International Conference on
Linear Algebra and its Applications
December 11-15, 2017

## REPORT

The Fourth DAE-BRNS Theme Meeting
on
Generation and Use of Covariance Matrices in the Applications of Nuclear Data

December 09-13, 2017

Editor
Shreemathi S. Mayya
Compiler
David Raj Micheal
January 15, 2018

MAHE honours save nature policy and limited number of this report are printed for internal circulation and for the communications with sponsors.

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# Overview of ICLAA 2017 \& Theme Meeting on Covariance Matrix 

International conference on Linear Algebra and its Applications-ICLAA 2017, third in its sequence, following CMTGIM 2012 and ICLAA 2014, held in Manipal Academy of Higher Education, Manipal, India in December 11-15, 2017.

Like its preceding conferences, ICLAA 2017 is also focused on the theory of Linear Algebra and Matrix Theory, and their applications in Statistics, Network Theory and in other branches of sciences. Study of Covariance Matrices, being part of Matrix Method in Statistics, has applications in various branches of


Chief Guest Address by Arjan Koning. Delegates on the dais: (from left) K. Manjunatha Prasad, Ravindra B. Bapat, Steve Kirkland, B. H. Venkataram Pai, S. Ganesan and Asha Kamath sciences. It plays crucial role in the study of measurement of uncertainty and naturally in the study of Nuclear Data. Theme meeting, which initially planned to be a preconference meeting, further progressed into an independent event parallel to ICLAA 2017, involving discussion on different methodology of generating the covariance information, training modules on different techniques and deliberations on presenting new research.

T.E.S. Raghavan delibrating on Nash-Equilibrium

About 167 delegates have registered for ICLAA 2017 alone ( 37 Invited +75 Contributory +04 Poster) and are from 17 different countries of the world. Interestingly, more than $80 \%$ are repeaters from the earlier conference and the remaining $20 \%$ are young students or scholars. In spite of a few dropouts due to unavoidable constraints, it is felt evident that the group of scholars with focus area of Linear Algebra, Matrix Methods in Statistics and Matrices and Graphs are not only consolidating, also growing as a society with a strong bond. ICLAA 2017 provided a platform for renowned Mathematicians and Statisticians to come together and discuss research problems, it provided ample of time for young scholars to present their contribution before eminent scholars. Every contributory speaker got not less than thirty minutes to present their results. Also, ICLAA 2017 was with several special lectures from senior scientists aimed at encouraging young scholars.

The sponsors of ICLAA 2017 are NBHM, SERB, CSIR and ICTP. Dr. Ebrahim Ghorbani and Dr. Zheng Bing are the two international participants benifited from ICTP grant for their international travel.

The conference was opened with an informal welcome and opening remark by K. Manjunatha Prasad (Organizing Secretary) and R. B. Bapat (Chairman, Scientific Committee). Invited talks and the special lectures were organized in 13 different sessions and contributory talks in 17 sessions. Poster presentation
was arranged on December 12, 2017.
A formal inaugural day joint function for ICLAA 2017 and DAE-BRNS theme meeting on covariance was held in the evening of December 11, 2017 starting at 7:15 PM, followed by the conference dinner. BHV Pai, Joint Director, MIT, MAHE, Manipal presided over the function. Professors Kirkland and Arjan Koning were the guests of honor. Asha Kamath, HOD, DOS delivered welcome address, Bapat, Chairman, Scientific Committee presented the overview of ICLAA 2017 and Ganesan, Chairman, Technical Committee, presented the overview of the Theme meeting. Following addresses by the chief guests, BHV Pai delivered the presidential address. Vote of thanks was offered by the organizing secretary.

(a) Carnatic Music (Vocal) by Sahana Udupa

(b) Yakshagana: Jatayu Moksha

## Cultural Programmes at Kota Shivarama Karantha Memorial Theme Park

Beside busy scientific schedule, a joint excursion to Kota Shivarama Karantha Memorial Theme Park was arranged on 13th evening, where a cultural program consisting of concert by Ms. Sahana Udupa and Yakshagana program by young artists guided by Mr. Narasimha Thunga held. The arrangements at Kota were with the cooperation of Kotathattu Grama Panchayat and its president Mr. Pramod Hande.

In an informal discussion, it has been consented by the present scientific committee and the organizing committee members that
(i) MAHE would continue to organize ICLAA 2020 in December 2020, the fourth in its sequence
(ii) Manjunatha Prasad would put up a proposal to organize ILAS conference in the earliest possible occasion (2022/23), in consultation with Kirkland
(iii) Manjunatha Prasad to initiate a dialog with the members in the present network to have Indian Society for Linear Algebra and its Application

ICLAA 2017 was concluded on December 15, 2017 with the valedictory session in which the participants have endorsed the idea of proceeding with the plan of ICLAA 2020. Dr. Asha Kamath (Head, Department of Statistics) welcomed the gathering, Dr. R.B. Bapat (Chairman, Scientific Committee) presided over the function, and chief guests of the function were Dr. K. P.S. Bhaskara Rao and Dr. Simo Puntanen. The tendative dates for the ICLAA 2020 have been scheduled as December 14-18, 2020.

## Invited Delegates: ICLAA 2017

1. RAFIKUL ALAM, Indian Institute of Technology Guwahati, INDIA
2. S. ARUMUGAM, Kalasalingam University, INDIA
3. OsKar Maria Baksalary, Adam Mickiewicz University, POLAND*

## *Absent


(a) R.B.Bapat

(c) Steve Kirkland

(e) Sharad S. Sane

(b) S. Ganesan

(d) Simo Puntanen

(f) Peter Schillebeeckx

Eminent Scientists are Honoured at Kota Shivarama Karantha Memorial Theme Park
4. R. BALAKRISHNAN, Bharathidasan University, INDIA*
5. RAVINDRA B. BAPAT, Indian Statistical Institute Delhi Centre, INDIA
6. B. V. RAJARAMA BHAT, Indian Statistical Institute Bangalore, INDIA
7. RAJENDRA BHATIA, Indian Statistical Institute, INDIA*
8. S. Parameshwara Bhatta, Mangalore University, india*
9. ZHENG BING, Lanzhou University, CHINA
10. Paritosh Biswas, von Karman Society, india*
11. ARUP BOSE, Indian Statistical Institute, INDIA*
12. SOMNATH DATTA, University of Florida, UNITED STATES
13. N. EAGAMBARAM, Former $D D G$, India
14. Ebrahim Ghorbani, K.N. Toosi University of Technology, iran, islamic republic of
15. Muddappa Seetharama Gowda, University of Maryland, Baltimore County, united states
16. Stephen John Haslett, Australian National University, australia
17. Jeffrey Hunter, Auckland University of Technology, new zealand
18. Stephen James Kirkland, University of Manitoba, Canada, canada
19. Bhaskara Rao Kopparty, Indiana University Northwest, united states
20. S. H. Kulkarni, Indian Institute of Technology Madras, india
21. Arbind Kumar Lal, Indian Institute of Technology Kanpur , indiA*
22. Helmut Leeb, TU Wien, Atominstitut, austria
23. André Leroy, Université $d^{\prime}$ Artois, france
24. Augustyn Markiewicz, Poznan University of Life Sciences, poland
25. S. K. Neogy, Indian Statistical Institute Delhi Centre, IndiA
26. SUKANTA PATI, Indian Institute of Technology Guwahati, INDIA
27. Simo Puntanen, University of Tampere, Finland
28. T. E. S. RaGhavan, University of Illinois at Chicago, united states
29. Sharad S. SANE, Indian Institute of Technology Bombay, INDIA
30. Bhaba Kumar Sarma, Indian Institute of Technology Guwahati, india*
31. Ajit IqBal Singh, The Indian National Science Academy, New Delhi, india
32. Martin Singull, Linköping University, sweden
33. K. C. Sivakumar, Indian Institute of Technology Madras, INDIA
34. Sivaramakrishnan Sivasubramanian, Indian Institute of Technology Bombay, india
35. Murali K. Srinivasan, Indian Institute of Technology Bombay, india
36. Ashish K. Srivastava, Saint Louis University, USA, united states*
37. Michael Tsatsomeros, Washington State University, united states

## Delegates Contributing Paper: ICLAA 2017

1. Adenike Olusola Adenidi, University of Abuja, Abuja, nigeria*
2. Fouzul Atik, Indian Statistical Institute, Delhi Centre, india
3. Mojtaba Bakherad, University of Sistan and Baluchestan, Zahedan, iran, islamic republic of*
4. SASMITA BARIK, Indian Institute of Technology Bhubaneswar, INDIA*
5. Debashis Bhowmik, Indian Institute of Technology Patna, india
6. Anjan Kumar Bhuniya, Visva-Bharati, Santiniketan, india*
7. Niranjan Bora, Dibrugarh University Institute of Engineering \& Technology, INDIA
8. MANAMI Chatterdee, Indian Institute of Technology Madras, india
9. Sriparna Chattopadhyay, NISER Bhubaneswar, india*
10. Kshittiz Chettri, SGC Tadong, Gangtok, india
11. Projesh Nath Choudhury, Indian Institute of Technology Madras, india
12. RANJAN KUMAR DAS, Indian Institute of Technology Guwahati, india
13. Pankaj Kumar Das, Tezpur University, india*
14. Soumitra Das, North Eastern Hill University, india
15. Rajaiah Dasari, Osmania University, india*
16. Biswajit Deb, Sikkim Manipal Institute of Technology, india
17. Amitav Doley, Dibrugarh University, india*
18. Dipti Dubey, Indian Statistical Institute Delhi Centre, india
19. SUPRIYO DUTTA, Indian Institute of Technology Jodhpur, IndiA*
20. Ramesh G., Indian Institute of Technology Hyderabad, india

