(X) \leq 0 \implies X = 0.$$

$T$  is said to possess the Jordan-w- $P$ -property if

$$X \circ L(X) \leq 0 \implies L(X) = 0.$$

Finally,  $T$  is said to satisfy the w- $P$ -property if

$$XL(X) = L(X)X, X \circ L(X) \leq 0 \implies L(X) = 0.$$

The three notions given in the previous para were proposed in [6]. The results stated earlier for the Lyapunov, Stein and the multiplicative transformations, have been shown to have analogues for operators satisfying any of the four latter properties defined above (see, for instance [4] and [6]).

In this talk, first we give a brief survey. We then present some new relationships between operators belonging to these classes.

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# Blowup-polynomials of graphs

Apoorva Khare

Indian Institute of Science; Analysis & Probability Research Group, Bangalore  
khare@iisc.ac.in

## Abstract

Given a finite simple connected graph  $G = (V, E)$ , we introduce a novel invariant which we call its *blowup-polynomial*  $p_G((n_v)_{v \in V})$ . To do so, we compute the determinant of the distance matrix of the graph blowup, obtained by taking  $n_v$  copies of the vertex  $v$ , and remove an exponential factor. First: we show that as a function of the sizes  $n_v$ ,  $p_G$  is a polynomial, is multi-affine, and is real-stable. Second: we show that the multivariate polynomial  $p_G$  is intimately related to the characteristic polynomial  $q_G$  of the distance matrix  $D_G$ , and that it fully recovers  $G$  whereas  $q_G$  does not. Third: we obtain a novel characterization of the complete multi-partite graphs, as precisely those whose “homogenized” blowup-polynomials are Lorentzian/strongly Rayleigh.

**Keywords:** Characteristic polynomial, Multivariate polynomial, Multi partite graph,

**AMS subject classifications.** 05C50, 05C31

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## Disease Invasibility on Networks: An Opportunity for Matrix Theory

Stephen J Kirkland

University of Manitoba, Canada  
stephen.kirkland@umanitoba.ca

**Hans Schneider Lecture sponsored by International Linear Algebra Society**

## Abstract

We consider the spread of an infectious disease, modeled as a network of patches (to reflect a heterogeneous environment) with movement between patches. The invasibility of the disease can be measured by the network reproduction number  $R_0$ , which turns out to be the spectral radius of a certain nonnegative matrix. In this talk we work with an approximation to  $R_0$ ; that approximation applies in the case that the time scale of movement is substantially larger than the time scale of the disease dynamics. We investigate how perturbations in the network structure affect the value of  $R_0$ , and discuss the changes to the network that yield the largest decrease in  $R_0$ . Throughout we use techniques from matrix analysis, and adopt perspectives from combinatorics.

**Acknowledgment.** This is a joint work with Zhisheng Shuai, Pauline van den Driessche and Xueying Wang.

# Matrices generated by conjugates of any matrix

**Surender Kumar Jain<sup>1</sup> and Andre Leroy<sup>2</sup>**

<sup>1</sup>Ohio University, United States of America  
jain@ohio.edu

<sup>2</sup>Université d'Artois, France  
andreleroy55@gmail.com

## Abstract

By using semigroup techniques, J. Araújo and F.C. Silva proved that a matrix  $B$  with coefficients in a division ring  $D$  is a product of conjugates of any matrix  $A$  with  $\text{rank}(B)$  smaller or equal to  $\text{rank}(A)$ . We prove this result over an algebraically closed field, in an elementary way suitable even for undergraduate students.

# On Characterization of $Q$ -matrices in Linear Complementarity Problem

**N. Eagambaram**

Former DDG, Indian Statistical Services, Govt of India  
ekambaramn@gmail.com

## Abstract

Given a matrix  $M$  and a vector  $q$ , the Linear Complementarity Problem ( $\text{LCP}(M, q)$ ) is to find a solution  $z$  to

$$Mz + q \geq 0, z^T(Mz + q) = 0, z \geq 0$$

Lemke's Algorithm applied to  $\text{LCP}(M, q + \theta p)$  fetches solutions to the LCP for all  $\theta \in [\theta_0, \infty)$  for some  $\theta_0$  and  $p > 0$ . We express  $\theta$  as a function of time index  $t$ . Solution  $z$  obtained from Lemke's Algorithm is a continuous function of  $t$ . A variant of Lemke's Algorithm is proposed using the continuity property. Some results on characterization of  $Q$ -matrices are obtained.

# Complexes from graphs

**Nandini Nilakantan**

Indian Institute of Technology Kanpur, India  
nandini@iitk.ac.in

## Abstract

Abstract: Lovasz, in his proof of the Kneser Conjecture had constructed a simplicial complex corresponding to a graph. In this talk I will discuss the neighborhood complex and briefly highlight the proof of the Conjecture. I will also talk about the Hom Complex associated to a pair of graphs.

# Generalized Euclidean distance matrices

**Balaji Ramamurthy**

Indian Institute of Technology Madras, India  
balaji5@iitm.ac.in

## Abstract

Let  $F = [f_{ij}]$  be an  $n \times n$  symmetric matrix. Define  $d_{ij} := f_{ii} + f_{jj} - 2f_{ij}$ . Now, the matrix  $D = [d_{ij}]$  is called a Euclidean distance matrix (EDM). EDMs have several interesting properties. We introduce a simple generalization of a EDM. Fix  $a, b > 0$ . Define

$$E = [a^2 g_{ii} + b^2 g_{jj} - 2ab g_{ij}],$$

where  $G = [g_{ij}]$  is a positive semidefinite matrix such that  $g\mathbf{1} = 0$ , where  $\mathbf{1}$  is the vector of all ones in  $\mathbb{R}^n$ . We call  $E$  a generalized EDM. Despite  $E$  being a non-symmetric matrix, many of the interesting properties of a EDM can be extended to a generalized EDM. All these properties will be discussed the talk.

**Keywords:** Euclidean Distance Matrix, Symmetric Matrix, Matrices and Graphs

# Orthogonality for bi-adjoints

**T.S.S.R.K. Rao**

Manipal Academy of Higher Education, Manipal, India  
tssrkr Rao@gmail.com

## Abstract

An important direction of investigation in Operator theory of Banach spaces, is, to perform standard Banach space theoretic operations on spaces of operators and ask if the resulting object is again a space of operators (possibly between different Banach spaces). In this talk, we tackle this problem for the well-known geometric operation, Birkhoff-James orthogonality. For non-reflexive Banach spaces,  $X, Y$ , for a closed subspace  $M$  of operators, we investigate Birkhoff-James orthogonality of an operator  $T$  to  $M$  with that of  $T^{**}$  with an appropriate subspace of  $M^{**}$ .

# On the Ryser Designs and Ryser Conjecture

**Sharad S. Sane**

Chennai Mathematical Institute Siruseri, Chennai-603103  
sharadsane@gmail.com

## Abstract

Ryser design can be described as a binary square matrix  $A$  for which  $A^t A$  equals the sum of a diagonal matrix and a multiple of  $J$ , the all 1 matrix with the additional requirement that  $A$  has at least two different column sums (equivalently two different row sums). This talk will discuss two different proofs of the Ryser-Woodall theorem (on Ryser designs), the first of which is by Ryser that is combinatorial in nature and the second by Ionin and M.S. Shrikhande that is linear algebraic. This talk will give an account of the known results on Ryser's conjecture and some recent work in this direction.



# Hack's Law in a Drainage Network Model: A Brownian Web Approach

**Anish Sarkar**

Indian Statistical Institute, Delhi Centre, India  
anish.sarkar@gmail.com

## Abstract

Hack (1957) while studying the drainage system in the Shenandoah valley and the adjacent mountains of Virginia, observed a power law relation  $l \sim a^{0.6}$  between the length  $l$  of a stream from its source to a divide and the area  $a$  of the basin that collects the precipitation contributing to the stream as tributaries. We study the tributary structure of Howard's drainage network model of headward growth and branching studied earlier by Gangopadhyay, Roy and Sarkar (2004). We show that the exponent of Hack's law is  $2/3$  for Howard's model. Our study is based on a scaling of the process whereby the limit of the watershed area of a stream is area of a Brownian excursion process. To obtain this, we define a dual of the model and show that under diffusive scaling, both the original network and its dual converge jointly to the standard Brownian web and its dual.

**Acknowledgment.** This is joint work with Rahul Roy and Kumarjit Saha.

# Inequalities among Two rowed Immanants of $q$ -Laplacian of trees and Odd height peaks in Generalized Dyck Paths

**Mukesh Kumar Nagar<sup>1</sup>, Arbind Kumar Lal<sup>2</sup>, and  
Sivaramakrishnan Sivasubramanian<sup>3</sup>**

<sup>1,2</sup>Department of Mathematics and Statistics, IIT Kanpur India  
<sup>1</sup>mukesh.kr.nagar@gmail.com

<sup>3</sup>Department of Mathematics, IIT Bombay, India  
siva.krishnan@gmail.com

## Abstract

Let  $T$  be a tree on  $n$  vertices and let  $\mathcal{L}_q^T$  be the  $q$ -analogue of its Laplacian. For a partition  $\lambda \vdash n$ , let the normalized immanant of  $\mathcal{L}_q^T$  indexed by  $\lambda$  be denoted as  $\overline{\text{Imm}}_\lambda(\mathcal{L}_q^T)$ . A string of inequalities among  $\overline{\text{Imm}}_\lambda(\mathcal{L}_q^T)$  is known when  $\lambda$  varies over hook partitions of  $n$  as the size of the first part of  $\lambda$  decreases. In this work, we show a similar sequence of inequalities when  $\lambda$  varies over two row partitions of  $n$  as the size of the first part of  $\lambda$  decreases. Our main lemma is an identity involving binomial coefficients and irreducible character values of  $\mathfrak{S}$  indexed by two row partitions.

Our proof can be interpreted using the combinatorics of Riordan paths and our main lemma admits a nice probabilistic interpretation involving peaks at odd heights in generalized Dyck paths or equivalently involving special descents in Standard Young Tableaux with two rows. As a corollary, we also get inequalities between  $\overline{\text{Imm}}_{\lambda_1}(\mathcal{L}_q^{T_1})$  and  $\overline{\text{Imm}}_{\lambda_2}(\mathcal{L}_q^{T_2})$  when  $T_1$  and  $T_2$  are comparable trees in the  $\text{GTS}_n$  poset and when  $\lambda_1$  and  $\lambda_2$  are both two rowed partitions of  $n$ , with  $\lambda_1$  having a larger first part than  $\lambda_2$ .

# **$q$ -Analogues of the Kac matrix and the adjacency matrix of the $n$ -cube**

**Subhajit Ghosh<sup>1</sup> and Murali K. Srinivasan<sup>2</sup>**

<sup>1</sup>Bar-Ilan University, Ramat-Gan, Israel  
gsubhajit@alum.iisc.ac.in

<sup>2</sup>Indian Institute of Technology Bombay, Mumbai, India  
murali.k.srinivasan@gmail.com

## **Abstract**

The adjacency matrix of the  $n$ -cube and the closely related tridiagonal matrix of Mark Kac have an elegant spectral theory that arise in a variety of applications. This paper defines  $q$ -analogues of these two matrices and studies their spectral theory.

We consider two applications: a  $q$ -analogue of the random walk on the  $n$ -cube and a  $q$ -analogue of the product formula for the number of rooted spanning trees of the  $n$ -cube.

## **Further Introduction to Philatelic Lattice Grids with Four-Sided Stamps (First international minisymposium on mathematical philately)**

**Garry Ka Lok Chu<sup>1</sup>, Simo Puntanen<sup>2</sup>, and George P.H. Styan<sup>3</sup>**

<sup>1</sup>Dawson College, Montreal, Quebec, Canada  
gchu@dawsoncollege.qc.ca

<sup>2</sup>Tampere University, Finland  
simo.puntanen@tuni.fi

<sup>3</sup>McGill University, Montreal, Quebec, Canada  
geostyan@gmail.com

## **Abstract**

SLIDES ARE APPENDED IN APPENDIX.

## ICLAA: Contributory Talks

# Chain Graph Sequences and Laplacian Spectra of Chain graphs

Abdullah Alazemi<sup>1</sup>, Milica Andjelić<sup>2</sup>, Kinkar Ch. Das<sup>3</sup>, and Carlos  
M. da Fonseca<sup>4</sup>

<sup>1,2</sup>Kuwait University, Kuwait

<sup>1</sup>abdullah.alazemi@ku.edu.kw

<sup>2</sup>milica.andelic@ku.edu.kw

<sup>3</sup>Sungkyunkwan University, Korea

kinkardas2003@gmail.com

<sup>4</sup>Kuwait College of Science and Technology, Kuwait

c.dafonseca@kcst.edu.kw

### Abstract

A graph is called a chain graph if it is bipartite and the neighbourhoods of the vertices in each colour class form a chain with respect to inclusion. We study the relation between the degree sequences and the Laplacian spectra of chain graphs. We provide explicit formulae for the Laplacian characteristic polynomial of a chain graph, and certain properties of the Laplacian spectrum that can be deduced from its degree sequence. Possible applications in control theory are outlined.

**Keywords:** chain graphs, graphic sequences, Laplacian spectrum, tridiagonal matrices, control theory

**AMS subject classifications.** 05C50; 93B05

**Acknowledgment.** The authors thanks the support of the Research Sector, Kuwait University, grant no. SM04/18.

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# Exponents of Primitive Symmetric Companion Matrices

**Satyanarayana Reddy Arikatla**

Shiv Nadar University, Uttar Pradesh, India  
satya.a@snu.edu.in

## Abstract

A *symmetric companion matrix*, we mean a matrix of the form  $A + A^T$ , where  $A$  is a companion matrix all of whose entries are in  $\{0, 1\}$  and  $A^T$  is the transpose of  $A$ . In this work, we found the total number of primitive and imprimitive symmetric companion matrices. We found formulas to compute the exponent of every primitive symmetric companion matrix. Hence the exponent set for the class of primitive symmetric companion matrices is completely characterized. We also obtain the number of primitive symmetric companion matrices with a given exponent for certain cases.

# Some Additive Inverse Eigenvalue Problems for Matrices whose Graphs are Trees

**Bijoya Bardhan**

National Institute of Technology Silchar, India  
bijoyab09@gmail.com

## Abstract

Inverse eigenvalue problem of a graph  $G$  refers to the problem of reconstructing a matrix from the prescribed eigendatas, where the off-diagonal entries of the matrix are governed by the adjacency of the vertices in  $G$ . In this paper, we have discussed two additive inverse eigenvalue problems. For a given graph of a tree  $G$  and a diagonal matrix  $D$ , the first problem is to construct a matrix  $A$  corresponding to  $G$ , when largest eigenvalues of each of the principal submatrices of the matrix  $A + D$  are given. And the second problem is to construct one matrix  $A$ , whose all diagonal elements are zero from the given eigeninformation of  $A + D$ . Recurrence relation between the characteristics polynomials of the leading principal submatrices of the  $A + D$  are used to get the conditions of the solvability of the problems.

**Keywords:** additive inverse eigenvalue problem, matrix of a tree

# A numerical solution for nonlinear Klein Gordon equation using domination polynomials

Wilma Laveena D'Souza<sup>1</sup>, and Chaitra V<sup>2</sup>

<sup>1</sup>St.Joseph's College(Autonomous), Bengaluru, India  
wilma.dsouza@sjc.ac.in

<sup>2</sup>B.M.S.College of Engineering, Bengaluru, India  
chaitrav.maths@bmsce.ac.in

## Abstract

We are proposing a numerical technique to solve nonlinear Klein Gordon equation. We have generated an operational matrix of integration of domination polynomials of complete graphs and solved the partial differential equation. The author off[1] generated a new operational matrix of integration using Clique polynomials of complete graphs and also introduced a new numerical technique to solve nonlinear Klein–Gordon equation. These equations describe a variety of physical phenomena. The author obtained an approximate solution for the nonlinear Klein–Gordon equation by transforming a system of nonlinear algebraic equations. We have modified this technique using domination polynomials.

