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Regression Analysis

A Useful Matrix Decomposition and Its Statistical Applications in Linear Regression

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It is well known that if $\mathbf{V}$ *is a symmetric positive definite* $n \times n$ *matrix, and* $(\mathbf{X} : \mathbf{Z})$ *is a partitioned orthogonal* $n \times n$ *matrix, then*

$$(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1} = \mathbf{X}'\mathbf{V}\mathbf{X} - \mathbf{X}'\mathbf{V}\mathbf{Z}(\mathbf{Z}'\mathbf{V}\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{V}\mathbf{X}. \qquad (*)$$

In this article, we show how useful we have found the formula $(*)$*, and in particular, its version*

$$\mathbf{Z}(\mathbf{Z}'\mathbf{V}\mathbf{Z})^{-1}\mathbf{Z}' = \mathbf{V}^{-1} - \mathbf{V}^{-1}\mathbf{X}(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}^{-1} := \dot{\mathbf{M}}, \qquad (**)$$

and present several related formulas, as well as some generalized versions. We also include several statistical applications.

Keywords BLUE; Frisch–Waugh–Lovell theorem; Löwner ordering; OLSE; Orthogonal projector; Partitioned linear model; Reduced linear model; Schur complement.

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1. Introduction

In this article, we consider the general linear model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \qquad (1.1)$$

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