[^0]21. Jadav Ganesh, Indian Institute of Technology Hyderabad, IndiA*
22. ARINDAM GHOSH, Indian Institute of Technology Patna, INDIA
23. Mahendra Kumar Gupta, Indian Institute of Technology Madras, india*
24. M. M. Holliyavar, K.L.E Society's Jagadguru Tontadarya College, india*
25. Akhlaq Husain, BML Munjal University Gurgaon, india*
26. Ahmad Jafarian, Islamic Azad university, Urmia, iran, islamic republic of*
27. Tanweer Jalal, National Institute of Technology, Srinagar, India*
28. SACHINDRANATH JAYARAMAN, IISER Thiruvananthapuram, india
29. P. SAM Johnson, National Institute of Technology Karnataka, indiA
30. Kamaraj K., Anna University, india
31. Mitra K., P. A. College of Engineering, india*
32. Nayan Bhat K., MAHE, Manipal, india
33. Debajit Kalita, Tezpur University, india*
34. M. RajEsh Kannan, Indian Institute of Technology Kharagpur, INDIA
35. Mounesha H. Kantli, K.L.E Society's Jagadguru Tontadarya College, india*
36. NiJara Konch, Dibrugarh University, india
37. MatJaz Kovse, Indian Institute of Technology Bhubaneswar, india
38. Sushobhan Maity, Visua-Bharati, Santiniketan, india*
39. Ranjit Mehatari, Indian Institute of Technology Kharagpur, india*
40. Vatsalkumar Nandkishor Mer, IISER Thiruvananthapuram, india
41. David Raj Micheal, mahe, Manipal, india
42. Ashma Dorothy Monteiro, MAHE, Manipal, india
43. Akash Murthy, Euprime, india*
44. Mukesh Kumar Nagar, Indian Institute of Technology Bombay, India
45. RaKESH NANDI, National Institute of Technology Raipur, INDIA*
46. Nupur Nandini, MAHE, Manipal, india
47. Mohammad Javad Nikmehr, K. N. Toosi University of Technology, iran, islamic republic of*
48. Divya Shenoy P., Manipal Institute of Technology, Manipal, india
49. Ramesh Prasad Panda, Indian Institute of Technology Guwahati, india
50. Rashmirekha Patra, Sambalpur University Institute of Information Technology, India*
51. Somnath Paul, Tezpur University, Assam, india*
52. Abhyendra Prasad, Indian Institute of Technology Patna, IndiA*
53. Rajkumar R., The Gandhigram Rural Institute - Deemed University, india*
54. B. R. Rakshith, University of Mysore, india
55. SonU Rani, Indian Institute of Technology Bhubaneswar, INDIA
56. Gokulraj S., Central University of Tamil Nadu, Thiruvarur, IndiA
57. Veeramani S., Indian Institute of Technology Hyderabad, indiA*
58. Deepak Sarma, Tezpur University, India*
59. Debashish Sharma, Gurucharan College, Silchar, india
60. Khalid Shebrawi, Al Balqa’Applied University, Jordan
61. Jyoti Shetty, Manipal Institute of Technology, Manipal, india
62. Adilson de Jesus Martins da Silva, University of Cape Verde, cape verde*
63. RANVEER SINGH, Indian Institute of Technology Jodhpur, INDIA
64. Manoj Solanki, S. V. College, (Autonomous), india*
65. M. A. Sriraj, Vidyavardhaka College of Engineering, Mysuru, india
66. LAVANYA SURIYAMOORTHY, Indian Institute of Technology Madras, INDIA
67. ANitha T., The Gandhigram Rural Institute - Deemed University, IndIA
68. Kurmayya Tamminana, National Institute of Technology Warangal, INDIA
69. Shendra Shainy V., Thiruvalluvar University, india
70. Balaji V., Thiruvalluvar University, INDIA
71. ANU Varghese, BCM College, Kottayam, india
72. Malathy Viswanathan, VIT University, india

## Delegates Presenting Poster: ICLAA 2017

1. Rajesh Kumar T. J., TKM College of Engineering, Kollam, Kerala, india
2. Mathew Varkey T. K., TKM College of Engineering, Kollam, Kerala, india
3. Sanjeev Kumar Maurya, Indian Institute of Technology (BHU) Varanasi, indiA*
4. Dhananjaya Reddy, Government Degree College, Puttur, india
5. P. G. Romeo, Cochin University of Science and Technology, India*

## Fourth DAE-BRNS Theme Meeting on Covariance Matrix

Fourth DAE-BRNS Theme Meeting on the generation and use of covariance matrices in the application of nuclear data, cosponsored by BRNS, started on December 09, 2017 with the welcome address by K. Manjunatha Prasad, Convener, Technical Committee and an opening remark by S. V. Suryanarayana, Technical Convener.

Technical sessions consisted of about 30 lectures, 3 tutorial sessions and plenary discussions. Lectures consisted of discussion on different methodologies such as applications of Least Square Methods, Generalized Least Square Methods, Bayesian Methods, Kalman Filter Methods and Total Monte Carlo Methods in generating covariance information of nuclear data measurements. Lectures and tutorials on TALYS nuclear model code were delivered.

In the panel discussion arranged on December 13, 2017, it has been observed that the culture of presenting covariance analysis of nuclear data measurements are to be encouraged further in the direction of India proceeding with developing its own evaluated nuclear data library. Though, the panel was satisfied with the initial progress in the development of subject, it is felt that more projects on the theme are to be encouraged to inculcate the tradition of presenting covariance evaluation in the measurements of nuclear data which would provide indigenous information for Indian nuclear reactor setup. It is also felt that a monograph on 'Matrix and Statistical Methods in the Measurements of Nuclear Data' which could serve as reference material for Master's and Doctoral programme. Members also felt that the time is more appropriate for working on a monograph instead of proceedings of conference.

Theme meeting was concluded with a valedictory session on December 13, 2017, welcomed by Asha Kamath, HOD, Department of Statistics, MAHE, presided by S. Ganesan, Chairman, Technical Committee, overview report by S. V. Suryanarayana, Technical Convener. Vote of thanks was delivered by Sripathi Punchithaya, Co convener, Technical Committee.

## Speakers in DAE-BRNS Theme Meeting

1. RUDRASWAMY B., Bangalore University, INDIA
2. SYLVIA BADWAR, North Eastern Hill University, INDIA
3. Abhishek Prakash Cherath, , india
4. Vidya Devi, IET Bhaddal Ropar Punjab, INDIA*
5. S. GANESAN, Bhabha Atomic Research Centre, INDIA
6. REETUPARNA GHOSH, North Eastern Hill University, INDIA
7. Betylda Jyrwa, North-Eastern Hill University, India*

## *Absent

8. Umasankari Kannan, Bhabha Atomic Research Centre, india*
9. Meghna Raviraj Karkera, MAHE, Manipal, india
10. Arjan Koning, IAEA, austria
11. ANEK KUMAR, Bhabha Atomic Research Centre, India
12. Rajeev Kumar, Bhabha Atomic Research Center, india
13. B. Lalremruata, Mizoram University, india
14. Helmut Leeb, TU Wien, Atominstitut, austria
15. Jayalekshmi M. Nair, VES Institute of Technology, india
16. Priyada Panikkath, Manipal Centre for Natural Sciences, india*
17. Sripathi Punchitaya K., Manipal Institute of Technology, Manipal, india
18. E. Radha, Indira Gandhi Centre for Atomic Research, IndiA*
19. Sangeetha Prasanna Ram, Vivekananand Education Society's Institute of Technology, india
20. Kallol Roy, Bharatiya Nabhikiya Vidyut Nigam Ltd, Kalpakkam, india*
21. Uttiyoarnab Saha, HBNI, IGCAR, india
22. Alok Saxena, Bhabha Atomic Research Centre, india
23. Peter Schillebeeckx, European Commission - Joint Research Centre, belgium
24. Y. Santhi Sheela, MAHE, Manipal, india
25. Henrik Sjöstrand, Uppsala University, sweden
26. S. V. Suryanarayana, Bhabha Atomic Research Centre, india


Theme Meeting Delegates

[^1]
## Acknowledgements

We acknowledge our sincere thanks to BRNS for being co sponsor for the event 'The Fourth DAE-BRNS Theme Meeting on Covariances'. We thank 'International Linear Algebra Society' for endorsing ICLAA 2017. We also acknowledge the support of National Board for Higher Mathematics (NBHM), Council of Scientific \& Industrial Research (CSIR), Science and Engineering Research Board (SERB) and Internatational Centre for Theoretical Physics (ICTP).


Our sincere thanks to G. Shankar Trust for their support. We thank Kota Grama Panchayat, particularly, its president Mr. Pramod Hande for making 'Kota Shivarama Karantha Memorial Theme Park' available for arranging cultural programme on December 13, 2017 evening.

We also thank the journals Special Matrices and Bulletin of Kerala Mathematical Association (BKMA) for their support by publishing special issues on the theme of the conference.

## Special Issues

## Special Matrices:

Articles in the focus area of (i) Linear Algebra, (ii) Matrices \& Graphs, and (iii) Matrix and Graph Methods in Statistics, not necessarily presented in the conference, may be submitted to a special issue of journal 'SPECIAL MATRICES' (https://www.degruyter.com/view/j/spma). Acceptance of the articles for the possible publication is subject to review norms set by the journal. For more details on the submission please visit the journal page given in the above link.

- All submissions to the Special Issue must be made electronically at http://www.editorialmanager.com/spma and will undergo the standard single-blind peer review systerm.
- The deadline for submission is April 15, 2018.
- Individual papers will be reviewed and published online as they arrive.
- Contributors to the Special Issue will benefit from:
- fair and constructive peer review provided by recognized experts in the field,
- Open Access to your article for all interested readers,
- no publication fees,
- convenient, web-based paper submission and tracking system - Editorial Manager,
- free language assistance for authors from non-English speaking regions;


## Bulletin of Kerala Mathematical Association:

All the articles submitted to ICLAA 2017 are eligible for the possible publication in a special issue of 'Bulletin of Kerala Mathematical Association' (indexed in MathSciNet), subject to review of its original scientific contribution. Full article may be submitted to any member of scientific advisory committee with the intention of submission of article for the special issue of BKMA.

- Article for the Special Issue may be submitted electronically at http://iclaa2017.com/submit-full-article-bkma/.
- Articles will undergo the standard single-blind peer review system.
- The template may be downloaded at www.iclaa2017.com
- The deadline for submission is February 15, 2018.
- Contributors to the Special Issue will benefit from:
- fair and constructive peer review provided by recognized experts in the field, no publication fees,
- no publication fees,
- convenient, web-based paper submission

We welcome every one for
ILA 2020.

(Dr. K. Manjunatha Prasad)


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5. Dr. Alok Saxena, Head, Nuclear Physics Division, BARC, Mumbai
6. Dr. K. C. Sivakumar, Indian Institute of Technology, Madras
7. Dr. Sivaramakrishnan Sivasubramanian, Indian Institute of Technology, Bombay

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Co-Chairman : Dr. Helmut Brand, Director, PSPH, MAHE, Manipal
Organizing Secretary : Dr. K. Manjunatha Prasad, MAHE, Manipal

## Members

1. Dr. Asha Kamath, MAHE, Manipal
2. Dr. Shreemathi S. Mayya, MAHE, Manipal
3. Dr. G. Sudhakara, Manipal Institute of Technology, Manipal
4. Dr. Vasudeva Guddattu, MAHE, Manipal
5. Prof. Ashma Dorothy Monteiro, MAHE, Manipal
6. Prof. Nilima, MAHE, Manipal
7. Dr. C. Ramesha, Manipal Institute of Technology, Manipal
8. Prof. Vinay Madhusudanan, Manipal Institute of Technology, Manipal
9. Mr. Alex Chandy, Director-Public Relations \& Media Communications, MAHE, Manipal
10. The Chief Warden, MIT Campus, MAHE, Manipal
11. The Chief Warden, Manipal Campus, MAHE, Manipal
12. Col. Badri Narayanan, Director, Purchase, MAHE, Manipal
13. Col. Prakash Chandra, General Services, MAHE, Manipal
14. Mr. B. P. Varadaraya Pai, Director-Finance, MAHE, Manipal
15. Ms. N. G. Sudhamani, MAHE, Manipal