**Keywords:** klein Gordan equation, domination polynomial, operational matrix

**AMS subject classifications.** 05C69

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# How perturbations propagate along the solutions of linear ordinary differential equations: a relative error analysis

Asma Farooq<sup>1</sup> and Stefano Maset<sup>2</sup>

University of Trieste, Italy

<sup>1</sup> Asma.farooq@phd.units.it

<sup>2</sup>maset@units.it

## Abstract

In this talk, we are going to present how perturbations in the co-efficient matrix  $A$  propagate along the solutions of  $n$ -dimensional linear ordinary differential equations

$$\begin{cases} y'(t) = Ay(t), & t \geq 0, \\ y(0) = y_0. \end{cases}$$

In other words we are considering the conditioning of the problem

$$(y_0, A) \mapsto e^{tA} y_0$$

and an asymptotic analysis of condition numbers, as  $t \rightarrow +\infty$ , will be given. The analysis is accomplished for the case where  $A$  is normal matrix.

We remark that conditioning of such problems attained less attention in literature. At the best of our knowledge there are only two papers [1] and [2] on this topic. These papers present computational aspects of the condition number. On the other hand our study is more on theoretical aspects of the condition number. It studies how this condition number depends on the time  $t$  and the initial data  $y_0$ . Also the asymptotic behavior of condition number as  $t \rightarrow +\infty$  is part of our study.

**Keywords:** Ordinary differential equation, Condition number, Normal matrix,

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# Impulse Controllability for Rectangular Descriptor Systems

**Mahendra Kumar Gupta<sup>1</sup> and Abhinav Kumar<sup>2</sup>**

National Institute of Technology Jamshedpur, India

<sup>1</sup>mahendra14389@gmail.com

<sup>2</sup>kabhinav774@gmail.com

## Abstract

In this work, the following form of descriptor system is considered.

$$E\dot{x} = Ax + Bu, \quad Ex(0_-) = Ex_0,$$

where  $x \in \mathbb{R}^n$ ,  $u \in \mathbb{R}^k$ , are the state vector and the input vector, respectively.  $E \in \mathbb{R}^{m \times n}$ ,  $A \in \mathbb{R}^{m \times n}$ , and  $B \in \mathbb{R}^{m \times k}$  are known constant matrices. System (??) is called regular descriptor system if  $m = n$  and  $\exists \lambda \in \mathbb{C}$  such that  $\det(\lambda E - A) \neq 0$ . Descriptor systems are also known as Differential-Algebraic Equations (DAEs).

In regular descriptor systems, terms impulse freeness and impulse controllability are well defined. For rectangular linear descriptor systems, however, there are various notions for impulse freeness and impulse controllability [1]. In this research, conditions for free of impulse, and no impulsive modes admitting arbitrary initial conditions are given in the form of Kronecker canonical form (KCF). Other equivalent conditions are also being investigated. Further, conditions for impulse controllability are explored, and a state feedback  $K$  is designed such that the closed-loop system  $(E, A + BK)$  has no impulsive modes and admits arbitrary initial conditions. Physical and academic examples illustrate the theoretical outcomes.

**Keywords:** Differential-Algebraic Equations, Impulse Controllability, Kronecker form

**AMS subject classifications.** 93B10; 93B05; 93B11; 93B52

**Acknowledgment.** The work is supported by Science and Engineering Research Board, New Delhi, under Grant No. SRG/2019/000451.

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# On Algebraic and Geometric Properties of $\mathcal{L}_+^n$ -semipositive Matrices

Aritra Narayana Hisabia<sup>1</sup>, and Manideepa Saha<sup>2</sup>

<sup>1,2</sup>National Institute of Technology Meghalaya  
Shillong, India

ahisabia@nitm.ac.in

<sup>2</sup> manideepa.saha@nitm.ac.in

## Abstract

For a proper cone  $K$  in  $\mathbb{R}^n$ , a square matrix  $A$  is said to be  $K$ -semipositive if there exists  $x \in K$  such that  $Ax \in \text{int}(K)$ . Semipositivity of matrices, taking  $K = \mathbb{R}_+^n$  (known as semipositive matrices), are well studied and has been found useful in various problems like (cone) linear complementarity problems (LCP), cone programming, stability of matrices, game theory, optimization problems etc. One of the simplest cones used in conic programming is the Lorentz cone  $\mathcal{L}_+^n$ , or also known as second order cone, or ice-cream cone, which is defined as

$$L_+^n = \left\{ x \in \mathbb{R}_+^n : x_n \geq 0, \sum_{i=1}^{n-1} x_i^2 \leq x_n^2 \right\}$$

The aim of the paper is to study a few algebraic and geometric properties  $\mathcal{L}_+^n$ -semipositive matrices. More specifically, for a given  $\mathcal{L}_+^n$ - semipositive matrix  $A$ , we study geometric properties of the two cones  $\mathcal{S}_{A, \mathcal{L}_+^n} = \{x : Ax \in \mathcal{L}_+^n\}$ , and  $\mathcal{K}_{A, \mathcal{L}_+^n} = \{x \in \mathcal{L}_+^n : Ax \in \mathcal{L}_+^n\}$ . Selfduality of  $\mathcal{L}_+^n$ -semipositive matrices are discussed.

Furthermore, we explain  $\mathcal{L}_+^n$ -semipositivity of diagonal and orthogonal matrices. In particular, we prove that  $\mathcal{L}_+^n$ -semipositivity of a diagonal and orthogonal matrix depends only on the sign of one entry of the given matrix.

**Keywords:** Lorentz Cone, Proper cone, Semipositive, Orthogonal matrix

**AMS subject classifications.** 15B10; 15B48; 52A20

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## On the construction of cospectral graphs

**M. Rajesh Kannan<sup>1</sup> and Shivaramkrishna Pragada<sup>2</sup>**

<sup>1</sup>Department of Mathematics, IIT Kharagpur  
Kharagpur, India  
rajeshkannan1.m@gmail.com

<sup>2</sup>Department of Aerospace Engineering, IIT Kharagpur  
Kharagpur, India  
rajeshkannan1.m@gmail.com

### Abstract

In [Steve Butler. A note about cospectral graphs for the adjacency and normalized Laplacian matrices. Linear Multilinear Algebra, 58(3-4):387-390, 2010.], Butler constructed a family of bipartite graphs, which are cospectral for both the adjacency and the normalized Laplacian matrices. In this article, we extend this construction for generating larger classes of bipartite graphs, which are cospectral for both the adjacency and the normalized Laplacian matrices. Also, we provide a couple of constructions of non-bipartite graphs, which are cospectral for the adjacency matrices but not necessarily for the normalized Laplacian matrices.

**Keywords:** Adjacency matrix, Normalized Laplacian matrix, Cospectral graphs.

**AMS subject classifications.** 05C50.

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## Condition eigenvectors of bounded linear operators

**Krishna Kumar G**

University of Kerala, Kariavattom Campus  
Thiruvananthapuram, Kerala, India  
krishna.math@keralauniversity.ac.in

### Abstract

The condition spectrum is an essential generalization of the spectrum. We define the eigenvector corresponding to condition spectral value of a matrix, bounded operator on a separable Hilbert space. The eigenvector corresponding to a condition spectral value of a matrix or an operator is called condition eigenvector. We develop various properties of condition eigenvectors of an operator. A sufficient condition for an operator to possess an almost invariant



subset is given using the condition eigenvector. We find common condition eigenvectors for almost commuting matrices and various classes of almost commuting pairs of operators.

**Keywords:** Condition spectrum, almost commutative, almost invariant, condition eigenvector

**AMS subject classifications.** Primary 47A15, 47A10; Secondary 47A55, 15A09, 15A18

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# On the spectral radius and the energy of eccentricity matrix of a graph

**Iswar Mahato<sup>1</sup>, R. Gurusamy<sup>2</sup>, M. Rajesh Kannan<sup>3</sup>, and S. Arockiaraj<sup>4</sup>**

<sup>1,3</sup>Indian Institute of Technology Kharagpur, Kharagpur, India

<sup>1</sup>iswarmahato02@gmail.com, <sup>3</sup>rajeshkannan1.m@gmail.com

<sup>2</sup>Mepco Schlenk Engineering College, Sivakasi, India

sahama2010@gmail.com

<sup>4</sup>Government Arts and Science College, Sivakasi, India

psarockiaraj@gmail.com

## Abstract

Let  $G$  be a finite, simple and connected graph with the vertex set  $V(G)$  and the edge set  $E(G)$ . The distance  $d(u, v)$  between the vertices  $u$  and  $v$  is the minimum length of the paths between them in  $G$  and the eccentricity  $e(u)$  of the vertex  $u$  is defined as  $e(u) = \max\{d(u, v) : v \in V(G)\}$ . The distance matrix of  $G$ , denoted by  $D(G)$ , is an  $n \times n$  symmetric matrix with  $(u, v)$ -entry is equal to  $d(u, v)$ . The eccentricity matrix  $\varepsilon(G)$  of graph  $G$  is obtained from the distance matrix

by retaining the largest distance in each row and each column, and setting the remaining entries as 0. To be more precise, the  $(u, v)$ -entry of  $\varepsilon(G)$  is defined as

$$\varepsilon(G)_{uv} = \begin{cases} d(u, v) & \text{if } d(u, v) = \min\{e(u), e(v)\}, \\ 0 & \text{otherwise.} \end{cases}$$

The largest eigenvalue of  $\varepsilon(G)$  is called the  $\varepsilon$ -spectral radius of  $G$  and is denoted by  $\rho(\varepsilon(G))$ . The eccentricity energy (or the  $\varepsilon$ -energy) of  $G$  is the sum of the absolute values of the eigenvalues of  $\varepsilon(G)$ . In this paper, we establish a necessary and sufficient condition for the eccentricity matrix of a tree to be invertible. Also, we obtain some bounds for the  $\varepsilon$ -spectral radius and characterize the extreme graphs. Two graphs are said to be  $\varepsilon$ -equienergetic if they have the same  $\varepsilon$ -energy. For any  $n \geq 5$ , we construct a pair of  $\varepsilon$ -equienergetic graphs on  $n$  vertices, which are not  $\varepsilon$ -cospectral.

**Keywords:** Eccentricity matrix,  $\varepsilon$ -spectral radius,  $\varepsilon$ -energy, Tree.

**AMS subject classifications.** 05C12; 05C50

## Interval eigenvalue problems and its applications

**Suman Maiti<sup>1</sup> and Snehashish Chakraverty<sup>2</sup>**

National Institute of Technology Rourkela, India

<sup>1</sup>smaiti2711@gmail.com

<sup>2</sup>sne\_chak@yahoo.com

### Abstract

We can enclose tiny errors by narrow closed and bounded intervals, whereas wide closed and bounded intervals are employed in guaranteed pathing for moving objects. Problems with both types of intervals can be dealt with by interval arithmetic. In problems with uncertainty, interval matrices appear, and consequently, we have to solve interval eigenvalue problems. Computing exact eigenvalues of complex interval matrices is an NP-hard problem. Our focus will be on the reduction of the overestimation of the eigenvalue bounds. We have derived theorems to obtain tighter eigenvalue bounds for complex interval matrices. These bounds are compared with Rohn [3], Hertz [1], Hladik [2] methods to see the tightness of the bounds. Eigenvalue bounds for symmetric interval matrices [4] are utilized to find these bounds. The sharper eigenvalue bounds are used for determining the stability of uncertain dynamical systems, also may be considered as the initial bounds for iterative methods for complex interval eigenvalue problems [5].

**Keywords:** interval matrices, interval eigenvalues, eigenvalue bounds

**AMS subject classifications.** 65G40; 15A18; 15A42

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## On the Seidel matrix of threshold graphs

Santanu Mandal<sup>1</sup> and Ranjit Mehatari<sup>2</sup>

<sup>1,2</sup>Department of Mathematics, NIT Rourkela, India

<sup>1</sup>santanu.vumath@gmail.com

<sup>2</sup>ranjitmehatari@gmail.com

### Abstract

Threshold graph has an important role in graph theory and several applied areas such as computer science, scheduling theory etc. Here threshold graphs with its binary string representation are considered. Let  $G$  be a connected threshold graph with adjacency and Seidel matrix  $A$  and  $S$  respectively. Then  $S = J - I - 2A$ . We study the spectral properties of  $S$ . A recurrence formula for characteristic polynomial of  $S$ , multiplicity of the eigenvalues  $\pm 1$  of  $S$  and eigenvalue bounds are obtained. Characterisation of threshold graphs with at most five distinct Seidel eigenvalue is shown also. We obtain several bounds on Seidel energy of  $G$ . It is shown that our bound is better than Haemers' bound in practical. Finally, we prove a very uncommon result for threshold graphs: two threshold graphs may be cospectral on Seidel matrix. Here we define a class of such threshold graphs.

**Keywords:** threshold graph, Seidel matrix, quotient matrix, Seidel energy.

**AMS subject classifications.** 05C50

**Acknowledgment.:** The author Santanu Mandal thanks to University Grants Commission, India for financial support under the beneficiary code BININ01569755.

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# Partial and Circulant Partial Hadamard matrices

**Pankaj Kumar Manjhi**

Vinoba Bhawe University, Hazaribag, India  
19pankaj81@gmail.com

## Abstract

Definitions, elementary properties and conjectures based on partial and circulant partial Hadamard matrices are discussed.

**Keywords:** Hadamard Matrix, Circulant matrix, Partial Hadamard Matrix.

**AMS subject classifications.** 05B15; 05B30

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# Derivatives of symplectic eigenvalues and a Lidskii type theorem

**Tanvi Jain<sup>1</sup>, and Hemant Kumar Mishra**

<sup>1,2</sup>Indian Statistical Institute, Delhi Center, India  
<sup>1</sup>tanvi@isid.ac.in  
hemant16r@isid.ac.in

## Abstract

Given a  $2n \times 2n$  positive definite matrix  $A$  there are  $n$  positive numbers associated, called the symplectic eigenvalues of  $A$ , and a basis of  $\mathbb{R}^{2n}$  called a symplectic eigenbasis of  $A$  corresponding to these numbers. Symplectic eigenvalues have applications in different areas of mathematics and physics such as symplectic geometry, quantum information, classical mechanics. In recent years, symplectic eigenvalues have gained much attention of physicists due

to their important applications in quantum information. In this talk we shall discuss differentiability and analyticity of symplectic eigenvalues and symplectic eigenvectors, and compute their derivatives. Using our analysis we derive a symplectic analogue of the Lidskii's theorem - a result in classical linear algebra for eigenvalues - that gives a majorization relation among the symplectic eigenvalues of the sum of two matrices and that of the individual matrices.

**Keywords:** Positive definite matrix, symplectic eigenvalue, derivative, analyticity, majorisation, Lidskii's theorem.

**AMS subject classifications.** 15A48; 15A18; 15A45; 15A90; 81P45; 81S10

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## Trees with Matrix Weights: Laplacian Matrix and Characteristic-like Vertices

Swetha Ganesh<sup>1</sup> and Sumit Mohanty<sup>2</sup>

<sup>1,2</sup>Gandhi Institute of Technology and Management, Vishakapatnam, India

<sup>2</sup>smohanty@gitam.edu

### Abstract

It is known that there is an alternative characterization of characteristic vertices for trees with positive weights on their edges via Perron values and Perron branches. In this talk, we will consider trees with matrix weights and discuss the existence of characteristic-like vertices in terms of Perron values and Perron branches.

## On different classes of prime hyperideals in hyperlattices

Pallavi P<sup>1</sup>, Kuncham S.P.<sup>2</sup>, Vadiraja G.R.B.<sup>3</sup>, and Harikrishnan Panackal<sup>4</sup>

<sup>1,2,3,4</sup>Department of Mathematics, Manipal Institute of Technology  
Manipal Academy of Higher Education, Manipal, Karnataka, India

<sup>1</sup>pallavipanjr@gmail.com

<sup>2</sup>syamprasad.k@manipal.edu<sup>2</sup>

<sup>3</sup>vadiraja.bhatta@manipal.edu

<sup>4</sup>

pk.harikrishnan@manipal.edu

### Abstract

Hyperlattices are the generalizations of a classical lattice, wherein one (or both) of the classical operations is/are replaced with hyperoperation/s. Wasadikar et. al. [4, 5, 6] studied the generalizations of a prime ideal in classical lattices. In this paper, we define the classes of prime hyperideals viz., 2-absorbing hyperideals, primary hyperideals, and their weak counterparts of hyperideals in join hyperlattices. Further, we investigate the properties of annihilators of these types of hyperideals and provide few results related to triple zero of weakly 2-absorbing hyperideals.

**Keywords:** Hyperlattices, hyperideals, prime hyperideals, lattices, 2-absorbing ideals.

**AMS subject classifications.** Primary 06D35, 20N20

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## Characteristic center of a connected graph

**Kamal Lochan Patra**

National Institute of Science Education and Research, Bhubaneswar, India

<sup>1</sup>klpatra@niser.ac.in

### Abstract

In this talk, we first define the characteristic center of a connected graph and discuss some of its centrality properties. We then prove that for a connected vertex transitive graph the characteristic center consists of all the vertices of it. We also prove that for any graph  $G$ , there exists a connected supergraph  $G_{ch}$  of  $G$  with the whole vertex set of  $G$  as the characteristic center of  $G_{ch}$ .

## Dynamics of nonnegative matrices

**Sachindranath Jayaraman<sup>1</sup>, Yogesh Kumar Prajapaty<sup>2</sup>, and  
Shrihari Sridharan<sup>3</sup>**

<sup>1,2,3</sup>Indian Institute of Science and Education Thiruvananthapuram, India

<sup>1</sup>sachindranthj@iisertvm.ac.in

<sup>2</sup>prajapaty0916@iisertvm.ac.in

<sup>3</sup>shrihari@iisertvm.ac.in

### Abstract

This talk aims to understand the dynamics of products of nonnegative matrices in max algebra. We will generalize a well-known consequence of Peron Fröbenius theorem on periodic points of a nonnegative matrix in a max algebra setting. We extend the result for a finite product associated with a  $p$ -lettered word on  $N$  letters from a finite collection of nonnegative matrices, each having its maximum circuit geometric mean at most 1.

**Keywords:** Products of nonnegative matrices, Max-algebras, Boolean matrices, Fröbenius normal form of nonnegative matrices, Circuit geometric mean, Periodic points.

**AMS subject classifications.** 15A80; 15B34; 37H12.

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## On Properties of Some Neighborhood Degree-Based Topological Indices of Graphs

Shivani Rai<sup>1</sup>, Biswajit Deb<sup>2</sup>, and Sourav Mondal<sup>3</sup>

<sup>1,2</sup>Sikkim Manipal Institute of Technology, SMU, Sikkim, India

<sup>1</sup>shivanirai866@gmail.com

<sup>2</sup>biswajittalk@gmail.com

<sup>3</sup>National Institute of Technology, Durgapur, West Bengal, India

souravmath94@gmail.com

### Abstract

Numerical parameters defined for graphs taking their topology into consideration are known as topological indices and these are well studied in literature. A topological index of a molecular graph of a chemical compound encrypts the structure of a chemical compound in the form of a real number and provides an insight into its properties. In this article, the bounds of the neighborhood modified first Zagreb index ( ${}^mNM1(G)$ ), neighborhood inverse degree index ( $NID(G)$ ), neighborhood zeroth order index ( $NZ(G)$ ) and the fifth geometric-arithmetic index ( $GA_5$ ) have been obtained in terms of graph invariants like the number of vertices, number of edges, minimum and maximum degree of a graph, and other degree-based indices. Also, extremal graphs with respect to these indices have been characterized.

**Keywords:** Molecular graph, topological index, neighborhood degree.

**AMS subject classifications.** 05C35; 05C07; 05C40

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# Partial order on matrix maps over generalized rings

Tapatee Sahoo<sup>1</sup>, Harikrishnan P.K.<sup>2</sup>, Kedukodi B. S.<sup>3</sup>, and  
Kuncham S.P.<sup>4</sup>

<sup>1,2,3,4</sup>Department of Mathematics, Manipal Institute of Technology  
Manipal Academy of Higher Education, Manipal, Karnataka, India

<sup>1</sup>tapateesahoo96@gmail.com

<sup>2</sup>pk.harikrishnan@manipal.edu

<sup>3</sup>babushrisrinivas.k@manipal.edu

<sup>4</sup>syamprasad.k@manipal.edu.

## Abstract

It is well known that an  $n \times n$  matrix over a ring  $R$  is considered as an endomorphism of an abelian group  $R^n$ , which is derived from the endomorphisms of the module  $R$  over itself. In this paper, we consider  $n \times n$  matrices over a generalized ring  $R$  (namely, nearring [4]), and due to non-linearity, these matrices are viewed as maps from  $R^n$  to  $R^n$ . The Partial order on rings and matrix rings are defined in several ways [5, 2]. However, we introduce a partial order on matrix maps wherein the entries are from a nearring. We consider a matrix nearring, denoted by  $M_n(R)$  with  $1 \in R$ , a positive cone in the  $M_n(R)$  module over  $R^n$  is defined and obtained its characterization. For a convex ideal  $I$  of an  $R$  module over itself, the corresponding ideal denoted by  $\mathcal{I}^n$  in  $M_n(R)$  module  $R^n$  is obtained; and conversely, if  $\mathcal{I}$  is a convex ideal in  $M_n(R)$ -module  $R^n$ , then the ideal  $\mathcal{I}_{**}$  is convex in  $R$  over itself. We prove subsequent results and properties.

**Keywords:** Nearring,  $N$ -group, matrix nearring, partial order.

**AMS subject classifications.** 16Y30

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# Gain distance matrices for complex unit gain graphs

Aniruddha Samanta<sup>1</sup> and M. Rajesh Kannan<sup>2</sup>

Indian Institute of Technology Kharagpur, India

<sup>1</sup>aniruddha.sam@gmail.com, <sup>2</sup>rajeshkannan@maths.iitkgp.ac.in

## Abstract

A complex unit gain graph ( $\mathbb{T}$ -gain graph)  $\Phi = (G, \varphi)$  is a graph where the function  $\varphi$  assigns a unit complex number to each orientation of an edge of  $G$ , and its inverse is assigned to the opposite orientation. A  $\mathbb{T}$ -gain graph is balanced if the product of the edge gains of each oriented cycle (if any) is 1. We propose two notions of gain distance matrices  $\mathcal{D}_{\prec}^{\max}(\Phi)$  and  $\mathcal{D}_{\prec}^{\min}(\Phi)$  of a  $\mathbb{T}$ -gain graph  $\Phi$ , for any ordering ' $\prec$ ' of the vertex set. We characterize the gain graphs for which the gain distance matrices are independent of the vertex ordering. We show  $\mathcal{D}_{\prec}^{\max}(\Phi) = \mathcal{D}_{\prec}^{\min}(\Phi)$  holds for the standard ordering of the vertices if and only if the same holds for any ordering of the vertices, and we call such  $\mathbb{T}$ -gain graphs as distance compatible gain graphs. We characterize the distance compatible gain graphs whose gain distance matrices are cospectral with the distance matrix of the underlying graph. Besides, we introduce the notion of positively weighted  $\mathbb{T}$ -gain graphs and establish an equivalent condition for the balance of a  $\mathbb{T}$ -gain graph. Acharya's and Stanić's spectral criteria for balance are deduced as a consequence. Besides, we obtain some spectral characterizations for the balance of a  $\mathbb{T}$ -gain graph in terms of the gain distance matrices. Finally, we characterize the distance compatible bipartite  $\mathbb{T}$ -gain graphs. We show a  $\mathbb{T}$ -gain graph  $\Phi$  is distance compatible if and only if every block of  $\Phi$  is distance compatible.

**Keywords:** Complex unit gain graph, Signed distance matrix, Distance matrix, Adjacency matrix.

**AMS subject classifications.** 05C22(primary); 05C50; 05C35(secondary)

## Inverses of non-bipartite unicyclic graphs with a unique perfect matching

Debajit Kalita and Kuldeep Sarma

Department of Mathematical Sciences, Tezpur University

Sonitpur, Assam, India

<sup>1</sup>kdebajit@tezu.ernet.in <sup>2</sup>kuldeep.sarma65@gmail.com

## Abstract

The class of non-bipartite unicyclic graphs with a unique perfect matching, denoted by  $\mathcal{U}$ , is considered in this article. This article describes the entries of the inverse of the adjacency matrix of graphs in  $\mathcal{U}$ . It is proved that the inverse graph of a graph in  $\mathcal{U}$  is always non-bipartite. The graphs in  $\mathcal{U}$  whose inverse graph is a mixed graph are characterized in this article. Among these graphs in  $\mathcal{U}$  whose inverse is quasi-bipartite are also obtained here. Furthermore, characterizations of unicyclic graphs in  $\mathcal{U}$  possessing unicyclic and bicyclic inverses are also provided in this article.

**Keywords:** Adjacency matrix, Alternating path, Corona, Inverse Graph, Mixed graph, Unique perfect matching.

**AMS subject classifications.** 05C50, 05C05, 15A18, 05C38, 05C70, 15A09.

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## Structure of the $\mathbf{w}$ -Solution Set of the Tensor Complementarity Problem

Sonali Sharma<sup>1</sup>, K. Palpandi, and Punit Kumar Yadav

Malaviya National Institute of Technology Jaipur, India

<sup>1</sup>2019RMA9072@mnit.ac.in

### Abstract

Motivated by the study of column competent and column adequate matrices in the standard linear complementarity problems, we define column competent tensor, column adequate tensor and study the underlying properties of these tensors. We establish equivalent conditions for a column adequate tensor within the class of symmetric tensors and Z-tensors. We then introduce the  $\mathbf{w}$ -solution set for the tensor complementarity problem and examine the uniqueness and finiteness of the  $\mathbf{w}$ -solution set. At last, we define column competent property and prove that under some additional condition, column competent property gives the finiteness of the  $\mathbf{w}$ -solution set of the corresponding tensor complementarity problem.

**Keywords:** Tensors, column adequate tensors, column competent tensors, positive semidefinite tensors, complementarity problems.

**AMS subject classifications.** 15A18, 15B48, 90C33

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# More on convergence of two-stage alternating iterative scheme

Vaibhav Shekhar

National Institute of Technology Raipur, India  
vaibhavshekhar29@gmail.com

## Abstract

Migallón et al. [Adv. Eng. Softw. 41:13-21, 2010] proposed alternating two-stage methods in which the inner iterations are accomplished by an alternating method. However, the convergence theory of an alternating two-stage iteration scheme in general proper cone setting is a gap in the literature. Especially, the same study for weak regular splittings of type II (even if in standard proper cone setting, i.e.,  $(K = R_+^n)$ ), is open. In this article, convergence theory of alternating two-stage iterative schemes for  $K$ -weak regular splittings of both types in the proper cone setting are proposed. Finally, numerical computation shows that the proposed method is faster than the simple two-stage method.

**Keywords:** Linear system; Alternating two-stage iteration; Convergence;  $K$ -nonnegativity;  $K$ -monotonicity;  $K$ -weak regular splittings.

**AMS subject classifications.** 13C10; 15A09; 15A24; 15B57

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# Graph Centrality Measures and Mobile Malware Detection

Roopak S.<sup>1</sup> and Tony Thomas<sup>2</sup>

<sup>1,2</sup>School of Computer Science and Engineering  
Kerala University of Digital Sciences, Innovation and Technology  
Kerala, India

<sup>2</sup> tony.thomas@duk.ac.in<sup>2</sup>

## Abstract

Nowadays, smart phones are widely used due to the availability of many new features and services, such as games, camera, Internet, location-based services etc. apart from conventional services, such as voice calls and SMS. Among smart phones, Android smart phones are more popular due to open source operating system (OS) and application programming interface (API). The increasing popularity of Android devices has attracted the attention of malware developers, and there is a big rise in the appearance of Android malware apps every day. Many new malware applications can bypass all the current anti-malware products which rely on static analysis techniques to detect the malicious behavior. Hence, it is essential to develop innovative dynamic analysis mechanisms for Android malware detection. It is known that, the malicious behavior of malware applications can get reflected in the system call trace generated by them. Existing system call based mechanisms depend only on the features derived from the system call counts for malware detection. These system call count related features are inadequate to capture many other useful characteristics related to the system call sequence. In order to overcome this limitation, we modeled the system call trace of an application as a digraph and compute various features in the form of centrality measures such as closeness centrality, betweenness centrality, and eigen vector centrality. Then, these centrality measures are used as feature vectors for various machine learning classifiers for malware detection. From the implementation results, we found that our mechanism can detect malware apps with an accuracy of 0.97.

## **The range column sufficiency and the pseudo-SSM property of linear transformations on Euclidean Jordan Algebra**

**Punit Kumar Yadav<sup>1</sup> and K. Palpandi<sup>2</sup>**

<sup>1,2</sup>Malaviya National Institute of Technology-Jaipur, India

<sup>1</sup> 2019rma9038@mnit.ac.in

<sup>2</sup>kpalpandi.maths@mnit.ac.in

### **Abstract**

Motivated by the range column sufficient matrix, we extend the range column sufficiency on Euclidean Jordan algebra. We also generalize the concept of the pseudo-SSM property on Euclidean Jordan algebra. We study the structure of the solution set of the linear complementarity problem on the range of linear transformation with the help of these properties. We show the equivalence between the pseudo-SSM property and the range cone column sufficiency property for some special transformations. In last we characterize the range column sufficiency and the pseudo-SSM property for the relaxation transformation.

**Keywords:** Euclidean Jordan Algebra, Linear Complementarity Problem, range column sufficiency, pseudo-SSM, Group inverse.

**AMS subject classifications.** 90C33, 17C55, 15A09.

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## On Generalized Core-nilpotent Decomposition

Savitha Varkady<sup>1</sup>, Manjunatha Prasad Karantha<sup>2</sup>, and  
Umashankara Kelathaya<sup>3</sup>

<sup>1,2,3</sup>Department of Data Science, MAHE, Manipal India

<sup>1</sup> savithavarkady@yahoo.co.in

<sup>2</sup> km.prasad@manipal.edu

<sup>3</sup> usksullia@gmail.com

### Abstract

Among several decompositions of real and complex matrices, the core-nilpotent decomposition is the important one. In the present talk we will discuss several generalizations of core-nilpotent decomposition with the help of generalized inverses and minus partial order. We investigate the generalization of core-nilpotent decomposition in the case of associative rings.

**Keywords:** core-nilpotent decomposition, g inverse, associative rings

**AMS subject classifications.** 15A09, 16E50

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# **Program: ICLAA 2021**



**December 15, 2021 (Wednesday, Day 01)**

**09:15 - 09:45 - Inauguration**

**09:45 - 10:10 Tea Break & Group Photo**

**SESSION 1 (10:15 – 12:00)**

*Chair Person: Ravindra B Bapat, Indian Statistical Institute, Delhi*

10:15 – 11:20 Sharad S Sane, Chennai Mathematical Institute, Chennai  
*On the Ryser Designs and Ryser Conjecture*

11:20 – 12:00 Murali K Srinivasan, Indian Institute of Technology Bombay, Mumbai  
 *$q$ -Analogues of the Kac matrix and the adjacency matrix of the  $n$ -cube*

**SESSION 2 (12:00 – 13:00)**

*Chair Person: Sharad S Sane, Chennai Mathematical Institute, Chennai*

12:00 – 12:50 Anish Sarkar, Indian Statistical Institute, Delhi  
*Hack's Law in a Drainage Network Model: A Brownian Web Approach*

**13:00 - 14:00 Lunch Break**

**SESSION 3 (14:00 – 14:50)**

*Chair Person: Simo Puntanen, Tampere University of Technology, Finland*

14:00 – 14:50 Arup Bose, Indian Statistical Institute, Kolkata  
*Random Matrices*

**SESSION 4 (15:00 - 15:50)**

*Balaji Ramamurthy, Indian Institute of Technology Madras, India*

15:00 - 15:50 Abraham Berman, Technion-Israel Institute of Technology, Israel  
*Completely Positive Matrices*

**16:00 – 17: 20 Tea Break & Break**

**SESSION 5 (17:20 – 18:00)**

*Chair Person: Sudhakara G, Manipal Institute of Technology, MAHE, Manipal, India*

17:20 – 18:00 Rafikul Alam, Indian Institute of Technology Guwahati, India  
*A trace-moment based method for solving holomorphic eigenvalue problems*

**Mini Symposium ( SESSION 6; 18:00 – 19:30)**

**Organizer: George PH Styan, McGill University, Canada**

**18:00 – 19:30**

Speakers: Garry Ka Lok Chu, Dawson College, Canada  
Simo Puntanen, Tampere University, Finland  
George PH Styan, McGill University, Canada

Title: A further introduction to Philatelic Lattice Grids (PLGs) with four-sided stamps  
(First International Mini-Symposium on Mathematical Philately)

**December 16, 2021 (Thursday, Day 02)**

**SESSION 7 (09:00 - 10:30)**

*Chair Person: Ravindra B Bapat, Indian Statistical Institute, Delhi, India*

09:00 – 09:45 Stephen J Kirkland, University of Manitoba, Canada  
*Disease Invasibility on Networks: An Opportunity for Matrix Theory*  
(Hans Schneider ILAS Lecture)