## Accommodation

1. Dr. C. Ramesha, Manipal Institute of Technology, Manipal
2. Dr. K. Sripathi Punchithaya, Manipal Institute of Technology, Manipal
3. Prof. Vipin N., MAHE, Manipal
4. Ms. Nupur Nandini, MAHE, Manipal
5. Dr. Nagaraj N. Katagi, Manipal Institute of Technology, Manipal

## Finance and Certificates

1. Dr. Ravi Shankar, MAHE, Manipal
2. Ms. N. G. Sudhamani, MAHE, Manipal
3. Prof. Divya P. Shenoy, Manipal Institute of Technology, Manipal
4. Ms. Y. Santhi Sheela, MAHE, Manipal
5. Ms. Maria Mathews, MAHE, Manipal

## Food and Events

1. Dr. Vasudeva Guddattu, MAHE, Manipal
2. Prof. Ashma Dorothy Monteiro, MAHE, Manipal
3. Dr. Shradha Parsekar, MAHE, Manipal
4. Ms. Amitha Puranik, MAHE, Manipal

## Registration and Conference Proceedings

1. Prof. Ashma Dorothy Monteiro, MAHE, Manipal
2. Prof. Nilima, MAHE, Manipal
3. Dr. Sujatha H. S., Manipal Institute of Technology, Manipal
4. Prof. K. Arathi Bhat, Manipal Institute of Technology, Manipal
5. Ms. Meghna Raviraj Karkera, MAHE, Manipal

## Souvenir and Pre-conference Organization

1. Dr. Shreemathi S. Mayya, MAHE, Manipal
2. Prof. Vinay Madhusudanan, Manipal Institute of Technology, Manipal
3. Prof. Purnima Venkat, MAHE, Manipal
4. Mr. David Raj Micheal, MAHE, Manipal

## Transportation

1. Dr. G. Sudhakara, Manipal Institute of Technology, Manipal
2. Dr. Jerin Paul, MAHE, Manipal
3. Mr. Nayan Bhat K., MAHE, Manipal
4. Mr. Kiran Bhandari, MAHE, Manipal

## Student Volunteers

1. Mr. Utsav Arandhara, MAHE, Manipal
2. Mr. Sachit Ganapathy, MAHE, Manipal
3. Mr. Veerendra Nayak, MAHE, Manipal
4. Mr. Mahesha, MAHE, Manipal
5. Mr. Shreeharsha B. S, MAHE, Manipal
6. Ms. Shefina Fathima Hussain, MAHE, Manipal
7. Ms. Shruti Mokashi, MAHE, Manipal
8. Ms. Kavya Nair H, MAHE, Manipal
9. Ms. Nimisha N, MAHE, Manipal
10. Ms. Sivapriya J. G, MAHE, Manipal
11. Ms. Pallavi, MAHE, Manipal
12. Ms. Ashni, MAHE, Manipal
13. Ms. B. P. Dechamma, MAHE, Manipal
14. Mr. Shreenidhi S. M, MAHE, Manipal
15. Mr. Aditya Joshi, MAHE, Manipal


Inaugural Day Func-
tion: December 11, 2017

Sharad S Sane on
'Some Linear Algebra related questions in the theory of Block Design'


Prof Stephen Haslett and Prof Simo Puntanen during the Valedictory session

## Message

It is great honor for the Department of Statistics to organize the International Conference on Linear Algebra and its Applications, 2017 and The Fourth DAE-BRNS Theme Meeting on Generation and Use of Covariance Matrices in the Applications of Nuclear Data from December 09 to 15, 2017.

The conference and theme meeting aim at providing scientific platforms to all the particpants to congregate and interact with subject experts. The ICLAA 2017 covers a number of plenary talks


Dr. Asha Kamath and oral presentations on recent advances in Linear Algebra and its applications to different specialities. Theme meeting covers several lectures, tutorials and presentations of new research on the methodology involving statistics and matrix theory in the applications of nuclear data.

I am sure that all the participants will have an enlightening and enriching experiences through the deliberations of this conference. It is noteworthy to mention that there is an overwhelming response to conference. About 200 delegates across the country and also from abroad are participating.

I am very thankful to our management and to all my colleagues for their unstinted help in organizing this conference.

## Dr. Asha Kamath

Associate Professor \& Head
Department of Statistics, PSPH
Manipal Academy of Higher Education, Manipal



# From the Desk of Chairman (DAE-BRNS Theme Meeting) 

The fourth DAE-BRNS Theme Meeting on "Generation and use of Covariance Matrices in the Applications of Nuclear Data", Dec. 9-13, 2017, being hosted by the Department of Statistics, Manipal University, Manipal, Karnataka is a very unique scientific event dealing with the DAE-BRNS sponsored foundational efforts in nuclear data science. Error analysis and propagation of errors are generic topics in all subject areas studied by human civilization. Basic sciences, applied sciences, engineering studies, health sciences, weather predictions, economic studies all should employ a non-adhoc assignment of errors in various attributes and in integral results that are encountered, as part of big data science. In the Indian context of Bhabha's 3 -stage nuclear programme, nurturing efforts towards indigenous evaluation of basic nuclear data, processing and integral testing are essential. These research and development efforts for safe and efficient operation of nuclear systems include specialized


Dr. S. Ganesan topics on error specifications. The specification of errors, by basic definition, is incomplete without specification of correlations. Progress achieved thus far, interesting scope and challenges to extend this important activity, in the Indian context, are expected to be intensely discussed in this Theme Meeting. As a result of the DAE-BRNS projects at Manipal, Mizoram, Vadodara, Calicut, Bangalore etc., in the Indian context, interestingly, more attention is now being given to covariance error analysis in some of the basic nuclear physics experiments performed in collaboration with BARC. These Indian covariance data are encouraged to be coded in the IAEA-EXFOR database. The foundational efforts needed to start making Indian evaluation of nuclear data include the ability to digest the covariance methodologies. India is new to the concept of nuclear data evaluation and is in the lower part of the learning curve but rapid progress is being made as can be seen from the papers in this Theme Meeting.

Confidence margins in integral design parameters of nuclear reactor plants need to be assessed and specified for regulatory purposes based on a non-ad hoc scientific approach based upon a firm scientific foundation. This strictly involves characterization of errors with correlations and their propagation. Covariance error matrices, their generation, processing and propagation in nuclear data thus play an important basic role. Methods, such as, Total Monte Carlo Approach, Unified Monte Carlo Approach in addition to covariance approach are being evolved around the world. The phrase "covariance methodology" has become a technical phrase to include all such studies in error characterization and propagation. In my assessment, the academic institutions and training in national laboratories in India across all scientific and engineering disciplines should include basic courses on error and their correlations in curricula, such as, in 1) regular Under Graduate and Post Graduate courses, 2) as foundation course in research methodologies for doctoral programs, and, 3) advanced electives (optional) for researchers in data science on error propagation with covariance, as part of big data science analytics.

I wish the theme meeting all success.

S. Ganesan<br>Formerly Raja Ramanna Fellow<br>Reactor Physics Design Division<br>Bhabha Atomic Research Centre, Mumbai, India

## Memories from Theme Meeting



Peter Schillebeeckx charing a session and introducing S. Ganesan


Arjan Koning on Exact uncertainty propagation from nuclear data to technology with Total Monte Carlo Method

Arjan Koning, Henrik Sjöstrand and Helmut Leeb at Malpe Beach on December 12, 2017 evening


## Technical Committee: DAE-BRNS Theme Meeting

1. Dr. S. Ganesan, Raja Ramanna fellow of the DAE, BARC, India (Chairman)
2. Dr. Umasankari Kannan, Head, RPDD, BARC, India
3. Dr. Manjunatha Prasad Karantha, Convener, MAHE, Manipal, India
4. Dr. Helmut Leeb, Atominstitut, Technische Universitat Wien, Austria
5. Dr. Alok Saxena, Head, NPD, BARC, India
6. Dr. S. V. Suryanarayana, NPD, BARC, India (Technical Convener)
7. Dr. Peter Schillebeeckx, European Commission, JRC, Belgium
8. Dr. K. Sripathi Punchithaya, MIT, MAHE, Manipal, India (Co-convener)


Alok Saxena delivering talk on 'An overview of nuclear data activities in India'

Rajeev Kumar delivering talk on 'Covariance analysis in reactor physics experiments'



## B. Lalremruta receiving

 momento from S. Ganesan. On the dias: (from left) Sripathi Punchithaya, SV Suryanarayana and Peter SchillebeeckxHelmut Leeb delivering talk on 'Generalized least squares method: reformulation suitable for large scale nuclear data evaluation,


Manjuatha Prasad, Helmut Leeb, Arjan Koning, Mohamed Musthafa and Henrik Sjöstrand discussing on the schedule during tea break

## Program: DAE-BRNS Theme Meeting

## December 09, 2017 (Saturday)

09:00-09:10 K. Manjunatha Prasad: Welcome Address
09:10-09:20 SV Suryanarayana: Opening Remarks
SESSION 1; Chair Person: Peter Schillebeeckx
09:20-10:10 Srinivasan Ganesan: Advances in nuclear data covariance in the Indian Context
10:10-11:00 Helmut Leeb: Bayesian evaluation methods and uncertainty determination I
11:00-11:20 Tea Break
SESSION 2; Chair Person: Srinivasan Ganesan
11:20-12:10 SV Suryanarayana: Surrogate nuclear reactions for determining compound nuclear reaction cross sections of unstable nuclei for fusion technology applications

12:10-13:00 B. Lalremruta: Measurement of neutron capture cross-sections for ${ }^{70} \mathrm{Zn}$ at spectrum averaged energies of $0.41,0.70,0.96$ and 1.69 MeV

13:00-14:30 Lunch Break
SESSION 3; Chair Person: Helmut Leeb
14:30-15:30 Peter Schillebeeckx: Neutron time-of-flight cross section measurement and its applications- I
15:30-16:00 Sripathi Punchithaya: Sensitivity analysis of estimation of efficiency of HPGe detector in the energy range of $0.050-1.500 \mathrm{MeV}$ using different linear parametric functions

16:00-16:20 Tea Break
16:20-18:00 MU Team: Tutorials on covariance generation in nuclear data

## December 10, 2017 (Sunday)

SESSION 4; Chair Person: Srinivasan Ganesan
09:00-10:00 Kallol Roy: Bayesian estimation and its application in data interpolation-I
10:00-10:50 Arjan Koning: Exact uncertainty propagation from nuclear data to technology with Total Monte Carlo Method-I

10:50-11:10 Tea Break

## SESSION 5; Chair Person: SV Suryanarayana

11:10-12:00 Helmut Leeb: Bayesian evaluation methods and uncertainty determination - II
12:00-13:00 Kallol Roy: Bayesian estimation and its application in data interpolation-II
13:00-14:30 Lunch Break
SESSION 6; Chair Person: Helmut Leeb

14:30-15:30 Arjan Koning: Exact uncertainty propagation from nuclear data to technology with Total Monte Carlo- II

15:30-16:00 Rajeev Kumar: Covariance analysis in reactor physics experiments
16:00-16:20 Tea Break