09:50 – 10:25 T.S.S.R.K. Rao, MAHE, Manipal, India  
*Orthogonality for bi-adjoints*

10:25– 10:40 **Tea Break**

**SESSION 8 (10:40 – 12:05)**

*Chair Person: Pradeep G Bhat, Manipal Institute of Technology, MAHE, Manipal, India*

10:40 – 11:30 S Arumugam, Kalasalingam Academy of Research and Education, Tamilnadu, India  
*Nullity of a Graph*

11:30 – 12:05 Balaji Ramamurthy, Indian Institute of Technology Madras, Chennai  
*Generalized Euclidean distance matrices*

**SESSION 9 (12:10 - 13:30)**

*Chair Person: K.C. Sivakumar, Indian Institute of Technology Madras, India*

12:10 – 12:45 N Eagambaram, Former DDG, CSO, MOSPI, Govt of India  
*On Characterization of  $Q$ -matrices in Linear Complementarity Problem*

12:50 – 13:30 Debajit Kalita, Tezpur University, Assam, India  
*Algebraic connectivity of graphs constructed with given Blocks*

13:30 - 14:15 **Lunch Break**

**SESSION 10 (14:15 – 16:00) : Contributory Session**

**SESSION 10.1 A (14:15 – 15:05)**

*Chair Person: Kuncham Syam Prasad, Manipal Institute of Technology, MAHE, Manipal*

Speakers Asma Farooq, University of Trieste, Italy  
*How perturbations propagate along the solutions of linear ordinary differential equations: a relative error analysis*  
Aritra Narayana Hisabia, National Institute of Technology Meghalaya, Shillong, India  
*On Algebraic and Geometric Properties of  $\mathcal{L}_+^n$ -semipositive Matrices*  
Hemant Kumar Mishra, Indian Statistical Institute, Delhi Center, India  
*Derivatives of symplectic eigenvalues and a Lidskii type theorem*

### **SESSION 10.1 B (14:15 – 15:05)**

*Chair Person: Debajit Kalita, Tezpur University, Assam, India*

- Speakers M. Rajesh Kannan, Indian Institute of Technology Kharagpur, India  
*On the construction of cospectral graphs*  
Bijoya Bardhan, National Institute of Technology Silchar, India  
*Some Additive Inverse Eigenvalue Problems for Matrices whose Graphs are Trees*  
Mahendra Kumar Gupta, National Institute of Technology Jamshedpur, India  
*Impulse Controllability for Rectangular Descriptor Systems*

### **SESSION 10.2 A (15:10 – 16:00)**

*Chair Person: Biswajit Deb, SMIT, Sikkim Manipal University, Sikkim, India*

- Speakers Pallavi P, Manipal Institute of Technology, MAHE, Manipal, India  
*On different classes of prime hyperideals in hyperlattices*  
Tapatee Sahoo, Manipal Academy of Higher Education, Manipal, India  
*Partial order on matrix maps over generalized rings*  
Savitha V, Manipal Academy of Higher Education, Manipal, India  
*On Generalized Core-Nilpotent Decomposition*

### **SESSION 10.2 B (15:10 – 16:00)**

*Chair Person: Sachindranath Jayaraman, IISER Thiruvananthapuram, India*

- Speakers Iswar Mahato, Indian Institute of Technology Kharagpur, Kharagpur, India  
*On the spectral radius and the energy of eccentricity matrix of a graph*  
Pankaj Kumar Manjhi, Vinoba Bhave University, Hazaribag, India  
*Partial and Circulant partial Hadamard matrices*  
Santanu Mandal, Department of Mathematics, NIT Rourkela, India  
*On the Seidel matrix of threshold graphs*

**16:00 – 16:30 Tea Break**

### **SESSION 11 (16:30 – 17:50)**

*Chair Person: Sivaramakrishnan Sivasubramanian, Indian Institute of Technology Bombay, India*

- 16:30 – 17:00 S.K. Jain, Ohio University, USA and André Leroy, Université d'Artois, France  
*Matrices Generated by Conjugates*  
17:05 – 17:50 Shaun M Fallat, University of Regina, Canada  
*On the minimum number of distinct eigenvalues of a graph*

**December 17, 2021 (Friday, Day 03)**

### **SESSION 12 (09:15 - 10:30)**

*Chair Person: S Arumugam, Kalasalingam Academy of Research and Education, Tamilnadu, India*

- 09:15 – 09:50 Apoorva Khare, Indian Institute of Science, Bangalore, India  
*Blowup-polynomials of graphs*

09:55 – 10:30 Sivaramakrishnan Sivasubramanian, Indian Institute of Technology Bombay, Mumbai  
*Inequalities among Two rowed Immanants of  $q$ -Laplacian of trees and Odd height peaks in Generalized Dyck Paths*

10:30 – 11:00 **Tea Break**

### **SESSION 13 (11:00 – 12:30) : Contributory Session**

#### **SESSION 13 A (11:00 – 12:30)**

*Chair Person: Arathi Bhat, Manipal Institute of Technology, MAHE, Manipal*

Speakers Shivani Rai, Sikkim Manipal Institute of Technology, SMU, Sikkim, India  
*On Properties of Some Neighborhood Degree-Based Topological Indices of Graphs*  
 Aniruddha Samanta, Indian Institute of Technology Kharagpur, India  
*Gain distance matrices for complex unit gain graphs*  
 Kuldeep Sarma, Tezpur University, Sonitpur, Assam, India  
*Inverses of non-bipartite unicyclic graphs with a unique perfect matching*  
 Suman Maiti, National Institute of Technology Rourkela, India  
*Interval eigenvalue problems and its applications*  
 Yogesh Kumar Prajapaty, Indian Institute of Science and Education Thiruvananthapuram, India  
*Dynamics of nonnegative matrices*

#### **SESSION 13 B (11:00 – 12:30)**

*Chair Person: M. Rajesh Kannan, Indian Institute of Technology Kharagpur, India*

Speakers Sonali Sharma, Malaviya National Institute of Technology Jaipur, India  
*Structure of the  $\mathbf{w}$ -Solution Set of the Tensor Complementarity Problem*  
 Vaibhav Shekhar, National Institute of Technology Raipur, India  
*More on convergence of two-stage alternating iterative scheme*  
 Punit Kumar Yadav, Malaviya National Institute of Technology-Jaipur, India  
*The range column sufficiency and the pseudo-SSM property of linear transformations on Euclidean Jordan Algebra*  
 Milica Andjelić, Kuwait University, Kuwait  
*Chain Graph Sequences and Laplacian Spectra of Chain graphs*  
 Krishna Kumar G, University of Kerala, Thiruvananthapuram, Kerala, India  
*Condition eigenvectors of bounded linear operators*

12:30 – 13:30 **Lunch Break**

#### **SESSION 14 (13:30 - 17:35)**

#### **Sessions in honor of Prof. Arbind Lal**

- Opening remarks, invitation to organizer and speakers of the sessions : K. Manjunatha Prasad, CARAMS, MAHE  
 Mourning of a minute in honor of Lal  
 Brief outline of life of Arbind K Lal : Sukanta Pati, Indian Institute of Technology Guwahati, Guwahati

13:45 – 15:40 **Technical Talks in Honor of Lal**

Chair Person Ravindra B Bapat, Indian Statistical Institute, Delhi

- Tirthankar Bhattacharya, Indian Institute of Science, Bangalore  
*On commuting isometries*  
Nandini Nilakantan, Indian Institute of Technology Kanpur, India  
*Complexes from graphs*

15:00 – 15:15 **BREAK**

15:15 – 16:35 **Dedicated Technical Talks from Lal's students**

Chair: Sukanta Pati, IIT Guwahati

Speakers: Satyanarayana Reddy Arikatla, Shiv Nadar University, Uttar Pradesh, India  
*Exponents of Primitive Symmetric Companion Matrices*  
Kamal Lochan Patra, National Institute of Science Education and Research,  
Bhubaneswar, India  
*Characteristic center of a connected graph*  
Sumit Mohanty, Gandhi Institute of Technology and Management, Vishakapat-  
nam, India  
*Trees with Matrix Weights: Laplacian Matrix and Characteristic-like Vertices*  
Tony Thomas, Kerala University of Digital Sciences, Innovation and Technology,  
India  
*Graph Centrality Measures and Mobile Malware Detection*

16:35 – 17:30 **TRIBUTES**

Bikash Bhattacharya, Indian Institute of Technology Guwahati, India  
Sivaramakrishnan Sivasubramanian, Indian Institute of Technology, Bombay  
Anish Sarkar, Indian Statistical Institute, Delhi Center, India  
Rajaram Bhat, Indian Statistical Institute, Bengaluru, India  
Any other among participants with the permission of Chair  
Ravindra B Bapat, Indian Statistical Institute, Delhi Center

Vote of thanks: Sukanta Pati, Indian Institute of Technology, Guwahati

### **17:35 - Valedictory Function**

Remarks: Participants

Closing Remarks: Steve Kirkland, Member, SAC  
Simo Puntanen, Member, SAC  
Ravindra B Bapat, Chair, SAC

Vote of Thanks: K. Manjunatha Prasad, Convener, SAC

## **Participants: ICLAA 2021**

1. LOC NGUYEN , Loc Nguyen's Academic Network, Vietnam  
ng\_phloc@yahoo.com
2. UROOJ ZEHRA , Al-Barkaat Public School  
zehrahusain27@gmail.com
3. RASHID A , National Institue of Technology Karnataka, India  
rashid441188@gmail.com
4. KAZEEM ADELEKE , Obafemi Awolowo University, Nigeria  
aadeleke@oauife.edu.ng
5. S. EJAJ AHMED , Affiliation  
sahmed5@brocku.ca
6. ABDULLAH AL AZEMI , Department of Mathematics, Kuwait University, Kuwait  
abdullah.alazemi@ku.edu.kw
7. RAFIKUL ALAM , Indian Institute of Technology Guwahati, India  
rafik@iitg.ernet.in; rafik@iitg.ac.in
8. MILICA ANDELIC , Kuwait University, Kuwait  
milica.andelic@ku.edu.kw
9. PRATHIGADAPA ANURADHA , S.R & B.G.N.R Government Degree College, Kammam, India  
anuprathigadapa@gmail.com
10. SATYANARAYANA REDDY ARIKATLA , Shiv Nadar University, Greater Noida, India  
satya.a@snu.edu.in
11. S. ARUMUGAM , Kalasalingam University, India  
s.arumugam.klu@gmail.com
12. MOHAMMAD ASHRAF , Aligarh Muslim University, UP, India  
mashraf80@hotmail.com
13. SOORYANSH ASTHANA , Indian Institute of Technology Delhi, India  
sooryanshasthana@gmail.com
14. PALLVI B , Indian Institute of Science Education and Research, Thiruvananthapuram, India  
pallavipoorna20@iisertvm.ac.in
15. BAYOWA BABALOLA , Kampala International University, Uganda  
teniola.babalola@eksu.edu.ng
16. NARAYANASWAMY BALAKRISHNAN , McMaster University, Hamilton, Ontario, Canada  
bala@mcmaster.ca
17. RAVINDRA B. BAPAT , Indian Statistical Institute, Delhi, India  
rbb@isid.ac.in
18. BIJOYA BARDHAN , National Institute of Technology Silchar, India  
bijoyab09@gmail.com
19. KESANG BASI , Sikkim Manipal Institute Technology, Sikkim, India  
kessang\_20211041@smit.smu.edu.in
20. ADI BEN-ISRAEL , Rutgers Business School, Newark, New Jersey, USA  
adi.benisrael@gmail.com
21. ANJALI BENIWAL , Indian institute of technology Kharagpur  
anjalibenawal1811@gmail.com
22. ABRAHAM BERMAN , Technion-Israel Institute of Technology, Israel  
bermanabraham@gmail.com
23. BISHAL BHANDARI , Sikkim Manipal Institute of Technology, Sikkim, India  
bishalbhandari211@gmail.com
24. ANAND BHARANGI , Indian Institute of Information Technology Dharwad, India  
apb4maths@gmail.com
25. ALOK BHARGAVA , Manipal University Jaipur, India

alok.bhargava@jaipur.manipal.edu

26. SHRIKRISHNA BHAT K , Pondicherry University, Puducherry, India  
skbhat.in@gmail.com
27. K NAYAN BHAT , St. Joseph's College (Autonomous), Bangalore, India  
bhatnayan@gmail.com
28. B. V. RAJARAMA BHAT , Indian Statistical Institute, Bangalore, India  
bhat@isibang.ac.in
29. S. PARAMESHWARA BHATTA , Mangalore University, Mangala Gangotri, India  
s\_p\_bhatta@yahoo.co.in
30. BIKASH BHATTACHARJYA , Indian Institute of Technology Guwahati, India  
b.bikash@iitg.ac.in
31. PARITOSH BISWAS , von Karman Society, India  
p.biswas1943@gmail.com
32. NIRANJAN BORA , Dibrugarh University Institute of Engineering & Technology, India  
niranjanbora11@gmail.com
33. ARUP BOSE , Indian Statistical Institute, Kolkata, India  
bosearu@gmail.com
34. BIPANCHY BUZARBARUA , Cotton University, Guwahati, India  
dasbipanchy123@gmail.com
35. FRANSISCO CAVALHO , Affiliation  
fpcarvalho@ipt.pt
36. RAMESHA C. , Manipal Institute of Technology, MAHE, Manipal  
ramesha.c@manipal.edu
37. ANKUR CHAKRABORTY , Indian Institute of Technology Dhanbad, West Bengal, India  
ankur.18dr0033@am.iitism.ac.in
38. WATHEK CHAMMAM , Majmaah University, Saudi Arabia  
w.chammam@mu.edu.sa
39. PREYANKA CHANDRAMOHAN , Surigao del Sur State University, Singapore  
sangcoresa18@gmail.com
40. GARRY KA LOK CHU , Dawson College, 3040 Sherbrooke Street West, Canada  
gchu@dawsoncollege.qc.ca
41. WILMA LAVEENA D SOUZA , St Joseph's College (Autonomous) Bengaluru, India  
wlaveena@gmail.com
42. PURAN DANGAL , Sikkim Manipal Institute Technology, Sikkim, India  
dangalpuran7@gmail.com
43. HIRAK JYOTI DAS , Tezpur University, Assam, India  
dashirakjyoti592@gmail.com
44. RAJAIAH DASARI , Kakthiya Institute of Technology & Science, Warangal, India  
dr.mh@kitsw.ac.in; dsreddy.hari@gmail.com
45. BISWAJIT DEB , Sikkim Manipal Institute of Technology, Majitar, India  
biswajit.d@smit.smu.edu.in; biswajittalk@gmail.com
46. DEEPA D , Department of Mathematics, University of Kerala, India  
deeparajk@yahoo.co.in
47. KALYANI DESIKAN , Vellore Institute of Technology, Chennai  
kalyanidesikan@vit.ac.in
48. GANESH DHARMADHIKARI , School of Mathematics, IISER Tiruananthapuram, India  
ganeshdharmadhikari42@gmail.com
49. N. EAGAMBARAM , Former DDG, MOSPI, Govt. of India, India  
ekambaramn@gmail.com



50. SHAUN FALLAT , *University of Regina, Canada*  
Shaun.Fallat@uregina.ca
51. ASMA FRAOOQ , *University of Trieste, Italy*  
asma.farooq@phd.units.it
52. KATARZYNA FILIPIAK , *Poznan University of Technology, Poland*  
katarzyna.filipiak@put.poznan.pl
53. KIRTHIKA G , *Bharathiar University of PG Extension and Research Centre, Erode, India*  
aisswarayakiruthika@gmail.com
54. S. GANESAN , *Bhabha Atomic Research Centre, Mumbai, India*  
ganesan555@gmail.com
55. SANDEEP GARG , *NMC*  
gargsandeep42@yahoo.com
56. SUBRATA GHANTY , *Santal Pargana College, Dumka, India*  
subrataghanty.sg.2001@gmail.com
57. SATTICK GHOSH , *Indian Institute of Technology Bhubaneswar, Orissa, India*  
sg53@iitbbs.ac.in
58. SUDHAKARA G. , *Manipal Institute of Technology, MAHE, Manipal, India*  
sudhakara.g@manipal.edu
59. VASUDEVA GUDDATTU , *Manipal Academy of Higher Education, Manipal, India*  
vasudev.guddattu@manipal.edu
60. MAHENDRA KUMAR GUPTA , *National Institute of Technology Jamshedpur*  
mahendra14389@gmail.com
61. SHRINATH HADIMANI , *Indian Institute of Science Education and Research Thiruvananthapuram, India*  
srinathsh33@gmail.com
62. SHAHNAZ ABDUL HAMEED , *University of Delhi, India*  
sciencefan19@gmail.com
63. STEPHEN JOHN HASLETT , *Massey University, Palmerston North, New Zealand*  
stephen.haslett@anu.edu.au
64. ARITRA NARAYANA HISABIA , *National Institute of Technology Meghalaya, India*  
aritrahisabia@gmail.com
65. JEFFREY HUNTER , *Auckland University of Technology, Auckland, New Zealand*  
jeffrey.hunter@aut.ac.nz
66. J ANITHA , *SRM Easwari Engineering College, Chennai, India*  
anitha.j@eec.srmrmp.edu.in
67. AISHA JABEEN , *Jamia Millia Islamia*  
ajabeen329@gmail.com
68. SURENDER KUMAR JAIN , *Ohio University, Ohio, United States*  
jain@ohio.edu
69. RAKESH JANA , *Indian Institute of Technology Guwahati, India*  
jana.rakesh.math@gmail.com
70. SACHINDRANATH JAYARAMAN , *IISER Thiruvananthapuram, India*  
sachindranathj@gmail.com
71. P. SAM JOHNSON , *National Institute of Technology Karnataka, Surathkal*  
nitksam@gmail.com
72. NEERAJ JOSHI , *Department of Statistics, University of Delhi, India*  
stats.joshi@gmail.com
73. MOHANA K. S. , *Mangalore University, Mangalore, India*  
krish305@gmail.com

74. DEBAJIT KALITA , Tezpur University, Assam, India  
k.debajit@gmail.com
75. KAMARAJ K. , Anna University, Arni, India  
krajkj@yahoo.com
76. ASHA KAMATH , Manipal Academy of Higher Education, Manipal  
asha.kamath@manipal.edu
77. RAJESH KANNAN M , Indian Institute of Technology Karaghpur, India  
rajeshkannan1.m@gmail.com
78. PRAJNA K. , Manipal Institute of Technology, MAHE, Manipal, India  
kprajna99@gmail.com
79. KARPAGAM ARCHITECTURE , Anna University, Coimbatore, India  
karpagamarchcbe@gmail.com
80. NEERAJ KAUSHIK , National Institute of Technology Kurukshetra  
kaushikneeraj@gmail.com
81. UMASHANKARA KELATHAYA , Manipal Academy of Higher Education, Manipal, India  
usksullia@gmail.com
82. HARSHA NARASINHA KELKAR , Hope Foundation's Finolex Academy of Management and  
Technology, Ratnagiri, India  
harsha.kelkar@famt.ac.in
83. MEGHA KHANDELWAL , Central University of Karnataka, Kalaburgi, India  
meghaghiya18@gmail.com
84. APOORVA KHARE , Indian Institute of Science, Bangalore, India  
khare@iisc.ac.in
85. PATRICK KAYUPE KIKODIO , Ibn Tofail University, Morocco  
kayupepatrick@gmail.com
86. VANISHREE KRISHNA KIREKOD , Technical University Kaiserslautern, Germany  
vSarkarrihegde@gmail.com
87. STEPHEN JAMES KIRKLAND , University of Manitoba, Canada  
Stephen.Kirkland@umanitoba.ca
88. DANIEL KLEIN , Pavol Jozef Safarik University in Košice, Slovakia  
daniel.klein@upjs.sk
89. BHASKARA RAO P. S. KOPPARTY , Indiana University Northwest, United States  
bkoppart@iun.edu
90. DILIP KUMAR , University of Kerala, Kerala  
dilipkumar@keralauniversity.ac.in
91. KRISHNA KUMAR G , BJM Government College, Chavara, Kollam, India  
krishna.math@gmail.com
92. KALPANARANI KUMAR , VIT University, Kamarajapuram, Chennai, India  
kalpana91085@yahoo.com
93. MYUNGHEE LEE , Deaprtment of Biostatistics, Samsung Medical Center, Republic of Korea  
yjmhlee@gmail.com
94. ANDRÉ LEROY , Université d'Artois, France  
andreleroy55@gmail.com
95. GNANASEKAR M , Rathinam college of Arts and Science, Coimbotore, India  
gnanasekar.kalam@gmail.com
96. THANGESHWARAN M , Periyar University, Dharmapuri, India  
thangeshmaths@gmail.com
97. VINAY MADHUSUDANAN , Manipal Institute of Technology, MAHE, Manipal, India  
vinay.m2000@gmail.com

98. ISWAR MAHATO , *Indian Institute of Technology Kharagpur, India*  
iswarmahato02@gmail.com
99. SAIKAT MAITY , *University of Culcutta, Kolkata, India*  
saikat.math1999@gmail.com
100. SUMAN MAITY , *National Institute of Technology Rourkela, India*  
smaiti2711@gmail.com
101. FIRDOUS AHMAD MALA , *Govt. Degree College Sopore, J&K, India*  
firdousmala@gmail.com
102. AIJAZ AHMAD MALLA , *University of Kashmir, Srinagar*  
ahaijaz99@gmail.com
103. MANNAT MAND , *D A V University, Jalandar, India*  
mandmannat@gmail.com
104. SANTANU MANDAL , *National Institute of Technology Rourkela, India*  
santanu.vumath@gmail.com
105. PANKAJ MANJHI , *Vinoba Bhave University, Hazaribag, India*  
19pankaj81@gmail.com
106. AUGUSTYN MARKIEWICZ , *Poznan University of Life Sciences, Poznan, Poland*  
amark@up.poznan.pl; augustyn.markiewicz@up.poznan.pl
107. MIKA MATTILA , *Tampere University, Tampere, Finland*  
mika.mattila@tuni.fi
108. SHREEMATHI MAYYA , *Manipal Academy of Higher Education, Manipal, India*  
shreemathi.mayya@manipal.edu
109. SHAFIQUL ISLAM MD. , *Bangladesh University of Engineering and Technology, Dakha, Bangladesh*  
mdshafiqulislam@ug.mme.buet.ac.bd
110. SUNIL MEENA , *Government College, Sirohi, India*  
meenasunil461@gmail.com
111. VIDHYA MEENAKSHI V , *Department of Statistics, Bharathidasan university, Trichy, India*  
vidhyameena5799@gmail.com
112. RUCHIKA MEHTA , *Manipal University Jaipur, India*  
ruchika.mehta@jaipur.manipal.edu
113. PARVATHI G MENON , *Sahrdaya College Of Advanced Studies, Kodakara, Thrissur*  
pgmenon174@gmail.com
114. DAVID RAJ MICHEAL , *Sikkim Manipal Institute of Technology, Majitar, India*  
davidddl2013@gmail.com
115. HEMANT MISHRA , *Indian Statistical Institute, Delhi Center, India*  
hemantmishra1124@gmail.com
116. RAJIV MISHRA , *Indian Institute of Technology Bhubaneswar, India*  
rm33@iitbbs.ac.in
117. SUBRAT KUMAR MISHRA , *Veer Surendra Sai University of Technology, Burla, India*  
subratmishra58135@gmail.com
118. SUYASH MISHRA , *Mjpru*  
suyashmishra840@gmail.com
119. VASANT KUMAR MISHRA , *Banaras Hindu University, India*  
vasantmishra99@gmail.com
120. SUMIT MOHANTY , *Gandhi Institute of Technology and Managemen, Vishakapatnam, India*  
smohanty@gitam.edu
121. DEBABROTA MONDAL , *Indian Institute of Technology Bhubaneswar, India*  
dm27@iitbbs.ac.in
122. YOGESH MUDGIL , *Lovely Professional University, Jalandar, India*

mudgilyogesh11@gmail.com

123. ARCHANA MURUGANANTHAM , *E.G.S. Pillay Engineering College, Nagapattinam, India*  
archu288m@gmail.com
124. SANGEETA NAHAR , *Krypt India, India*  
sg30@iitbbs.ac.in
125. SACHIN NAIK , *National Institute of Technology Karnataka, India*  
sachinchittar@gmail.com
126. NUPUR NANDINI , *Manipal Academy of Higher Education, Manipal, India*  
n.nandini048@gmail.com
127. PALASH NATH , *Tezpur University, India*  
mathspalash30@gmail.com
128. S. K. NEOGY , *Indian Statistical Institute, Delhi Centre, India*  
skn@isid.ac.in
129. PALLAVI P , *Manipal Institute of Technology, MAHE, Manipal, India*  
pallavipanjr@gmail.com
130. FAYIS THAZHETHE PALLIYALI , *KLM College of Engineering, Kollam, India*  
fayistp@tkmce.ac.in
131. SARVESH PANDEY , *Banaras Hindu University, India*  
pandeysarvesh798@gmail.com
132. ROMEO PARACKAL , *Department of Mathematics, CUSAT Kochi*  
romeopg@cusat.ac.in
133. SAMIRON PARUI , *Indian Institute of Science Education and Research, Kolkata, India*  
samironparui@gmail.com
134. SUKANTA PATI , *Indian Institute of Technology Guwahati, India*  
pati@iitg.ernet.in; pati@iitg.ac.in
135. KAMAL LOCHAN PATRA , *National Institute of Science Education and Research, Bhubaneswar, India*  
klpatra@niser.ac.in
136. SOMNATH PAUL , *Tezpur University, Assam, India*  
sommath.paul60@gmail.com
137. C PERIYASAMY , *Presidency College, Chennai, India*  
periyasamyc1995@gmail.com
138. RAKSHA POOJARY , *Manipal Academy of Higher Education, Manipal, India*  
Rakshayogeesh@gmail.com
139. ROOPA PRABHU , *S. J. Govt Polytechnic College, Bengaluru, India*  
roopa\_prabhu@rediffmail.com
140. SUMATHI K. PRABHU , *Manipal Institute of Technology, MAHE, Manipal, India*  
k.sumathi@manipal.edu
141. BARSHA PRADHAN , *Sikkim Manipal Institute Technology, Sikkim, India*  
barshapradhan09@gmail.com
142. YOGESH KUMAR PRAJAPATY , *Indian Institute of Science and Education, Thiruvananthapuram*  
Prajapaty0916@iisertvm.ac.in
143. PUNAM KUMARI PRASAD , *Sikkim Manipal Institute Technology, Sikkim, India*  
psikkim@ymail.com
144. KUNCHAM SYAM PRASAD , *Manipal Institute of Technology, MAHE, Manipal, India*  
kunchamsyamprasad@gmail.com
145. SIMO PUNTANEN , *Tampere University, Finland*  
simo.puntanen@tuni.fi

146. HARSHITH R A , Department of Data Science, PSPH, MAHE, Manipal, India  
harshith.a@learners.manipal.edu
147. T. E. S. RAGHAVAN , University of Illinois at Chicago, United States  
terctu@gmail.com
148. SHIVANI RAI , Sikkim Manipal Institute of Technology , Sikkim, India  
shivaniirai866@gmail.com
149. CHEPURI RAJITA , Kaktiya University, Warangal, India  
rajithachepuri341@gmail.com
150. G S RAJU K , Indian Institute of Technology Ropar, India  
raju@iitrpr.ac.in
151. BALAJI RAMAMURTHY , Indian Institute of Technology Madras, Chennai, India  
balaji5@iitm.ac.in
152. ARNI S R SRINIVASA RAO , Medical College of Georgia, Augusta University, USA  
arni.rao2020@gmail.com
153. T S S R K RAO , Manipal Academy of Higher Education, Manipal; Ashoka University, Sonipat,  
Haryana, India  
tssrkr Rao@gmail.com
154. JITA RASTOGI , Not available  
Jitirastogi@gmail.com
155. BILAL AHMAD RATHER , University of Kashmir, Srinagar, India  
bilalahmadrr@gmail.com
156. DHANANJAYA REDDY , Government Degree College, Puttur, India  
djreddy65@gmail.com
157. KONDAM RAVINDER REDDY , Kakatiya University, Warangal, India  
ravindhar.iitm18@gmail.com
158. TEKALIGN REGASA , Jimma University, Ethiopia  
tekalignregasa3@gmail.com
159. NATÁLIA MARIA DE BESSA REGO , 178995274  
natarego@gmail.com
160. RAMESH RENIKUNTA , KITS Warangal, Telangana, India  
rameshrenikuntla@gmail.com
161. DIETRICH VON ROSEN , Swedish University of Agricultural Sciences, Sweden  
Dietrich.von.Rosen@slu.se
162. NIRMAL CHANDRA ROUT , National Institute of Technology Raipur, India  
nirmal.rout1989@gmail.com
163. AALEKH ROY , Affiliation  
mathematicianacademic@gmail.com
164. ARITRA ROY , Banaras Hindu University, India  
97aritra@gmail.com
165. SAPTARSHI ROY , Indian Institute of Technology Kanpur  
saptarshi.ndp@gmail.com
166. GOKULRAJ S. , Central University of Tamil Nadu, Thiruvavur, India  
gokulrajs93@gmail.com
167. RITIKA SACHDEV , Department of Computer Science, Mumbai University, India  
ritikasachdev1411@gmail.com
168. JAJATI KESHARI SAHOO , BITS Pilani KK Birla Goa Campus, India  
jksahoo@goa.bits-pilani.ac.in
169. CHAPALA SAHOO , Department of Mathematics, VSSUT, Burla, India  
chumulu2013@gmail.com

170. SHANTANU KUMAR SAHOO , Banki College, Banki, India  
sahooshantanu799@gmail.com
171. TAPATEE SAHOO , Manipal Institute of Technology, MAHE, Manipal, India  
tapateesahoo96@gmail.com
172. NASRIN SAIKIA , PUB Kamroop College, Assam  
nnasrinsaikia@gmail.com
173. DEEPAK SAINI , CCS University, Meerut, India  
deepaksaini1168@gmail.com
174. ANIRUDDHA SAMANTA , Indian Institute of Technology Kharagpur, India  
aniruddha.sam@gmail.com
175. SHARAD S. SANE , Chennai Mathematical Institute, Chennai, India  
sharadsane@gmail.com, ssane@cmi.ac.in
176. RESA MAE SANGCO , Surigao del Sur State University, Philippines  
sangcoresa18@gmail.com
177. NIRAJ SAPKOTA , Sikkim Manipal Institute Technology, Sikkim, India  
niraj.sapkota13@gmail.com
178. KULDEEP SARMA , Tezpur University, Assam, India  
kuldeep.sarma65@gmail.com
179. ATHIRA SATHISH K , National Institute of Technology Karnataka, India  
athirask.187ma001@nitk.edu.in
180. DEENA SCARIA , Mahatma Gandhi University, Kerala, India  
deenacelinescaria@gmail.com
181. IRENE SCIRIHA , University of Malta, Malta  
isci1@um.edu.mt
182. NITU SHARMA , Indian Institute of Technology Delhi, India  
nitusharma911@gmail.com
183. DEBASHISH SHARMA , Gurucharan College, Silchar, India  
debashish0612@gmail.com
184. RUHI SHARMA , GNDU  
sharmaruhi235@gmail.com
185. SONALI SHARMA , Malaviya National Institute of Technology , Jaipur, India  
2019rma9072@mnit.ac.in
186. VAIBHAV SHEKHAR , National Institute of Technology Raipur, India  
vaibhavshekhar29@gmail.com
187. SHIHAB P. M. , KLM College of Engineering, Kollam, India  
shihabpm@tkmce.ac.in
188. RAVI PRATAP SINGH , Banaras Hindu University, India  
ravibhu002@gmail.com
189. MARTIN SINGULL , Linköping University, Linköping, Sweden  
martin.singull@liu.se
190. BIKAS KUMAR SINHA , Indian Statistical Institute, Kolkata, India  
bksinha@isical.ac.in
191. K. C. SIVAKUMAR , Indian Institutes of Technology Madras, Chennai, India  
kcskumar@iitm.ac.in
192. SIVARAMAKRISHNAN SIVASUBRAMANIAN , Indian Institute of Technology Bombay, India  
siva.krishnan@gmail.com
193. PAYAL SOLANKI , Visvesvaraya National Institute Of Technology, Nagpur  
solankipayal2906@students.vnit.ac.in
194. RACHANA SONI , Bennett University, Uttar Pradesh, India