## SESSION 7; Chair Person: Alok Saxena

16:20-16:50 Reetuparna Ghosh: Measurement of and uncertainty propagation of the $(\gamma, n)$ reaction cross section of ${ }^{58} \mathrm{Ni}$ and ${ }^{59} \mathrm{Co}$ at 15 MeV bremsstrahlung

16:55-17:25 Anek Kumar: Introduction to covariance files in ENDF/B library
17:30-18:00 B. Rudraswamy: Efficiency calibration of HPGe detector and covariance analysis

## December 11, 2017 (Monday)

SESSION 8; Chair Person: Alok Saxena
09:00-10:00 Arjan Koning: TALYS nuclear model code TENDL evaluated nuclear data library- Part I
10:00-10:50 Henrik Sjostrand: Adjustment of nuclear data libraries using integral benchmarks 10:50-11:10 Tea Break

SESSION 9; Chair Person: Mohamed Musthafa
11:10-12:00 Peter Schillebeeckx: Neutron time-of-flight cross section measurement and its applications-II
12:00-13:00 Arjan Koning: TALYS nuclear model code TENDL evaluated nuclear data library-II
13:00-14:30 Lunch Break

## SESSION 10; Chair Person: Asha Kamath

14:30-15:30 Simo Puntanen: On the role of the covariance matrix in the linear statistical model
15:30-16:00 Alok Saxena: An overview of nuclear data activities in India
16:00-16:20 Tea Break
16:20-18:30 Arjan Koning: Tutorial
19:15-20:00 Inaugural Day Function of ICLAA 2017
20:00-21:00 DINNER

## December 12, 2017 (Tuesday)

## SESSION 11; Chair Person: B. Lalremruta

09:00-09:40 Henrik Sjostrand: Choosing nuclear data evaluation techniques to obtain complete and motivated covariances

09:40-10:30 Y Santhi Sheela: Covariance analysis in neutron Activation Measurements of ${ }^{59} \mathrm{Co}(n, 2 n){ }^{58} \mathrm{Co}$ and ${ }^{59} \mathrm{Co}(n, \gamma){ }^{60} \mathrm{Co}$ reactions in the MeV region

10:30-11:00 Jayalekshmi Nair: Error propagation techniques
11:00-11:30 Tea Break
SESSION 12; Chair Person: SV Suryanarayana
11:20-12:20 Peter Schillebeeckx: Adjustment of model parameters by a fit to experimental data
12:20-13:00 Uttiyornab Saha: Covariance matrices of DPA Cross Sections from TENDL-2015 for Structural Elements with NJOY-2016 and CRaD Codes

13:00-14:30 Lunch Break

## SESSION 13; Chair Person: Helmut Leeb

14:30-15:00 Sangeetha Prasanna Ram: A stochastic convergence analysis of random number generator as applied to error propogation using Monte Carlo method and unscented transformation technique

15:00-15:30 Abhishek Cherath: A case study on the cross section data of ${ }^{232} T h(n, 2 n)^{231} T h$ : A look, with a covariance analysis at the 1961 data of Butler and Santry (EXFOR ID 12255)

15:30-16:00 Meghna R Karkera: To be announced
16:00-16:20 Tea Break

## SESSION 14; Chair Person: Srinivasan Ganesan

16:20-16:50 Betylda Jyrwa: Measurement of Neutron Induced Reaction Cross Sections for ${ }^{64} \mathrm{Ni}(n, \gamma){ }^{65} \mathrm{Ni}$ and ${ }^{96} \mathrm{Zr}(n, \gamma){ }^{97} \mathrm{Zr}$ at $\mathrm{En}=0.025 \mathrm{eV}$

16:50-18:20 MU Team: Tutorials on covariance generation in nuclear data

## December 13, 2017 (Wednesday)

## SESSION 15; Chair Person: Srinivasan Ganesan

09:00-10:00 Helmut Leeb: Generalized least squares method: reformulation suitable for large scale nuclear data evaluation

10:00-10:30 Photo Session
10:30-11:00 Vidya Devi: Calculating efficiencies and their uncertainties propagation in efficiency
11:00-11:30 Tea Break

## 11:30-13:00 Panel Discussion

Title: In the Indian context, the current status and road map for the generations and use of covariance matrices in nuclear data
Panel Members: Dr. Helmut Leeb, Dr. Peter Schillebeeckx, Dr. S. Ganesan, Dr. Alok Saxena, Dr. K. Manjunatha Prasad, Dr. Sreekumaran Nair, Dr. Suryanarayana, Dr. Arjan Koning, Dr. B K Nayak, Dr. Sripathi Punchithaya

13:00-14:30 Lunch Break
14:30-16:00 VALEDICTORY
16:00-19:00 Cultural Program at Karantha Bhavan, KOTA
19:00-20:00 Dinner at Karantha Bhavan, KOTA


Asha Kamath welcoming the delegates on the dais: (from left) Sripathi Punchithaya, S. Ganesan, S.V. Suryanarayana and Peter Schillebeeckx

# Abstracts: DAE-BRNS Theme Meeting 

# Efficiency calibration of HPGe detector and covariance analysis 

B.Rudraswamy, ${ }^{1}$ Imran Pasha ${ }^{2}$, S. Ganesan ${ }^{3}$, E.Radha ${ }^{4}$ and B.S Shivashankar ${ }^{5}$<br>1,2 Dept.of Physics, Bangalore University, Bengaluru-560056, Inida.<br>${ }^{1}$ brudraswamy@gmail.com, ${ }^{2}$ imranp $905 @$ gmail.com<br>${ }^{3}$ Raja Ramanna Fellow, Reactor Physics Division, Reactor Design and Development Group Bhabha Atomic Research Centre, Mumbai - 400085, India. ganesan555@gmail.com<br>${ }^{4}$ RPD,ROMG, IGCAR, Tamilnadu, 603102, India. radha@igcar.gov.in<br>${ }^{5}$ Department of Statistics, Manipal University, Manipal, India. shivu1982@hotmail .com


#### Abstract

Energy-efficiency calibration of the HPGe detector and corresponding covariance analysis may be considered as an integral parts in the determination of nuclear cross-section. In the present work, gamma spectroscopy measurement using HPGe detector (DSG-German) coupled to a PC-based 16-K channel MultiportII MCA(Canberra), efficiency calibration and corresponding covariance analysis have been investigated. The standard calibration sources considered for the analysis are ${ }^{133} \mathrm{Ba},{ }^{22} \mathrm{Na},{ }^{137} \mathrm{Cs}$ and ${ }^{60} \mathrm{Co}$. The covariance information obtained for the efficiencies of the HPGe detector with respect to $\gamma$-lines of standard calibration sources is further employed in the covariance analysis of efficiencies of the HPGe detector with respect to characteristic $\gamma$-lines of the reaction product ${ }^{116 m} \mathrm{In}$.

The efficiency ( $\epsilon$ ) of detector has been estimated for various energies of $\gamma$ - lines of the calibration source $\left(E_{\gamma}\right)$ with the inclusion of correction factor for coincidence summing $K_{c}[1]$ by the standard expression $$
\begin{equation*} \epsilon=\epsilon\left(E_{\gamma}\right)=\frac{C K_{c}}{I_{\gamma} A_{0} e^{-\lambda t}} \tag{7.1} \end{equation*}
$$


The uncertainty in efficiency ( $\Delta \epsilon_{i}$, where $i=1$ to 6 corresponds to $\epsilon_{1}\left(E_{\gamma 1}\right)$ to ( $E_{\gamma 6}$ ) respectively) is obtained using partial uncertainties ( $e_{i}(r)$ ), where attribute number $r=1,2,3$, and 4 corresponds to the attributes $C, I_{\gamma}, A_{0}$ and $\lambda$ respectively [2], [3]

$$
\left(\Delta \epsilon_{i}\right)^{2}=\left(\frac{\Delta C_{i}}{C_{i}} \epsilon_{i}\right)^{2}+\left(\frac{\Delta I_{\gamma i}}{I_{i}} \epsilon_{i}\right)^{2}+\left(\frac{\Delta A_{o i}}{A_{o i}} \epsilon_{i}\right)^{2}+\left(\frac{\Delta \lambda_{i}}{\lambda_{i}} \epsilon_{i}\right)^{2}
$$

The presence of common errors in attributes 3 and 4 affect the uncertainties in $\epsilon_{i}$ and $\epsilon_{j}$ simultaneously. Therefore it is mandatory to consider covariance matrix

$$
V_{\epsilon i j}=\sum_{r=1}^{4} e_{i}(r) S_{i j}(r) e_{j}(r) ; i, j=1,2, \ldots, 6
$$

where $S_{i j}$ is micro-correlation within the attribute. The macro-correlation matrix corresponding to correlation between errors in $\epsilon_{i}$ and $\epsilon_{j}$ is given by

$$
\begin{equation*}
C_{\epsilon i j}=\frac{V_{\epsilon i j}}{\Delta \epsilon_{i} \Delta \epsilon_{j}} \tag{7.2}
\end{equation*}
$$

The efficiency $\epsilon_{i}$ and correlation matrix $C_{\epsilon i j}$ for various $\gamma$ - line energies of the calibration sources have been obtained by substituting the data sequentially in Eq. (7.1) and Eq. (7.2).

These results are further utilized to obtain efficiency of the detector with respect to characteristic $\gamma$-photons of energy $E_{\gamma c}$ and correlation matrix $C_{\gamma c}$ of the reaction product ${ }^{116 m} I n$. The formalism is
as follows; Consider the log transformation of Eq. (7.1) $z_{i}=\ln \left(\epsilon_{i}\right)$. Then elements of the covariance matrix $V_{z}$ are of form $V_{z i j}=\frac{V_{i i j}}{\epsilon_{i} \epsilon_{j}}$. The log transformed efficiencies can be reproduced using the fitting function $z_{i} \approx \sum_{k}^{m} p_{k}\left(\ln \left(E_{\gamma i}\right)\right)^{k-1}$ where $p_{k}$ is the $k^{t h}$ fitting parameter. In matrix notation, the fitting function can be conveniently represented as $z \approx A P$, where $A$ is an $n \times m$ matrix, whose elements are $A_{i k}=\left(\ln \left(E_{\gamma i}\right)\right)^{k-1}$. The least square approach to obtain best fit parameters $P$ is to minimize $\chi^{2}=[Z-A P]^{T} V_{z}^{-1}[Z-A P]$. The corrected efficiency w.r.t reaction product ${ }^{116 m}$ In has been obtained by incorporating the gamma ray self attenuation factor in the present study [4].
Keywords: covariance, correlation
AMS subject classifications. $62 \mathrm{H} 20 ; 62 \mathrm{~J} 10$

## References

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# Advances in nuclear data covariance in the Indian context 

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#### Abstract

Error analysis is generic to all subject areas studied by human civilization. Basic science, applied science and engineering studies should all employ a non-adhoc assignment of errors in various attributes and in integral results that are encountered, as part of big data science. In the Indian context of Bhabha's 3 -stage nuclear programme [1], nurturing efforts towards indigenous evaluation of basic nuclear data, processing and integral testing are essential [2]. These research and development efforts for safe and efficient operation of nuclear systems include specialized topics on error specifications. The specification of errors, by basic definition, is incomplete without specification of correlations. Progress achieved thus far, interesting scope and challenges to extend this important activity, in the Indian context, are presented. In the Indian context, interestingly, more attention is now being given to covariance error analysis in some of the basic nuclear physics experiments. See, for instance, Refs. [2]-[7]. These Indian covariance data are encouraged to be coded in the IAEA-EXFOR [8] database. The foundational efforts needed to start making Indian evaluation of nuclear data are described. Keywords: nuclear data covariance, errors and correlations, big data science, Indian nuclear power programme, EXFOR compilations, generalized least squares, evaluated nuclear data files, error propagation studies, confidence margins, advanced nuclear power plant designs


AMS subject classifications. 62P35, 62J12, 62 J 10

## References

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[7] R. Ghosh et al.. Measurement of photo-neutron cross-sections of Gd and Ce using bremsstrahlung with an end-point energy of 10 MeV . J. Radioanal. Nucl. Chem, 2017. (https://doi.org/10.1007/s10967-017-5535-0)
[8] N. Otuka et al.. Towards a more complete and accurate experimental nuclear reaction data library (EXFOR): International collaboration between nuclear reaction data centres (NRDC). Nuclear Data Sheets, 120:272-276, 2014.