rachanasoni007@gmail.com

195. MURALI K. SRINIVASAN , Indian Institute of Technology Bombay, India  
murali.k.srinivasan@gmail.com
196. GEORGE P H STYAN , McGill University, Canada  
geostyan@gmail.com
197. SUMATHI SUBBARAYAN , Vellur Institute of Technology, Chennai, India  
sumathi.manivannan04@gmail.com
198. SUBBULAKSHMI P , Department of Mathematics, Stella Maris College, Chennai, India  
subbulakshmi216@gmail.com
199. LAVANYA SURIYAMOORTHY , Sri Ramachandra Institute for Higher Education and Research,  
Chennai India  
mathlavi@gmail.com
200. SWATI , Department of Data Science, PSPH, MAHE, Manipal, India  
swathi.muma1@learner.manipal.edu
201. VENKATESWARARAO TEEGA , Indian Institute of Technology Hyderabad, India  
ma19reschi1001@iiith.ac.in
202. MANJU BHASHINI THIYAGARAJAN , Vivekananda college of technology for women, Tamil-  
nadu, India  
manjutmpv13@gmail.com
203. TONY THOMAS , Kerala University of Digital Sciences, Innovation and Technology, Kerala,  
India  
tony.thomas@duk.ac.in
204. REZWAN UL SHABAN , University of Kashmir, Kasmir, India  
rezwanbhat21@gmail.com
205. SAVITHA V , Manipal Academy of Higher Education, Manipal, India  
savithavarkady@yahoo.co.in
206. MATHEW VARKEY T. K. , TKM College Of Engineering, Kollam, India  
mathevarkeytk@gmail.com
207. PIYUSH VERMA , Indian Institute of technology, Bhubaneswar, India  
pv18@iitbbs.ac.in
208. JASVINDER VIRDI , Veer Surendra Sai University of Technology (VSSUT) Odisha  
jsvirdi@gmail.com
209. PRATEEK KUMAR VISHWAKARMA , Indian Institute of Science, Bangalore, India  
prateekv@iisc.ac.in
210. JULIA VOLAUFOVA , Louisiana State University Health Sciences Center, New Orleans (USA)  
jvolau@lsuhsc.edu
211. HITESH WANKHEDE , Department of Mathematics, IISER Pune, India  
hitesh.wankhede@students.iiserpune.ac.in
212. HANS JOACHIM WERNER , Affiliation  
hjlw.de@uni-bonn.de
213. YIMIN WEI , Fudan University, Shanghai, China  
yimin.wei@gmail.com; ymwei@fudan.edu.cn
214. PUNIT KUMAR YADAV , Malaviya National Institute of Technology Jaipur, India  
punitjrf@gmail.com
215. PRADEEP KUMAR YADAV , Central University of Allahabad, India  
pradeepyaduvanshi28295@gmail.com
216. Y. SANTHI SHEELA , National Institute of Technology Karnataka, India  
krishsanthi76@gmail.com
217. BING ZHENG , Lanzhou University, Lanzhou, China

bzheng@lzu.edu.cn



# **ACTGMG 2021**

International Workshop on  
Algorithms for Cooperative TU  
Games using Matrices and Graphs  
December 20-31, 2021

## About ACTGMG

CARAMS, MAHE is very happy to organize a two-week workshop, carrying a credit of 2 points, on the topic 'Algorithms for Cooperative TU Games using Matrices and Graphs' during December 20-31, 2021. This is in continuation of Prof. T E S Raghavan's effort of organizing 'Gurukulam' for the last few years. The course will be organized in in-person/hybrid format depending on the prevailing COVID situation. For the same reason, the dates also remain to be flexible.

The objective of the workshop is to provide an advanced input on integrating the following:

- Matrices associated with graphs such as Incidence, Adjacency and Laplacian matrices, Distance matrix of a tree and its generalizations, Resistance distance, Proof of the sensitivity conjecture, and the Algorithmic aspects of Cooperative Game Theory which involve many basic combinatorial developments like: Max-flow min-cut, Longest paths ending in a graph, Edmond's maximum matching in a general graph.
- Completely mixed games and Perron Frobenius Theorem, Matrix games with payoff  $M$  that guarantee the solvability of the associated LCP constructively via the Lemke Howson algorithm, Totally positive matrices with their combinatorial connections and the spectral properties, Jacobi matrices and their implications to continuous time stationary Markov processes in discrete state space, and Determinantal properties of Sylvester, Gantmacher and MG Krein.



TES Raghavan



Ravindra B Bapat

The course will be delivered through lectures on the topic in-depth and tutorials. A limited number (up to 20) of seriously interested students (Doctoral/Post-doctoral) with a good background of Linear Algebra and Calculus will be selected for participation in the workshop. They having basic knowledge of Markov Chains and Continuous Time Discrete Stochastic Process is appreciated.

It may be noted that certificates will be issued only to the participants who complete the tutorial assignments.

A few students (up to 6) will be partially sponsored by CARAMS, MAHE under Scientific Social Responsibility of different projects. Selection depends on the Supervisor/HOD's recommendation describing the possible benefits to the candidate. Candidates aspiring the support may write to the Coordinator of CARAMS (km-prasad63@gmail.com) or to T. E. S. Raghavan (terctu@gmail.com) immediately after the registration, attaching the necessary recom-

mendation.

Some memories from 'Gurukulam'



Collective Memories



Memories from Gurukulam 1 (2017)





Memories from Gurukulam 2 (2018)



Memories from Gurukulam 3 (2019)

# Appendix

# **Mathematical Philately Souvenir (MPS) with extended abstracts for the International Mini-Symposium on Mathematical Philately in the 28th International Workshop on Matrices & Statistics<sup>1</sup>**

*edited by* Ka Lok Chu<sup>2</sup>, Simo Puntanen<sup>3</sup> & George P. H. Styan<sup>4</sup>

November 27, 2021

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<sup>1</sup> IWMS-2020/2021 Mini-Symposium (MS 02) Session 16 organized by Ka Lok Chu, Simo Puntanen & George P. H. Styan to be held in the Centre for Advanced Research in Applied Mathematics and Statistics (CARAMS), Manipal Academy of Higher Education (MAHE), Manipal (Karnataka State), India (Wednesday 15 December 2021: 18h20–19h40 = 07h50–09h10 EST).

<sup>2</sup> Ka Lok Chu, Dawson College, Westmount/Montréal (Québec), Canada H3Z 1A4: gchu@dawsoncollege.qc.ca

<sup>3</sup> Simo Puntanen, Faculty of Information Technology and Communication Sciences, FI-33014 Tampere University, Finland: simo.puntanen@tuni.fi

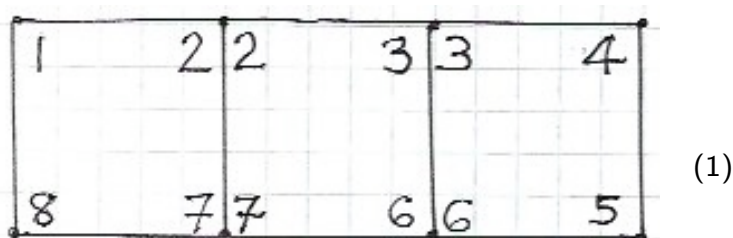
<sup>4</sup> *Corresponding author:* George P. H. Styan, Dept. of Mathematics & Statistics, McGill University: geostyan@gmail.com

In this Mathematical Philately Souvenir (MPS)<sup>5</sup> with extended abstracts for the International Mini-Symposium on Mathematical Philately in the 28th International Workshop on Matrices & Statistics (IWMS-2020/2021) we present extended abstracts for the following three beamer files

[MPS-1/PLG] “A further introduction to Philatelic Lattice Grids (PLGs) with four-sided stamps” by Ka Lok Chu, Simo Puntanen & George P. H. Styan, McGill Report 2021-05, our pdf file PLG-26nov21a 52pp in progress. See also our display (1).

[MPS-2/TG] “More breaking news for ‘When Math meets Art’ illustrated philatelically and with emphasis on Tangrams and Japanese classical poetry” by Ka Lok Chu, Simo Puntanen & George P. H. Styan, McGill Report 2021-03, our pdf file TG-26nov21a 71pp in progress.

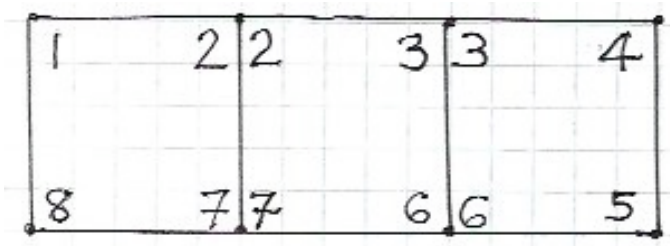
[MPS-3/RK] “An introduction to the random knight’s tour on a rectangular chessboard illustrated philatelically and with emphasis on Markov chains and Edgar Allan Poe” by Ka Lok Chu, Awani Kumar, Simo Puntanen & George P. H. Styan, McGill Report 2021-01, our pdf file RK-26nov21b 64pp in progress.



<sup>5</sup> McGill Report 2021-06, our pdf file MPS-27nov21 30pp in progress

## “A further introduction to Philatelic Lattice Grids (PLGs) with four-sided stamps” by Ka Lok Chu, Simo Puntanen & George P. H. Styan

**Extended Abstract** This talk builds on “A short introduction to Philatelic Lattice Grids (PLGs) with four-sided stamps” by Ka Lok Chu, Simo Puntanen & George P. H. Styan published in the “Farewell issue” of *Philamath: A Journal of Mathematical Philately* (vol. 41, no. 4, pp. 31–34: April 2020). We define a **Philatelic Lattice Grid** (PLG) as a grid with  $h$  four-sided stamps and  $n$  vertices. Our PLG is defined by a grid of se-tenant (or almost se-tenant) stamps as in displays (2), (4) below. Our “PLG-problem” is adapted from the *Jisuguimundo*: “Hexagonal Tortoise Problem” (HTP) posed and solved by Choi Seok-jeong (1646–1715). In Choi’s HTP each of the integers from 1 to  $n$  is assigned one integer each to the  $n$  vertices of the grid. In our PLG with  $h$  four-sided stamps the four vertex entries in each “stamp box” add to the same “magic sum”  $m$ , often the “central magic sum”  $\bar{m} = 2(n + 1)$ . Depicted here in display (2) left panel is our “Korean Scientists philatelic strip” of  $h = 3$  four-sided stamps and  $n = 8$  vertices in the “Science in Korea” series (2019) and a “PLG-paired solution” with central magic sum  $\bar{m} = 2(n + 1) = 18$  and paired entries  $(1, 8), (2, 7), \dots, (4, 5)$  each pair adding to  $n + 1 = \frac{1}{2}\bar{m} = 9$ .



(2)



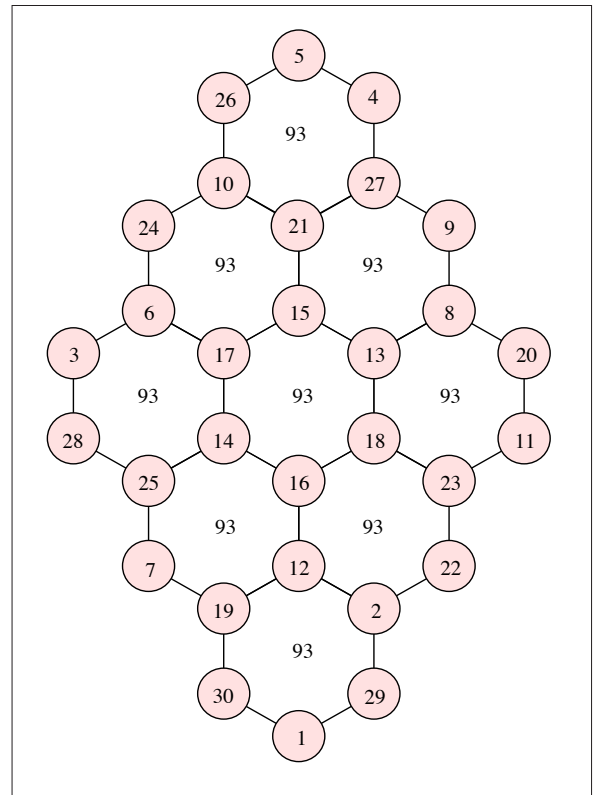
Our study of PLGs was motivated by the *Jisuguimundo*: Hexagonal Tortoise Problem (HTP) first posed and solved (c. 1700) by the Korean **mathematician** CHOI Seok-jeong (1646–1715).

In Choi's solution (3) to his HTP, each of the integers from 1 to 30 is assigned one each to the  $n = 30$  vertices and to the six vertex entries in each of the  $h = 9$  hexagons so that the 6 vertex entries all have the same "magic sum"  $m$ . Here

$$m = 93 = 3(n + 1) = p(n + 1)/2,$$

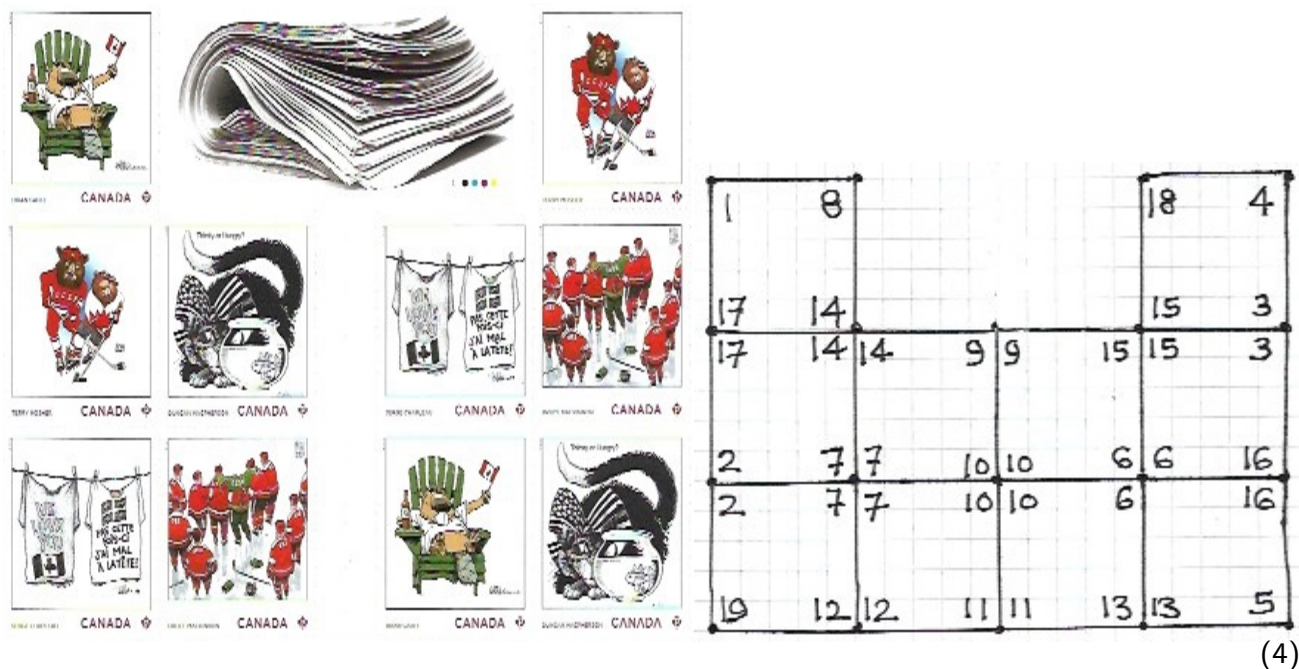
with  $p = 6$  here since hexagons are six-sided.

It seems that **Choi's HTP is still unsolved in general**. Indeed we believe it received no attention until 1989 with the publication "Properties and solution-finding algorithm of *Jisuguimundo* (Turtle-shape Diagram)" by KIM Dong Jin & OH Yung Hwan (1989).



(3)

A larger example of a PLG is defined by the recent 10-stamps booklet, display (4) left panel, featuring cartoons by five Canadian editorial cartoonists (Serge Chapleau, Brian Gable, Bruce MacKinnon, Duncan Macpherson & Terry Mosher/Aislin), issued by Canada (2021) with each of 5 different stamps appearing twice. Our PLG-solution with  $h = 10$  stamps and  $n = 19$  vertices in display (4) right panel has central magic sum  $\bar{m} = 2(n + 1) = 40$ .





A PLG example with  $h = 11$  stamps and  $n = 20$  vertices is given in the sheetlet, display (5) here, “In Celebration” of the 95th Birthday of Her Majesty Queen Elizabeth II: “Devoted to your service” issued jointly by Ascension Island, Bahamas, British Antarctic Territory, British Virgin Islands, Falkland Islands, Gibraltar, Guernsey, Isle of Man, Jersey, South Georgia and the South Sandwich Islands & Tristan da Cunha. PLG-solution (6) has central magic sum  $\bar{m} = 2(n + 1) = 42$ .

2	19	19	17	17	4
20	1	1	5	5	16
20	1			5	16
13	8			9	12
13	8	8	9	9	12
3	18	18	7	7	14
3	18	18	7	7	14
15	6	6	11	11	10

(6)

In addition to the joint  
sheetlet display (6) above

“In Celebration” of the 95th  
Birthday of Her Majesty  
Queen Elizabeth II  
(*b.* 21 April 1926):

“Devoted to your service”

issued jointly by  
Ascension Island,  
Bahamas,  
British Antarctic Territory,  
British Virgin Islands,  
Falkland Islands,  
Gibraltar,  
Guernsey,  
Isle of Man,  
Jersey,  
South Georgia and the South  
Sandwich Islands  
& Tristan da Cunha,

we have featured  
philatelic items from  
Botswana,  
Canada,  
Christmas Island,  
Croatia,  
Dominica,  
Finland,  
India,  
Korea,  
Kyrgyzstan,  
Poland  
& Ukraine.

## Key words and phrases

- AHN Dong-hyuk (1907–2004)  
 Archimedes of Syracuse (c.287–c.212 BC)  
   stamps from Ukraine  
 Stefan Batory (1533–1586)  
   stamps from Poland  
 “Botswana moths” stamps from Botswana  
 Canadian editorial cartoonists stamps booklet  
 “Cats” stamps from Dominica  
 “Chinatown Gates” stamps from Canada  
 CHOI Seok-jeong (1646–1715)  
 Christmas Island stamps  
 René Descartes (1596–1650)  
   stamps from Ukraine  
 Pierre de Fermat (1607–1665)  
   stamps from Ukraine  
 four-sided stamps  
 Johann Carl Friedrich Gauß (1777–1855)  
   stamps from Ukraine  
 “Happy New Year!” stamps from Kyrgyzstan  
 “Historic Covered Bridges”  
   stamps from Canada  
*Jisuguimundo*:  
   Hexagonal Tortoise Problem (HTP)  
 Korea Post’s “Science in Korea” stamps  
 Elias Lönnrot (1802–1884)  
   stamps from Finland  
 Mathematical Philately  
 “NHL Teams & fans wear colours proudly”  
   stamps from Canada  
 Philatelic Lattice Grids (PLGs)  
 Queen Elizabeth II: Joint sheetlet  
   “In Celebration” of her 95th Birthday  
 Johan Vilhelm Snellman (1806–1881)  
   stamps from Finland  
 “Spices of India” stamps from India  
 Franjo Tudjman (1922–1999)  
   stamps from Croatia  
 “Weather phenomena” stamps from Canada  
 YI Soon-ji (1406–1465)

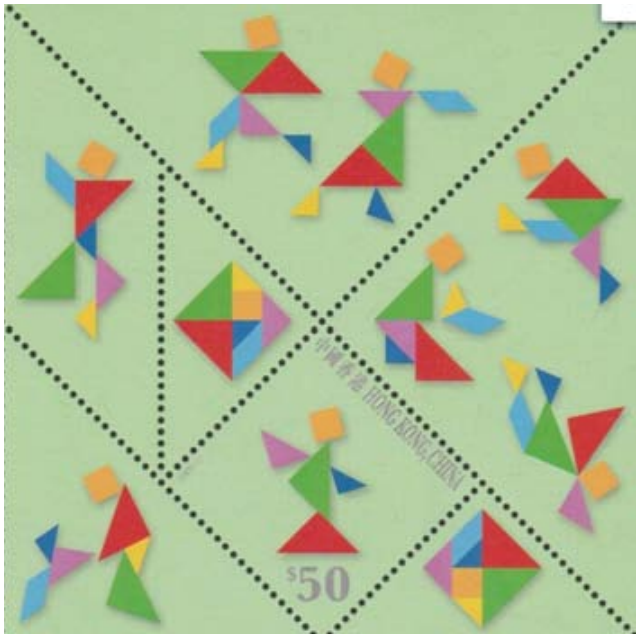


**[MPS-2] “More breaking news for ‘When Math meets Art’ illustrated philatelically and with emphasis on Tangrams and Japanese classical poetry”**  
 by Ka Lok Chu, Simo Puntanen & George P. H. Styan



Philatelic tangrams with the same 7-polygon configuration, left to right:  
 Finland (2000) “Heureka Science Centre, Vantaa” 3 stamps and Sierpiński triangle label  
 South Africa (2010) “Bridging the digital divide” 5 stamps for “southern Africa small letter”.

(7)

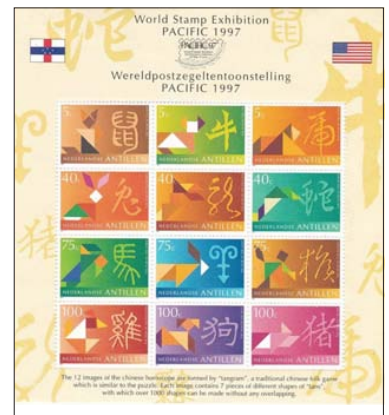


Hong Kong China 2009: 23rd Asian International Stamp Exhibition;  
Netherlands Antilles 1997: World Stamp Exhibition PACIFIC 1997: Scott 811a image A234 with 12 stamps featuring “stylized designs” for tangram-friendly “Signs of the Chinese Calendar”.

- Finland 2000: Millenium hologram sheetlet for the Heureka Science Centre in Vantaa (Finland) Scott 1140 image A595 with 3 stamps and tangram-friendly postmark in First Day Cover (30 May 2000)
- South Africa 2010: Hologram sheetlet for “Bridging the digital divide” with 5 stamps for “southern Africa small letter”
- Hong Kong China 2009: 23rd Asian International Stamp Exhibition with 1 stamp and tangram-friendly postmark in First Day Cover (14 May 2009)
- Netherlands Antilles 1997: World Stamp Exhibition PACIFIC 1997: Netherlands Antilles Scott 811a image A234 with 12 stamps featuring “stylized designs” for tangram-friendly “Signs of the Chinese Calendar”.

From left to right:

Finland 2000, South Africa 2010, Hong Kong China 2009, Netherlands Antilles 1997

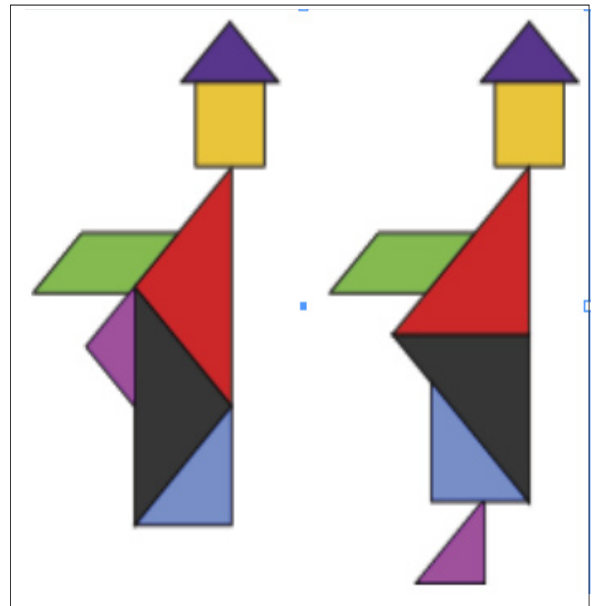




A "tangram paradox" is a dissection fallacy: Two figures composed with almost the same set of 7 pieces, with one set seeming to be a proper subset of the other.

One famous tangram paradox, displayed in (8), is that of the two monks, attributed to Henry Ernest Dudeney (1857–1930), which consists of two similar shapes, one with and the other missing a foot.

The area of the foot is compensated for in the the footless body by a subtly larger body. Overlaying the bodies shows that the footless body is larger by the foot's area. The change in area is often unnoticed as  $\sqrt{2} \simeq 1.5$ .



(8)

Text and image here copied from the "Tangram" article in *Wikipedia* (last edited on 5 April 2021, at 17:05 UTC).  
 See also Problem 169 "A Tangram Paradox" (problem & discussion: pp. 43–46, solution: pp. 178–179) in  
*Amusements in Mathematics* by Henry Ernest Dudeney (pub. Nelson 1917, reprinted by Dover 1958/1970)  
 and "Tangram paradox explained" by SiamMandalay, online at  
<https://www.siammandalay.com/blogs/puzzles/90769478-tangram-paradox-explained>



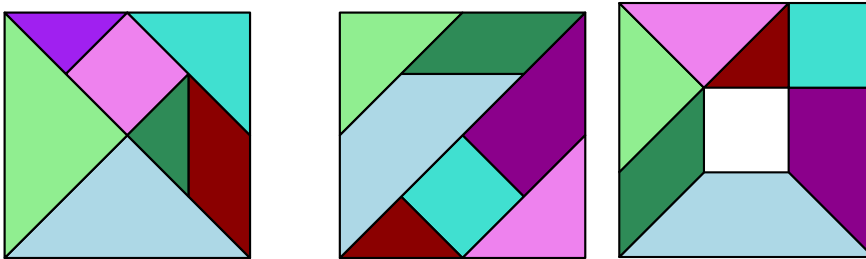
*tangram designs: woman skating, squirrel?, chicken, candle*



*tangram designs: goose, the Hatter, woman with headdress, teapot*



*tangram designs: rooster, cat, messenger, running person*



(10)

As observed by Fox-Epstein & Uehara (2014) and by Katsumata & Uehara (2015) in their excellent recent survey articles the Tangram and the ‘Sei Shōnagon Chie no Ita’ are two popular dissection puzzles, each consisting of 7 pieces. The two puzzles can each be formed by identifying edges from 16 identical right isosceles triangles. In the leftmost panel of (10) the Tangram, and in the other two panels the “Sei Shōnagon Chie no Ita,” are shown in square configuration.

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“The convex configurations of “Sei Shōnagon Chie no Ita” and other dissection puzzles” by Eli Fox-Epstein & Ryuhei Uehara, *Proceedings of the 26th Canadian Conference on Computational Geometry*, CCCG 2014, Halifax, Nova Scotia, Canada (11–13 August 2014), our file FoxEpstein 4 pp. <http://www.cccg.ca/proceedings/2014/papers/paper56.pdf> (11)

“The convex configurations of dissection puzzles with seven pieces” by Kazuho Katsumata & Ryuhei Uehara, Information Processing Society of Japan (IPSJ) SIG Technical Report vol. (12) 2015-AL-152 no. 9 (2015/3/3), our file Katsumata 5 pp.



(13)

Featured in (13) here in each of the 5 rows are a label and 2 stamps for

(row 1) **Murasaki Shikibu** (c. 973/978–1031), novelist, poet and lady-in-waiting best known as the author of *The Tale of Genji*, widely considered to be one of the world's first novels.

(row 2) **Fujiwara no Sanekata** (d. 998), poet listed as one of the *Late Classical Thirty-Six Immortals of Poetry*. One of his poems was included in the *Ogura Hyakunin Isshu*.

(row 3) **Sei Shōnagon** (c. 966–1017/1025), author, poet, and court lady who served the Empress Teishi (Sadako) around the year 1000 during the middle Heian period. She is the author of *The Pillow Book*.

(row 4) **Fujiwara no Kintō** (966–1041), also known as **Shijo-dainagon**, poet mentioned in works by Murasaki Shikibu and Sei Shōnagon.

(row 5) **Izumi Shikibu** (976–1030), considered by many to have been the greatest woman poet of the Heian period with 242 poems.

## Key words and phrases

advertising stamps for Suchard chocolate

“Bridging the digital divide”

sheetlet from South Africa

“The convex configurations of *Sei Shōnagon Chie no Ita* and other dissection puzzles”

by Eli Fox-Epstein & Ryuhei Uehara

“The convex configurations of dissection puzzles with seven pieces”

by Kazuho Katsumata & Ryuhei Uehara

dissection fallacy

Henry Ernest Dudeney (1857–1930)

Dudeney’s “two-monks tangram paradox”

“Tales with Tangrams” by Henry Ernest

Dudeney, *The Strand Magazine* (1908)

Heureka Science Centre, Vantaa (Finland)

hologram sheetlets from Finland, South Africa

Japanese classical poetry

Philatelic tangrams from

Finland, Hong Kong China,

Netherlands Antilles, South Africa

Sei Shōnagon (c.966–1017/1025)

stamp from Japan

Sei Shōnagon’s *The Pillow Book*

*Sei Shōnagon Chie no Ita* dissection puzzle

Murasaki Shikibu (c.973/978–1031)

stamp from Japan

Murasaki Shikibu’s *The Tale of Genji*

stamps from Tuvalu

Wacław Franciszek Sierpiński (1882–1969)

stamp from Poland

Sierpiński triangles stamp from Macau China

tangram designs

tangram-friendly

“Signs of the Chinese Calendar”

“When Math meets Art”

**[MPS-3] “An introduction to the random knight’s tour on a rectangular chessboard illustrated philatelically and with emphasis on Markov chains and Edgar Allan Poe”  
by Ka Lok Chu, Awani Kumar, Simo Puntanen & George P. H. Styan**

In a personal communication from Awani Kumar to George P. H. Styan (11 December 2020) Awani commented that **A joint article on Knight’s tour is a nice idea.**

“To the best of my knowledge, there has been no published work on “Random walk of Knight” much akin to the well known problem of “Random walk”. All the literature on Knight’s tour<sup>6</sup> is anything but “random tour”.

As the name suggests, “random walk of Knight” will be its random jump over boards of various sizes and its mean distance from the starting cell after a particular number of jumps; average number of jumps needed to reach a particular distance, etc.

Later, it can also be extended to 3-D or even higher dimensions. It is an unexplored field

with ample scope for research. This problem has been lurking in my mind for years.”

**In this report we now begin with references for “Markov chains underlying a Knight’s random walk”.**

**A Markov chain is a stochastic model for a sequence of possible events in which the probability of each event depends only on the state attained in the previous event as defined by its transition probability matrix.**

The earliest reference that we have found for “Markov chains underlying a Knight’s random walk” is dated **22 April 1996** in *Reversible Markov Chains and Random Walks on Graphs* by David J. Aldous & James Allen Fill (1999/2002), who refer to a Knight’s random walk as a **“classic homework problem”** “.

<sup>6</sup> In the Knight’s Tour: “The challenge is to move the Knight over an empty chessboard in such a way that it covers all the squares in successive jumps without visiting a square twice”. [From “Tour of Knight: the eternal and evergreen puzzle” by

Here is a classic homework problem for an undergraduate Markov chains course.

Start a knight at a corner square of an otherwise-empty chess-board. Move the knight at random, by choosing uniformly from the legal knight-moves at each step. What is the mean number of moves until the knight returns to the starting square?

It’s a good question, because if you don’t know Markov chain theory it looks too messy to do by hand, whereas using Markov chain theory it becomes very simple. The knight is performing random walk on a graph (the 64 squares are the vertices, and the possible knight-moves are the edges). It is not hard to check that the graph is connected, so by the elementary Lemma 3.5, for a corner square  $v$  the mean return time is

$$E_v T_v^+ = \frac{1}{\pi_v} = \frac{2|\mathcal{E}|}{d_v} = |\mathcal{E}|,$$

and by drawing a sketch in the margin the reader can count the number of edges  $|\mathcal{E}|$  to be 168.

(14)

*Reversible Markov Chains and Random Walks on Graphs* by David J. Aldous & James Allen Fill: unfinished monograph (1999/2002), recompiled (2014). [From pp 64–65: “Example 3.6 Chess moves — copied here from the start of Example 18 of Chapter 5 (4/22/96 version).”]

An example of the Markov chain underlying a Knight's random walk on the  $3 \times 4 = 12$  minimal-rectangular chessboard with  $12 \times 12$  transition probability matrix  $\mathbf{P}$  and  $3 \times 4$  "coding matrix"  $\mathbf{Q}$  as given in (15):

$$\mathbf{P} = \{p_{i,j}\} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{Q} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}. \quad (15)$$

The  $12 \times 12$  transition probability matrix  $\mathbf{P} = \{p_{i,j}\}$  is **centro-symmetric**<sup>7</sup> since

$$p_{i,j} = p_{n-i+1,n-j+1} \quad i, j = 1, \dots, n = 12.$$

Our Knight starts at cell # $i$  and can move to cell # $j$  with probability  $p_{i,j}$ .