# Measurement of neutron induced reaction cross-sections for ${ }^{64} \mathbf{N i}(\mathbf{n}, \gamma){ }^{65} \mathbf{N i}$ and ${ }^{96} \mathbf{Z r}(\mathbf{n}, \gamma){ }^{97} \mathbf{Z r}$ at $\mathbf{E n}=\mathbf{0 . 0 2 5} \mathbf{e V}$ 

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#### Abstract

Neutron induced reaction cross-sections for structural materials Zr and Ni are basic data for evaluation of the processes in materials under irradiation in nuclear reactors. The reaction cross-sections for ${ }^{64} \mathrm{Ni}(\mathrm{n}$, $\gamma)^{65} \mathrm{Ni}$ and ${ }^{96} \mathrm{Zr}(\mathrm{n}, \gamma){ }^{97} \mathrm{Zr}$ at $\mathrm{En}=0.025 \mathrm{eV}$ have been experimentally determined using activation and offline $\gamma$-ray spectrometric technique. Nuclear reactors are the major neutron sources. The thermal neutron energy of 0.025 eV was used from the reactor Critical Facility at BARC, Mumbai. The reactor is designed for a nominal fission power of 100 W with an average flux of $10^{8} \mathrm{n} / \mathrm{cm}^{2} / \mathrm{s}$. The experimentally determined reactions cross-sections from present work are compared with the existing literature data available in IAEA-EXFOR along with the evaluated nuclear data libraries of ENDF/B-VII.1, CENDL-3.1 and JEFF-3.2 and are found to be in close agreement. This work also includes the covariance analysis of efficiency calibration of HPGe detector using the ${ }^{152} E u$ standard sources. The sources of errors such as source activity, gamma ray abundance, gamma ray counts and half-life of radioactive nuclide are carefully accounted for in the propagation of errors and the correlations between these measurements are considered to derive


the covariance information for efficiency of HPGe detector at different $\gamma$-ray energies. Covariance analysis and generation of covariance matrix of the measurement of reaction cross section ${ }^{64} \mathrm{Ni}(\mathrm{n}, \gamma){ }^{65} \mathrm{Ni}$ and ${ }^{96} \mathrm{Zr}(\mathrm{n}$, $\gamma)^{97} \mathrm{Zr}$ at $\mathrm{En}=0.025 \mathrm{eV}$ is still in continuation.
Keywords: reaction cross section, nuclear data libraries
AMS subject classifications. 81V35

# Exact uncertainty propagation from nuclear data to technology with Total Monte Carlo 

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#### Abstract

A revolutionary nuclear data system is presented which connects basic experimental and theoretical nuclear data to a large variety of nuclear applications. This software system, built around the TALYS nuclear model code, has several important outlets: - The TENDL nuclear data library: complete isotopic data files for 2808 nuclides for incident gamma's, neutrons and charged particles up to 200 MeV , including covariance data, in ENDF and various processed data formats. In 2017, TENDL has reached a quality nearing, equaling and even passing that of the major data libraries in the world. It is based on reproducibility and is built from the best possible data from any source - Total Monte Carlo: an exact way to propagate uncertainties from nuclear data to integral systems, by employing random nuclear data libraries and transport, reactor and other integral calculations in one large loop. This can be applied to criticality, damage, medical isotope production, etc. - Automatic optimization of nuclear data to differential and integral data simultaneously by combining the two features mentioned above, and a combination of Monte Carlo and sensitivity analysis.

Both the differential quality, through theoretical-experimental comparison of cross sections, and the integral performance of the entire system will be demonstrated. The impact of the latest theoretical modeling additions to TALYS on differential nuclear data prediction will be outlined, and the effect on applications. Comparisons with the major world libraries will be shown. The effect of various uncertainty methods on the results will be discussed.


Keywords: nuclear data, nuclear reactions, TALYS, TENDL, Total Monte Carlo
AMS subject classifications. 62P35, 81V35

# TALYS nuclear model code and TENDL evaluated nuclear data library <br> A. J. Koning <br> Nuclear Data Section, IAEA. A.Koning@iaea. org 


#### Abstract

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Both the differential quality, through theoretical-experimental comparison of cross sections, and the integral performance of the entire system will be demonstrated. The impact of the latest theoretical modeling additions to TALYS on differential nuclear data prediction will be outlined, and the effect on applications. Comparisons with the major world libraries will be shown. The effect of various uncertainty methods on the results will be discussed.


Keywords: nuclear data, nuclear reactions, TALYS, TENDL, Total Monte Carlo
AMS subject classifications. 62P35, 81V35

# Introduction to covariance files in ENDF/B library 

Anek Kumar

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#### Abstract

One of the important aspects of nuclear data and of cross sections in particular is that the various data tend to be correlated to an important degree through the measurement processes and the different corrections made to the observable quantities to obtain the microscopic cross sections. In many applications when one is interested in estimating the uncertainties in calculated results due to the cross sections, the correlations among the data play a crucial role.

In principle, the uncertainties in the results of a calculation due to the data uncertainties can be calculated, provided one is given all of the variances in and covariances among the data elements. The formalism and formats for representing data covariances in ENDF/B-V were extended to cover all neutron cross section data in the files. The format of covariances data in ENDF/B formatted nuclear data library will be discussed in the paper.


Keywords: nuclear data, covariance files, ENDF/B library
AMS subject classifications. 81V35, 62P35

# Covariance analysis in reactor physics experiments 

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#### Abstract

Experimental reactor physics is an essential element of physics design of a nuclear reactor and plays an important role in the safe design and operation of nuclear reactors. Approximations in modelling the reactor using computer codes and the 'uncertainty in the nuclear data' that goes as input into these codes contribute to the uncertainty of the theoretically computed design parameters. Reactor physics experiments provide estimates of the uncertainty in the design by comparing the measured and computed values of these parameters.

Error propagation in the nuclear data evaluation is carried out properly by doing the covariance analysis. Availability of new neutron cross section covariance data have allowed the quantification of the impact of current nuclear data uncertainty on the design parameters of advanced reactors for example Gen-IV reactors. Also, uncertainty propagation using covariance matrices in nuclear data results covariance matrices of the desired set of computed integral parameters of reactor design. Since the computed design parameters are compared with the measurement, hence it is desirable that uncertainty in the measured data obtained by carrying out the reactor physics experiments should be expressed in covariance matrices.

A thorium fuel cycle based advanced heavy water reactor (AHWR) is being designed in Reactor Physics Design Division, BARC. A zero power critical facility (CF) was commissioned to generate the experimental data for physics design validation of AHWR. A number of experiments were carried out in CF which includes the measurement of differential/integral parameters and various reaction rates. The covariance analysis of these measurement will be carried out to generate the relevant covariance matrices.


Keywords: nuclear data covariance, error propagation studies
AMS subject classifications. 62P35, 62J12, 62J10

# Measurement of neutron capture cross-sections for ${ }^{70} \mathbf{Z n}$ at spectrum averaged energies of $0.41,0.70,0.96$ and 1.69 MeV 

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#### Abstract

The cross sections of the ${ }^{70} \mathrm{Zn}(n, \gamma)^{71} \mathrm{Zn}^{m}\left(T_{1 / 2}=3.96 \pm 0.05 \mathrm{hrs}\right)$ reaction have been measured relative to the ${ }^{197} A u(n, \gamma){ }^{198} A u$ cross sections at four incident energies $\langle E n\rangle=0.41,0.70,0.96$ and 1.69 MeV using a ${ }^{7} \operatorname{Li}(p, n){ }^{7} B e$ neutron source and activation technique. The experiment was performed at the Folded Tandem Ion Accelerator (FOTIA) Facility, Nuclear Physics Division, Bhabha Atomic Research Centre (BARC), Mumbai. The protons at $2.25,2.6,2.80$ and 3.50 MeV after passing through a beam collimator ( 0.5 cm in diameter) bombarded $\sim 2.0-\mathrm{mg} / \mathrm{cm}^{2}(37.4 \mu \mathrm{~m})$ thick natural lithium target to produce neutrons through the ${ }^{7} L i(p, n)^{7} B e$ reaction ( $E_{t h}=1.881 \mathrm{MeV}$ ). The proton beam energy spread is $\pm 0.02 \mathrm{MeV}$. The cross section of this reaction has been measured for the first time in the MeV region. Detail data analysis procedure, uncertainty analysis and comparison of the newly measured cross sections with theoretical cross sections predicted by TALYS-1.8 and evaluated data libraries will be presented.


Keywords: neutron capture cross section, ${ }^{7} \operatorname{Li}(p, n)^{7} B e$ reaction, activation technique
AMS subject classifications. 81V35

## References

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# Bayesian evaluation methods and uncertainty determination: an overview of recent methods 

H. Leeb

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#### Abstract

The aim of nuclear data evaluation is the generation of consistent and reliable sets of nuclear data and associated uncertainties which comprise reaction cross section, decay rates, fission yields and related properties of atomic nuclei. The evaluation process should combine the available experimental data with up-to-date nuclear theory in order to assess our best knowledge of these quantities and their uncertainties. This request is best satisfied by evaluation methods based on Bayesian statistics. In this presentation an overview of the available Bayesian methods in nuclear data evaluation is given. In recent years there is increasing awareness about the importance of the inclusion of so-called model defects for reliable evaluations and uncertainty estimates. Therefore current attempts to account for model defects will be discussed. In this context a recently developed Bayesian evaluation method with statistically consistent treatment of model defects will be presented in more detail.


Keywords: Bayesian evaluation technique, data analysis
AMS subject classifications. 62P35, 62P30

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# Generalized least square method: reformulation suitable for large scale data evaluations 

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#### Abstract

The increase of computational power and the availability of large storage enable the simultaneous evaluation of great sets of data in science and economics. In general these sets of observed data are not sufficiently dense and must be complimented by a-priori knowledge, usually described by models. Frequently practitioners use the generalized least square method (GLS) which allows a consistent combination of observations and a-priori knowledge. The GLS is a special form of Bayesian evaluation technique and requires for its application the construction of a prior covariance matrix for all observables included in the evaluation. For large scale evaluations this may result in a prior covariance matrix of intractable size. Therefore a mathematically equivalent formulation of the GLS-method was developed which does not require the explicit determination of the prior [1]. The modified GLS-method can deal with an arbitrary number of data. The proposed scheme allows updates with new data and is well suited as a building block of a database application providing evaluated data. The capability of the modified GLS-method is demonstrated in a nuclear data evaluation involving three million observables using the TALYS code.

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Keywords: general least square method, large scale evaluation
AMS subject classifications. $62 \mathrm{P} 35,65 \mathrm{~K} 10$

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# Error propagation techniques 

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#### Abstract

The propagation of errors through nonlinear systems using different error propagation techniques are discussed in this lecture. The Sandwich methodology of error [9] propagation is widely used in many useful computation in the analysis of data uncertainties. However it involves the linearity assumptions. Unscented transformation, an efficient, consistent and unbiased transformation procedure suggested by Julier \& Uhlmann [2] can be used for error propagation studies. UT method is superficially similar to Monte Carlo method but uses a small deterministically chosen set of sample points which are selected according a specific deterministic algorithm. It was shown [3] that this deterministic method of UT produces better results compared to that of sandwich formula, for nonlinear error propagation.


Keywords: error propagation, unscented transform
AMS subject classifications. 60G06

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# Bayesian estimation \& its application in data interpolation 

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#### Abstract

Estimation of unmeasured states and monitoring of changes in the statistical parameters of the residues/innovations, form an important approach towards model-based fault detection \& diagnosis (FDD). This requires the formulation of system dynamics in the state-space framework $$
\begin{aligned} & x_{k}=A_{K \mid k-1} x_{k-1}+B_{k-1} u_{k-1}+w_{k-1} \\ & z_{k}=H_{k} x_{k}+D_{k} u_{k}+v_{k} \end{aligned}
$$


wherein the conditional probability density function (pdf) of the state-vector ( $\mathbf{X}$ ), conditioned on the measurement, $z p\left(x_{k} \mid z_{k}\right)$, is propagated through a predictor-corrector process to obtain the optimum estimate of the state while minimizing its error covariance

$$
E\left[\left(\hat{x}_{k}-x_{k}\right)^{T}\left(\hat{x}_{k}-x_{k}\right)\right]=E\left[\tilde{x}_{k}^{T} \widetilde{x}_{k}\right]
$$

The Bayesian formulation yields the conditional pdf of the $k^{\text {th }}$ state, which is equated to the likelihood function \& the prior

$$
\text { posterior }=p\left(x_{k} \mid z_{1: k}\right)=\frac{p\left(z_{k} \mid x_{k}\right) \cdot p\left(x_{k} \mid z_{1: k-1}\right)}{p\left(z_{k} \mid z_{1: k-1}\right)}=\frac{\text { likelihood.prior }}{\text { evidence }}
$$

and it is this formulation which governs the Bayesian estimation methodology.
Here an overview of the Bayesian estimation problem is presented, which discusses the formulation of the Kalman filter as a Bayesian estimator resulting in a closed form solution, provided the dynamics are linear and the uncertainties are Gaussian. The sequential Monte-Carlo filters (SMC), or particle filters, which addresses both non-linear \& non-Gaussian problems, but do not offer a closed form solution, are also introduced.

The model-based data interpolation problem, by study of the behavior of the estimated states, $X_{k} \&$ the residues ( $z_{k}-H \hat{x}_{k}^{-}$) along with the convergence of the error covariance matrix $P_{k}=\left(1-K_{k} H\right) P_{k}^{-}$and by use of multiple-model filtering, GLR (generalized likelihood ratio) methods, sequential probability ratio tests (SPRT) on the residues, etc. are explained, along with typical applications in engineering data processing/interpolation.
Keywords: Bayesian estimation, Kalman filter
AMS subject classifications. 62P35

# Overview of nuclear data activities in India 

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#### Abstract

The nuclear data activities in India has been coordinated by Nuclear Data Physics Centre of India (NDPCI), which operated under Board of Research of Nuclear Sciences, Department of Atomic Energy. It consisted of scientists and faculties from various divisions of DAE units and universities. Detailed and accurate nuclear data are required from design and safety point of view for India's three stage nuclear power programme, accelerator shield design, personal dosimetry, radiation safety, production of radioisotopes for medical applications, radiation damage studies, waste transmutation etc. The NDPCI has coordinated projects / collaborations with universities and various units of department of atomic energy (DAE) across India involving physicist, radio-chemists, reactor physicists and computer engineers. It has provided a platform for coordinated efforts in all aspects of nuclear data, viz., measurements, analysis, compilation and evaluation involving national laboratories and universities in India. NDPCI has organized many theme meetings cum workshops on various topics of interest. NDPCI has contributed more than 350 entries to EXFOR database of IAEA on nuclear reactions. We are maintaining the mirror website of nuclear data section of IAEA. NDPCI scientists have carried out many experiments related to nuclear data using BARC-TIFR pelletron facility, FOTIA, electron accelerator at Khargar, Dhruva, CERN n-TOF facility, Legnaro national laboratory, electron accelerator, Pohang Korea. There are number of computer simulation studies which were carried out using the various nuclear data libraries for sensitivity studies and benchmarking for nuclear reactor applications. There are number of students, part of DAE-BRNS projects of NDPCI, who participated in collaborative experiments using DAE facilities. The NDPCI scientists are participating in IAEA activities through CRPs and NRDC and INDC meetings. NDPCI has contributed to the increased awareness about the nuclear data activities among the teaching institutes and organization of schools/workshops under the NDPCI banner has also led to more students/faculty taking part in nuclear data programmes. The present talk will give a glimpse of these activities.


Keywords: nuclear data, nuclear data libraries
AMS subject classifications. 81V35

# Neutron time-of-flight cross section measurements and its applications 

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#### Abstract

Neutron induced reaction cross sections are essential nuclear data for a wide variety of nuclear technology applications and other disciplines ranging from fundamental physics, medicine, security, archaeology to astrophysics. The majority of the cross sections of neutron induced reactions that are recommended in evaluated data libraries are parameterized in terms of nuclear reaction theory. Unfortunately no nuclear reaction theory exist that can predict the model parameters from first principles. Therefore, they can only be determined from an adjustment to experimental data. In the resolved resonance region (RRR) the Rmatrix theory is employed, while in the unresolved resonance region (URR) cross sections are described by


the Hauser-Feshbach theory including width fluctuations. At higher energies the optical model in addition to statistical and pre-equilibrium reaction theory is used. The production of cross section data in the resonance region will be discussed. In addition, the use of resonances to characterise materials and objects will be explained.

Experimentally, the neutron cross sections in the resonance region are best studied at a pulsed white neutron source that is optimised for time-of-flight (TOF) measurements [1]. The resonance parameters in the RRR are derived from a RSA at a TOF-spectrometer with an extremely good energy resolution. Such an analysis requires a good understanding of the response functions of the TOF spectrometer. In addition a set of complementary independent experimental observables is required [1]. These experimental observables result from transmission and reaction cross section measurements [1].

The different components affecting the TOF response will be studied. The impact of the TOF response function and Doppler broadening on the determination of resonance parameters will be explained. Detection techniques for the measurement of total and reaction cross section together with their specific data reduction and analysis procedures will be presented. In addition examples of a RSA to derive parameters in the RRR will be given [2]-[3] and problems related the treatment of cross section data in the URR will be explained [3]-[8]. In addition, the use of experimental data to produce evaluated cross section data will be discussed [9]-[11].

Most of the material will be best on results of experiments carried out at the TOF facility GELINA installed at the JRC Geel (B) [12]. This facility has been designed to study neutron-induced reactions in the resonance region. It is a multi-user facility, providing a pulsed white neutron source, with a neutron energy range between 10 meV and 20 MeV and a time resolution of 1 ns . Results obtained at GELINA will be compared with results of similar measurements at other TOF facilities.

Finally the use of resonance structures to study properties of materials and objects will be presented [13]. These resonance structures are the basis of two analytical methods, Neutron-Resonance-CaptureAnalysis (NRCA) and Neutron-Resonance-Transmission-Analysis(NRTA), which have been developed at the JRC Geel. NRTA and NRCA are non-destructive analysis (NDA) methods which are applicable to almost all stable elements and isotopes; determine the bulk elemental composition; do not require any sample taking or surface cleaning and result in a negligible residual radioactivity. They have been already been applied to determine the elemental composition of an archaeological objects and to characterize nuclear reference materials and nuclear waste. Due to the expertise with NRTA and NRCA, the JRC Geel has been invited by the JAEA (Japan Atomic Energy Agency) to assist them in the development of a NDA method to quantify nuclear material in particle-like debris of melted fuel [13], [14]. It is also being investigated as an analytical technique to determine the nuclide vector of spent nuclear fuel pellets and solutions.
Keywords: neutron resonances, resonance parameters, cross section, NDA, neutron resonance analysis, time-of-flight
AMS subject classifications. 81V35; 82D75; 93E24

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# Adjustment of model parameters by a fit to experimental data 

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#### Abstract

Cross sections of neutron induced reactions in evaluated data libraries are parameterized in terms of nuclear reaction theory. Unfortunately no nuclear reaction theory exist that can predict the model parameters from first principles. Therefore, they can only be determined from an adjustment to experimental data. In the resolved resonance region the R-matrix theory is employed, while in the unresolved resonance region cross sections are described by the Hauser-Feshbach theory including width fluctuations. At higher energies the optical model in addition to statistical and pre-equilibrium reaction theory is used.

In this presentation principles to derive model parameters and their covariance in a fit to experimental data are discussed with an emphasis on the analysis of cross section data in the resolved and unresolved resonance region. The basic principles of least squares fitting are reviewed. Bias effects related to weighted least square adjustments are discussed and the reason for extreme low uncertainties of cross sections in


the resolved resonance region that are recommended in evaluated data file is verified. The presentation is strongly based on the work of Refs. [1] and [2].

A full Bayesian statistical analysis reveals that the level to which the initial uncertainty of the experimental parameters propagates, strongly depends on the experimental conditions. In the resolved resonance region the uncertainties of the model parameters due to the background can become very small for high precision data, that is, for high counting statistics. Also for thick sample measurements and high precision data the covariance of the normalisation does not fully propagate to the resonance parameters. These conclusions are independent of the method that is applied to propagate the experimental covariance of the experimental parameters. By adjusting the model parameters to experimental data based on a maximum likelihood principle one supposes that the model used to describe the experimental observables is perfect. In case the quality of the model cannot be verified a more conservative method based on a renormalization of the covariance matrix should be applied to propagate the experiment recommended.