Then assuming the probability of leaving a vertex along a particular edge of the underlying graph is independent of the edge chosen, it follows that if our Knight starts at cell #1 then it can move only to cell #7 or #10 each with probability  $p_{1,7} = p_{1,10} = \frac{1}{2}$ .

<sup>7</sup> Selected references for centro-symmetric matrices are given on pages RK-14a & RK-14b. Our definition of "centro-symmetric matrix" coincides with the definition of a "cross-symmetric matrix" given by Graybill (1969, p. 361), see also Graybill (1983, pp. 287–289), Weaver (1985), Venkatesh (2005), Seber (2008, p. 160).



The Markov chain underlying the Knight's random walk on an  $8 \times 8$  chessboard is **reversible** with  $64 \times 64$  **centro-symmetric** transition probability matrix  $P =$

$$\frac{1}{24} \begin{pmatrix} 0 & P_1 & P_2 & 0 & 0 & 0 & 0 & 0 \\ P_3 & 0 & P_3 & P_4 & 0 & 0 & 0 & 0 \\ P_5 & P_6 & 0 & P_6 & P_5 & 0 & 0 & 0 \\ 0 & P_5 & P_6 & 0 & P_6 & P_5 & 0 & 0 \\ 0 & 0 & P_5 & P_6 & 0 & P_6 & P_5 & 0 \\ 0 & 0 & 0 & P_5 & P_6 & 0 & P_6 & P_5 \\ 0 & 0 & 0 & 0 & P_4 & P_3 & 0 & P_3 \\ 0 & 0 & 0 & 0 & 0 & P_2 & P_1 & 0 \end{pmatrix}. \quad (16)$$

With centro-symmetric submatrices  $P_1, P_2, P_3 =$

$$\begin{pmatrix} 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 \end{pmatrix},$$

and centro-symmetric submatrices  $P_4, P_5, P_6 =$

$$\begin{pmatrix} 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 8 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 \end{pmatrix}.$$



Somalia 1996

In **“Markov chains or the game of structure and chance: from complex networks to language evolution, to musical compositions”**

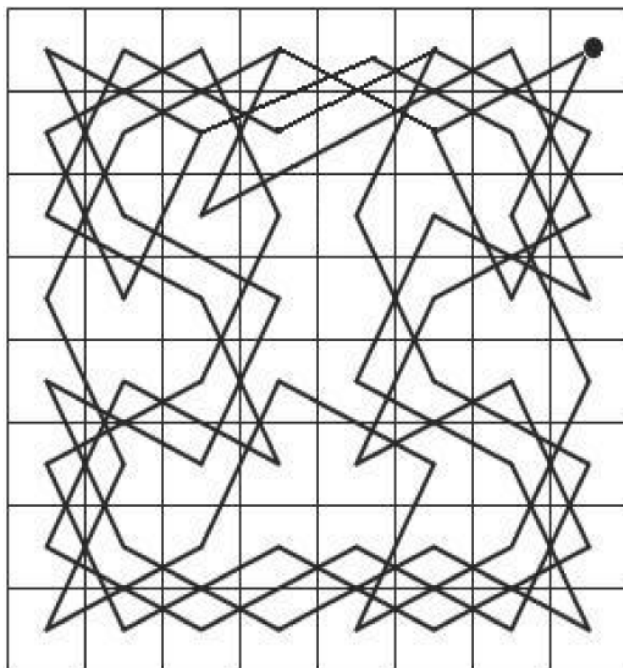
by Ph. Blanchard, J. R. Dawin & D. Volchenkov, *The European Physical Journal Special Topics*, vol. 184, pp. 1–82 (June 2010) we found

**“Random tours of a knight on a chessboard starting from a corner square”**

as depicted in their Fig.1 (their page 6) and our display (17) here and their solution to the **“famous knight tour problem”**

“... the expected number of steps before the knight passes through the same corner square equals the number of edges in such a graph,  $M = 168$ .”

**“Markov chains provide us with a powerful tool for studying the structure of graphs and databases in detail.”**



(17)

Moreover, in addition to “Random tours of a knight on a chessboard starting from a corner square”, Blanchard *et al.* in “Markov chains or the game of structure and chance” (*op. cit.* 2010, page 2) observed that

“In the short story *The Mystery of Marie Rogêt* (1842/1843) **Edgar Allan Poe** based the details on a real crime and Poe’s detective character, the **Chevalier C. Auguste Dupin**, used his analytical abilities and the public newspaper reports to get into the mind of the murderer.

The meager light of information practically confined to times of visits and the approximate duration between the successive visits of characters was enough for the detective to unravel the mystery of the assassination of a young girl named *Marie Rogêt*. It was not surprising that the affair was regarded as little less than miraculous acquiring for *Chevalier’s* skills of ratiocination the credit of intuition.”

“There are few persons, even among the calmest thinkers, who have not occasionally been startled into a vague yet thrilling half-credence in the supernatural, by coincidences of so seemingly marvellous a character that, as mere coincidences, the intellect has been unable to receive them.

Such sentiments — for the half-credences of which I speak have never the full force of thought — such sentiments are seldom thoroughly stifled unless by reference to the doctrine of chance, or, as it is technically termed, the **Calculus of Probabilities**.

Now this Calculus is, in its essence, purely mathematical; and thus we have the anomaly of the most rigidly exact in science applied to the shadow and spirituality of the most intangible in speculation.”

Edgar Allan Poe, “The Mystery of Marie Rogêt”, *The Ladies’ Companion, a Monthly Magazine* (November & December 1842, February 1843), pub. William W. Snowden, New York. Poe’s text here copied from Blanchard *et al.* (*op. cit.* page 1).

The narrative of *The Mystery of Marie Rog t* is based upon the actual murder of Mary Cecilia Rogers, who disappeared on October 4, 1838, in New York City. Three years later, on July 25, 1841, she disappeared again. Her body was found floating in the Hudson River on July 28 in Hoboken, New Jersey. The details surrounding the case suggested she was murdered. Months later, the inquest still ongoing, her fian   was found dead, an act of suicide. By his side, a remorseful note and an empty bottle of poison were found.

Writing about Rogers, Poe tried to solve the aforementioned enigma by creating a murder mystery. As Poe wrote in a letter in 1842: "under the pretense of showing how Dupin ... unravelled the mystery of Marie's assassination, I, in fact, enter into a very rigorous analysis of the real tragedy in New York." Although there was intense media interest and immortalizing of a sort by Poe, the crime remains one of the most puzzling unsolved murders of New York City.

Text here from *Wikipedia*.




In 1942 Universal Pictures produced the gothic mystery film *The Mystery of Marie Roget* based on the Edgar Allan Poe story. Directed by Phil Rosen, the film starred Patric Knowles, Maria Ouspenskaya and Maria Montez.

Edgar Allan Poe's detective character, the Chevalier C. Auguste Dupin, appeared for the first time in "The Murders of the Rue Morgue" (1841). **With this story Edgar Allan Poe created the first fictional detective.**

*Dupin* was an outlandish romantic who inhabited candle-lit quarters in Paris. He was poor but of noble lineage, very experienced, a great smoker who wore green-tinted glasses.

Although some forty countries issued stamps commemorating the 50th Anniversary of Interpol (1923–1973), the International Police Organization, most issues were of a single

stamp with the Interpol symbol or a picture of the Headquarters Building. Only Nicaragua issued a large set with fictional detectives: "The 12 Most Famous Fictional Detectives" with *Dupin*: Scott C806, image AP104 (1972).

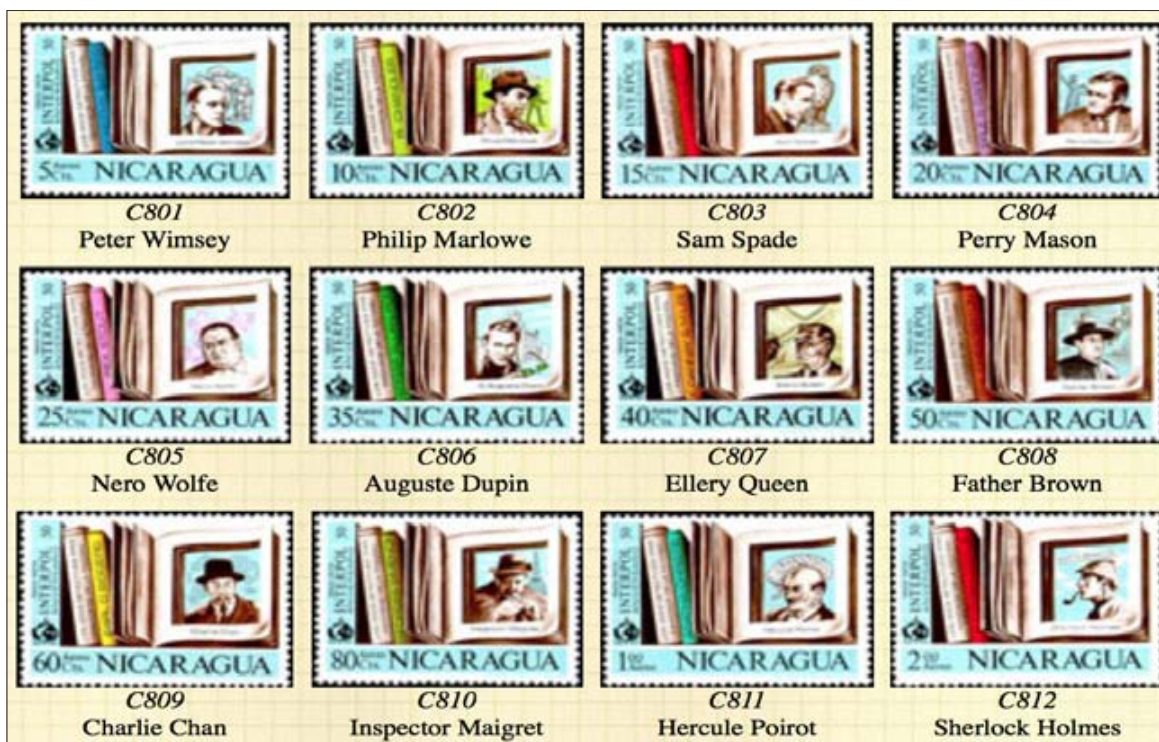


Lord Peter Wimsey, by Dorothy L. Sayers AP104

Designs (Book and): 10c, Philip Marlowe, by Raymond Chandler. 15c, Sam Spade, by Dashiell Hammett. 20c, Perry Mason, by Erle S. Gardner. 25c, Nero Wolfe, by Rex Stout. 35c, Auguste Dupin, by Edgar Allan Poe. 40c, Ellery Queen, by Frederick Dannay and Manfred B. Lee. 50c, Father Brown, by G. K. Chesterton. 60c, Charlie Chan, by Earl Derr Biggers. 80c, Inspector Maigret, by Georges Simenon. 1cor, Hercule Poirot, by Agatha Christie. 2cor, Sherlock Holmes, by A. Conan Doyle.

1972, Nov. 13 Litho. Perf. 14x13½			
C801	AP104	5c blue & multi	.30 .20
C802	AP104	10c blue & multi	.30 .20
C803	AP104	15c blue & multi	.30 .20
C804	AP104	20c blue & multi	.30 .20
C805	AP104	25c blue & multi	.30 .20
C806	AP104	35c blue & multi	.40 .25
C807	AP104	40c blue & multi	.40 .25
C808	AP104	50c blue & multi	.50 .30
C809	AP104	60c blue & multi	.70 .40
C810	AP104	80c blue & multi	.85 .50
C811	AP104	1cor blue & multi	1.10 .65
C812	AP104	2cor blue & multi	2.25 1.25





(18)

Display (??) from page 1 of "Detective Fiction on Stamps: Nicaragua (1972): 50th Anniversary of Interpol, 1923–1973" our 14 pp pdf file DetFic copied from <https://www.trussel.com/detfic/nicarag.htm> Scott C801–C812 (13 November 1972). Inscriptions on back, printed on top of gum, give thumbnail sketch of character and author. File updated 30 December 2017.



What game is being played on this  $7 \times 11$  “chessboard” in Matisse’s *La famille du peintre* (1911)?

Henri Émile Benoît Matisse (1869–1954), a French artist, was known for both his use of colour and his fluid and original draughtsmanship.

Awani Kumar (in a personal communication to George P. H. Styan) gave us “this interesting (and challenging to compose) Knight’s Tour. We see that the first eight perfect squares (1, 4, 9, . . . , 64) are in sequence clockwise from cell 1. They are also all a Knight’s move apart. All odd perfect squares and all even perfect squares are equidistant from the centre of the  $7 \times 11$  board, a *Figured Knight’s Tour* with aesthetic appeal.”



Niger 1998

45	66	41	6	73	4	39	18	53	10	13
42	63	44	1	40	7	52	9	12	17	54
65	46	67	74	5	72	3	38	19	14	11
62	43	64	71	2	75	8	51	16	55	20
47	68	33	60	35	50	37	76	23	26	15
32	61	70	49	30	77	58	25	28	21	56
69	48	31	34	59	36	29	22	57	24	27

Awani Kumar\* gave us this closed semi-magic Knight's Tour for the  $6 \times 10$  chessboard as depicted on a clown's suitcase and noted that

"There is no magic tour on the  $6 \times 10$  board. The smallest rectangular board on which a magic tour is possible is  $4 \times 18$  or  $6 \times 12$  that is a 72-cell board."

All column and row totals are semi-primes: all column totals are  $1830/10 = 183 = 3 \times 61$  and all row totals are a prime multiple of 5 with  $1830 = (60 \times 61)/2 = 6 \times 305$ .

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\*Personal communication from Awani Kumar to George P. H. Styan (15 September 2020).



"Clown on the town" (Lima, Peru) from the NAVIGATOR (Findings, facts and FIGURES) NP section: *Montreal Gazette*, page NP2 (5 August 2020).



## Key words and phrases

centro-symmetric matrix	Edgar Allan Poe (1809–1849) stamps from
famous fictional detectives	Bulgaria, Hungary, Monaco, USA
stamps from Nicaragua	<i>The Murders of the Rue Morgue</i> (1841)
Sir Arthur Conan Doyle (1859–1930)	<i>The Mystery of Marie Rogêt</i> (1842/1843)
stamp from Monaco	random tours of a knight
Chevalier C. Auguste Dupin	on a rectangular chessboard
famous knight tour problem	knight's random walk
figured Knight's Tour	with philatelic item from Somalia
Knight's Tour on clown's $6 \times 10$ chessboard	<i>Reversible Markov Chains and</i>
Andrei Andreyevich Markov (1856–1922)	<i>Random Walks on Graphs</i>
stamp from Liechtenstein	by David J. Aldous & James Allen Fill
Markov chains	single/double-flip property
Henri Émile Benoît Matisse (1869–1954)	"A Stamp A Day" collection of Mark Jochim
Matisse's <i>La famille du peintre</i> (1911)	transition probability matrix
stamp from Niger	YouTube video: PBS Infinite Series
	"Can a chess piece explain Markov chains?"
	by Kelsey Houston-Edwards

In our beamer files MPS-2 and MPS-3 we featured philatelic items from Argentina, Brazil, Bulgaria, Cambodia, Czechoslovakia, Finland, Hong Kong China, Germany (*Deutsche Post*), Great Britain (Royal Mail), Hungary, Italy, Japan, Liechtenstein, Macau China, Mali, Monaco, Netherlands Antilles, Nicaragua, Niger, Poland, Romania, St. Vincent & The Grenadines, San Marino, São Tomé & Príncipe, Somalia, South Africa, Tunisia, Turkey, Tuvalu, USA

as well as Gerald King Wonderland, Nichtsburg & Zilchstadt, and we recall that in beamer file MPS-1 we featured philatelic items from Botswana, Canada, Christmas Island, Croatia, Dominica, Finland, India, Korea, Kyrgyzstan, Poland, Ukraine.

The sheetlet “In Celebration”, display (6) above, of the 95th Birthday of Her Majesty Queen Elizabeth II was issued jointly by Ascension Island, Bahamas, British Antarctic Territory, British Virgin Islands, Falkland Islands, Gibraltar, Guernsey, Isle of Man, Jersey, South Georgia and the South Sandwich Islands, Tristan da Cunha.