In the unresolved resonance region an additional complication appears whden average resonance parameters are derived from an adjustment to the data applying the Hauser-Feshbach theory including width fluctuations. Due to the remaining resonance structure in the data the model cross section will be underestimated when a normalization uncertainty is introduced based on the experimental values. This bias effect is similar to the one observed in Peelle's Pertinent Puzzle. It appears when the data are weighted by factors which are not consistent with the model that is applied. A recipe to avoid such problems will be given.
Keywords: nuclear reaction models, resonance parameters, least squares adjustment, Bayesian theory, Peelle's Pertinent Puzzle
AMS subject classifications. 62F15;81V35; 82D75; 93E24

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# On the role of the covariance matrix in the linear statistical model 

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#### Abstract

In this talk we consider the linear statistical model $\mathbf{y}=\mathbf{X} \boldsymbol{\beta}+\boldsymbol{\varepsilon}$, which can be shortly denoted as the triplet $M=\{\mathbf{y}, \mathbf{X} \boldsymbol{\beta}, \mathbf{V}\}$. Here $\mathbf{X}$ is a known $n \times p$ fixed model matrix, the vector $\mathbf{y}$ is an observable $n$-dimensional random vector, $\boldsymbol{\beta}$ is a $p \times 1$ vector of fixed but unknown parameters, and $\varepsilon$ is an unobservable vector of random errors with expectation $E(\boldsymbol{\varepsilon})=\mathbf{0}$, and covariance matrix $\operatorname{cov}(\boldsymbol{\varepsilon})=\mathbf{V}$, where the nonnegative definite matrix $\mathbf{V}$ is known. In our considerations it is essential that the covariance matrix $\mathbf{V}$ is known; if this is not the case the statistical considerations become much more complicated.

An extended version of $M$ can be obtained by denoting $\mathbf{y}_{*}$ a $q \times 1$ unobservable random vector containing "new future" unknown observations. These new additional observations are assumed to come from $\mathbf{y}_{*}=$


$\mathbf{X}_{*} \boldsymbol{\beta}+\boldsymbol{\varepsilon}_{*}$, where $\mathbf{X}_{*}$ is a known $q \times p$ matrix, $\boldsymbol{\beta}$ is the same vector of unknown parameters as in $M$, and $\boldsymbol{\varepsilon}_{*}$ is a $q$-dimensional random error vector. The covariance matrix of $\boldsymbol{\varepsilon}_{*}$ as well as the cross-covariance matrix between $\boldsymbol{\varepsilon}_{*}$ and $\boldsymbol{\varepsilon}$ are assumed to be known.

Our main focus is to define and introduce in the general form, without rank conditions, the concepts of best linear unbiased estimator, BLUE, and the best linear unbiased predictor, BLUP. With the BLUE of $\mathbf{X} \boldsymbol{\beta}$ we mean the estimator $\mathbf{G y}$ which is unbiased and it has the smallest covariance matrix (in the Löwner sense) among all linear unbiased estimators of $\mathbf{X} \boldsymbol{\beta}$. Correspondingly, a linear unbiased predictor By is the BLUP for $\mathbf{y}_{*}$ whenever the covariance matrix of the prediction error, i.e., $\operatorname{cov}(\mathbf{y}-\mathbf{G y})$ is minimal in the Löwner sense.

This talk is concentrating on statistical properties of the covariance matrix in the general linear model, skipping thereby the main topic of the Theme Meeting. For the references we may mention [1], [2], and [3]. Keywords: BLUE, BLUP, linear statistical model, Löwner partial ordering, generalized inverse AMS subject classifications. 62J05; 62J10

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# Adjustment of nuclear data libraries using integral benchmarks 

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#### Abstract

Integral experiments can be used to adjust nuclear data libraries and consequently the uncertainty response in important applications. In this work we show how we can use integral experiments in a consistent way to adjust the TENDL library. A Bayesian method based on assigning weights to the different random files using a maximum likelihood function [1] is used. Emphasis is put on the problems that arise from multiple isotopes being present in an integral experiment [2]. The challenges in using multiple integral experiments are also addressed, including the correlation between the different integral experiments.

Methods on how to use the Total Monte Carlo method to select benchmarks for reactor application will further be discussed. In particular, in respect to the so-called fast correlation coefficient and the fast-TMC method [14].


Keywords: Total Monte Carlo, nuclear data evaluation, integral experiments
AMS subject classifications. 62P35; 81V35; 62-07

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# Choosing nuclear data evaluation techniques to obtain complete and motivated covariances 

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#### Abstract

The quality of evaluated nuclear data and its covariances is affected by the choice of the evaluation algorithm. The evaluator can choose to evaluate in the observable domain or the parameter domain and choose to use a Monte Carlo- or deterministic techniques [1]. The evaluator can also choose to model potential model-defects using, e.g., Gaussian Processes [2]. In this contribution, the performance of different evaluation techniques is investigated by using synthetic data. Different options for how to model the model-defects are also discussed.

In addition, the example of a new Ni-59 is presented where different co-variance driven evaluation techniques are combined to create a final file for JEFF-3.3 [3].


Keywords: Total Monte Carlo, Nuclear data evaluation
AMS subject classifications. 62P35, 81V35, 62-07

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# Surrogate nuclear reactions for determining compound nuclear reaction cross sections of unstable nuclei for fusion technology applications ${ }^{1}$ 

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#### Abstract

In D-T fusion reactor, large amount radio nuclides will be produced during reactor operation as well as after shutdown. These radio, nuclides will interact with slow and fast neutrons and produce large amount of hydrogen and helium which leads to the swelling and embrittlement of the structural and wall materials. These radio nuclides may also affect neutronics of the reactor, whereas fusion neutronics studies so far considered only the stable isotopes ofCr, $\mathrm{Fe}, \mathrm{Ni}$, because these elements and $\mathrm{Mn}, \mathrm{Co}, \mathrm{Nb}$ are main constituents of structural materials. The radiological hazard comes from the following radio nuclides in the mass region $\sim 50-60,{ }^{53} \mathrm{Mn}\left(T_{\frac{1}{2}}=3.74 \times 10^{6} y\right),{ }^{54} \mathrm{Mn}\left(T_{\frac{1}{2}}=312.03 \mathrm{~d}\right),{ }^{56} \mathrm{Mn}\left(T_{\frac{1}{2}}=2.5789 \mathrm{~h}\right)$, ${ }^{55} \mathrm{Fe}\left(T_{\frac{1}{2}}=2.73 y\right),{ }^{60} \mathrm{Fe}\left(T_{\frac{1}{2}}=1.5 \times 10^{6}\right.$ year), ${ }^{59} \mathrm{Fe}\left(T_{\frac{1}{2}}=44.6 d\right)^{57} \mathrm{Co}\left(T_{\frac{1}{2}}=271.74 \mathrm{~d}\right),{ }^{58} \mathrm{Co}\left(T_{\frac{1}{2}}=70.86 \mathrm{~d}\right)$, ${ }^{60} \mathrm{Co}\left(T_{\frac{1}{2}}=5.27 y\right),{ }^{57} \mathrm{Ni}\left(T_{\frac{1}{2}}=35.60 \mathrm{~h}\right),{ }^{59} \mathrm{Ni}\left(T_{\frac{1}{2}}=7.6 \times 10^{4} y\right),{ }^{63} \mathrm{Ni}\left(T_{\frac{1}{2}}=100.1 y\right),{ }^{51} \mathrm{Cr}\left(T_{\frac{1}{2}}=27.7025 \mathrm{~d}\right)$, ${ }^{65} \mathrm{Zn}\left(T_{\frac{1}{2}}=244 \mathrm{~d}\right)$ and ${ }^{94} \mathrm{Nb}\left(T_{\frac{1}{2}}=2.03 \times 10^{4} y\right)$; they originate from transmutation reactions of neutrons with the elements in the initial SS composition. Therefore, we need data of $(n, p),(n, \alpha),(n, d),(n, t),\left(n,{ }^{3} \mathrm{He}\right)$ reaction cross section on these radio targets and isotopic systematics as a function of mass number covering stable to radio nuclides from 1 MeV to 20 MeV . For many of these isotopes, EXFOR data does not exist or in some cases very sparsely measured. The nuclear reaction codes Talys and Empire predict the cross sections only approximately, due to insufficient systematics over radio nuclides. The measured reaction cross sections can benchmark the potentials, level density options in various mass regions, also provide critical input to test the evaluated nuclear data libraries. For measuring the cross sections, we adopt Surrogate reaction approach (SRA), specifically Surrogate Ratio Method (SRM). This SRA/SRM may be useful when mono energetic neutron beam of desired energy is not available, do not have a target of stable/unstable nuclei, target nucleus does not have sufficient abundance, enriched targets are very costly, off-line gamma method is not possible owing to very short half lives or products are stable, target nuclei are produced only transiently in reactor operation, when the targets are very difficult to handle due to high activity

Following SRA/SRM, we measured cross sections for reactions ${ }^{55} \mathrm{Fe}(n, p)$ by using it surrogate reaction ${ }^{52} \mathrm{Cr}\left({ }^{6} \mathrm{Li}, d\right){ }^{56} \mathrm{Fe}{ }^{*} \rightarrow{ }^{55} \mathrm{Fe}+p ;{ }^{55} \mathrm{Fe}(n, \alpha)$ reaction by ${ }^{52} \mathrm{Cr}\left({ }^{6} \mathrm{Li}, d\right){ }^{56} \mathrm{Fe}{ }^{*} \rightarrow{ }^{55} \mathrm{Fe}+\alpha ;{ }^{59} \mathrm{Ni}(n, p)$ reaction by ${ }^{56} \mathrm{Fe}\left({ }^{6} \mathrm{Li}, d\right){ }^{60} \mathrm{Ni}^{*} \rightarrow{ }^{59} \mathrm{Co}+p$, by measuring $(d, p)$ and $(d, \alpha)$ coincidence events. We are preparing to measure cross sections for ${ }^{53} \mathrm{Mn}(n, p),{ }^{55} \mathrm{Mn}(n, p)$ by SRA/SRM approach. These measurements details will be presented in the talk. Further, we will discuss some case studies of TALYS model calculations for 14 MeV neutron induced reactions on ${ }^{65} \mathrm{Zn},{ }^{59} \mathrm{Ni},{ }^{63} \mathrm{Ni},{ }^{57} \mathrm{Co},{ }^{58} \mathrm{Co},{ }^{60} \mathrm{Co},{ }^{55} \mathrm{Fe},{ }^{59} \mathrm{Fe}$ etc.. The SRA experiments for some 14 MeV neutron induced reactions are given below, one sample case will be presented. ${ }^{65} \mathrm{Zn}(n, p)$ using enriched ${ }^{63} \mathrm{Cu}: \alpha-p$ coincidence measurements for in ${ }^{63} \mathrm{Cu}\left({ }^{7} \mathrm{Li}, \alpha\right){ }^{66} \mathrm{Zn} \rightarrow{ }^{65} \mathrm{Cu}+p$ ${ }^{65} \mathrm{Zn}(n, p)$ using enriched ${ }^{62} \mathrm{Ni}, d+p$ coincidence with enriched target ${ }^{62} \mathrm{Ni}\left({ }^{6} L i, d\right){ }^{66} \mathrm{Zn} \rightarrow{ }^{65} \mathrm{Cu}+p$ ${ }^{65} \mathrm{Zn}(n, \alpha)$ using enriched ${ }^{62} \mathrm{Ni}: d+\alpha$ coincidence ${ }^{62} \mathrm{Ni}\left({ }^{6} L i, d\right){ }^{66} \mathrm{Zn} \rightarrow{ }^{62} \mathrm{Ni}+\alpha$ ${ }^{63} \mathrm{Ni}(n, x)$ and ${ }^{59} \mathrm{Fe}(n, p)$ reactions are not feasible by SRA, as surrogate pairs are difficult to get. ${ }^{57} \mathrm{Co}(n, p)$ reactions using enriched ${ }^{56} \mathrm{Fe}: \alpha+p$ coincidence for ${ }^{56} \mathrm{Fe}\left({ }^{6} L i, \alpha\right){ }^{58} \mathrm{Co} \rightarrow{ }^{57} \mathrm{Fe}+p$


[^2]${ }^{58} \mathrm{Co}(n, p)$ reactions using enriched ${ }^{57} \mathrm{Fe}:{ }^{57} \mathrm{Fe}\left({ }^{6} L i, \alpha\right){ }^{59} \mathrm{Co} \rightarrow{ }^{58} \mathrm{Fe}+p$
${ }^{60} \mathrm{Co}(n, p)$ reactions using enriched ${ }^{58} \mathrm{Fe}:{ }^{58} \mathrm{Fe}\left({ }^{7} \mathrm{Li}, \alpha\right){ }^{61} \mathrm{Co} \rightarrow{ }^{60} \mathrm{Co}+p$
Keywords: nuclear reaction cross section, EXFOR
AMS subject classifications. 62P35

# A case study on the cross section data of ${ }^{232} T h(n, 2 n)^{231} T h$ : A look, with a covariance analysis at the 1961 data of Butler and Santry (EXFOR ID 12255) 

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#### Abstract

We examined, as a case study, the experimental values of ${ }^{232} T h(n, 2 n)^{231} T h$ nuclear reaction published by J.P.Butler \& D.C. Santry [1]. The numerical data are available in the EXFOR compilation [2, 3], EXFOR ID 12255 . This is one of the best data of this nuclear reaction measured very carefully at that time and considered even today as a very valuable data in the process of creating modern evaluated nuclear data files. In this student exercise, we have attempted to estimate Butler and Santry's experimental data with a covariance analysis. Butler and Santry used the monitor cross sections of ${ }^{32} S(n, p)^{32} P$ reaction by L. Allen et al. [4], which is considered even today as very high-quality dosimetry data available for nuclear data evaluators. We noticed that Butler and Santry have used [1] the monitor reaction cross section values of ${ }^{32} S(n, p)^{32} P$ but do mention, in their Table II for their results, the monitor (Allen's) data without the errors available in Allen's data. We are inclined to believe that the errors in the monitor cross sections provided in [4] which were available that time were not taken into account by Butler and Santry. [2] In the EXFOR entry (ID 12255), the text in EXFOR entry under keywords "ERR-ANALYS" and "METHOD" also mentions [3] for \#ENTRY $12255 \mathrm{~L}=2$, "ERR-ANALYS (DATA-ERR) Quoted errors do not include any errors in the monitor cross section.", which agrees with our subjective understanding. Therefore, in this work, a cubic B-spline fit is first performed to fit the monitor ${ }^{32} S(n, p){ }^{32} P$ reaction cross section data based on numerical data reported by Allen et al., [4] and to obtain through the fit the covariance matrix associate with those fitted data. The so obtained monitor reaction data with covariance matrix are then used to estimate the cross sections of ${ }^{232} T h(n, 2 n)^{231} T h$ nuclear reaction with the covariance error matrix. We also present discussions on the subjective understanding that influences this "re-estimation" process of old EXFOR data. The work presented in the paper is for illustrative and learning purposes. A complete and comprehensive renormalization for purpose of a professional nuclear data evaluation would require more work with considerable subjective and objective analysis involving all attributes in each of the experiments in the EXFOR database.


Keywords: Nuclear reactions, ${ }^{232} T h(n, 2 n)^{231} T h$, EXFOR database, covariance, error propagation, regression analysis, cubic B-spline fit, monitor reaction, evaluated nuclear data files
AMS subject classifications. 62P35

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# Calculating efficiencies and their uncertainties propagation in efficiency 

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#### Abstract

It is difficult to transform a probability density function (PDF) through a general nonlinear function that is why uncertainty propagation is also difficult. In this abstract we will briefly present some methods such as Sandwich formula, Unscented transform technique and Monte Carlo method for the determination of the Uncertainty propagation. We generate and present the covariance information by taking into account various attributes influencing the uncertainties and also the correlations between them.


Keywords: uncertainty propagation, Monte Carlo method
AMS subject classifications. 62P35

# Measurement and uncertainty propagation of the $(\gamma, n)$ reaction cross-section of ${ }^{58} \mathrm{Ni}$ and ${ }^{59} \mathrm{Co}$ at 15 MeV bremsstrahlung 

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#### Abstract

Activation cross-section of photon-induced reaction on structural materials ${ }^{58} \mathrm{Ni}$ and ${ }^{59} \mathrm{Co}$ was measured at the bremsstrahlung endpoint energy 15 MeV from an S band electron linac. The uncertainties in the $(\gamma, n)$ reaction cross-section of both ${ }^{58} \mathrm{Ni}$ and ${ }^{59} \mathrm{Co}$ were estimated by using the concept of covariance analysis. The cross-section of ${ }^{58} N i(\gamma, n)^{57} N i$ reaction in the present work is slightly lower than the previous experimental data and the TENDL-2015 data. The cross-section of ${ }^{59} \mathrm{Co}(\gamma, n)^{58} \mathrm{Co}$ reaction has been measured for the first time. However, the present experimental data of ${ }^{59} C o(\gamma, n){ }^{58} C o$ reaction is very low in comparison to the TENDL-2015 and JENDL/PD-2004 data.


Keywords: covariance, cross-section
AMS subject classifications. 62P35, 81V35

# Estimation of efficiency of the HPGe detector and its covariance analysis 

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#### Abstract

In the present paper efficiency of the HPGe detector is determined at characteristic gamma energies 0.08421 MeV and 0.7433 MeV obtained in the reactions ${ }_{90}^{232} T h(n, 2 n)_{90}^{231} \mathrm{Th}$ and ${ }_{90}^{232} \mathrm{Th}(n, f)_{40}^{97} Z r$ using the least square method. ${ }_{56}^{133} B a$ and ${ }_{63}^{152} \mathrm{Eu}$ are used as standard sources whose gamma energy ranges from 0.05316 MeV to 1.4080 MeV . Energy-efficiency model is well represented by an empirical formula. The energy range spanned in this model does not extend much below 0.2 MeV . The principle of least squares is used in sequence to find the covariance and correlation matrices and the variation of efficiency is plotted. Keywords: least square method, ${ }_{56}^{133} \mathrm{Ba},{ }_{63}{ }^{152} \mathrm{Eu}$ AMS subject classifications. 62 P 35


# A stochastic convergence analysis of random number generators as applied to error propagation using Monte Carlo method and unscented transformation technique 

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#### Abstract

This paper compares the stochastic convergence of the Uniform Random number generators of two simulation software namely Matlab and Python and establishes the significance in choosing the right random number generator for error propagation studies. It further discusses about the application of Gaussian type of these random number generators to nonlinear cases of Error propagation using the Monte Carlo method and unscented transformation technique by means of a nonlinear transformation of one dimensional random variable of nuclear data.


Keywords: Monte Carlo method, unscented transformation, stochastic convergence, random number generators, nuclear data
AMS subject classifications. $60 \mathrm{G} ; 60 \mathrm{H} ; 60 \mathrm{~J} ; 62 \mathrm{M} ; 68 \mathrm{U}$

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# Covariance matrices of DPA cross sections from TENDL-2015 for structural elements with NJOY-2016 and CRaD codes 

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#### Abstract

In the recently concluded IAEA-CRP on Primary Radiation Damage Cross Sections [1] and the studies made at IGCAR, it has been observed that there is a spread in the neutron damage and heating cross sections computed using various basic evaluated nuclear data libraries, such as ENDF/B-VII.1, TENDL2015, JENDL-4.0 etc., available from the IAEA (Ref: wwwnds. indcentre.org.in). This spread in the derived quantities reflect the non-uniqueness or nonconvergence of evaluated nuclear data from various sources, the non-uniqueness arising due to differences in the procedures in basic data evaluations, wherein the measured data with their associated experimental errors and correlations of results from nuclear model based calculations are employed. Since such differences in the basic evaluated nuclear reaction cross sections result from various causes including mainly the uncertainties in nuclear model parameters input to nuclear model codes (such as TALYS or EMPIRE) within their distributions, a new approach based on Total Monte Carlo (TMC) [2] [3] has been recently developed and used for uncertainty propagation in the derived quantities. In the present work, neutron damage energy cross sections of few isotopes of structural elements are computed from a large set of TMC based random ENDF-6 files in TENDL 2015 [3] with NJOY 2016 [4] and indigenously developed CRaD [5] codes. The statistical uncertainties involved are quantified and compared through the calculation of covariance and correlation matrices in a fine energy group structure ( 175 group VITAMIN-J).


Keywords: derived quantities, neutron heating, neutron damage, random, Total Monte Carlo
AMS subject classifications. 62

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# Covariance analysis in neutron activation measurements of ${ }^{59} \mathrm{Co}(\mathrm{n}, 2 \mathrm{n}){ }^{58} \mathrm{Co}$ and ${ }^{59} \mathrm{Co}(\mathrm{n}, \gamma){ }^{60} \mathrm{Co}$ reactions in the MeV region 

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#### Abstract

Uncertainties in any measurement is inevitable so is in the case of nuclear data measurements. Estimating the measurements with uncertainty as accurate as possible is very important for the reasons of safety and economy. In the process of estimation of nuclear data, it is necessary to identify different sources of uncertainty associated with all the attributes involved, which propagates the error in the estimation. Using law of error propagation, in the present work, we generalize the methodology of Smith [1] used for obtaining covariance matrix of $n$ measurements derived from observations of $m$ attributes in $n$ experiments, where the observations of different attributes are uncorrelated.

In the work, we consider all possible attributes which influence the measurements, correlations between them, and identify different steps of error propagations in the process of measurements and demonstrate the same in finding the cross sections of ${ }^{59} \mathrm{Co}(\mathrm{n}, 2 \mathrm{n}){ }^{58} \mathrm{Co},{ }^{59} \mathrm{Co}(\mathrm{n}, \gamma){ }^{60} \mathrm{Co}$ reactions at effective neutron energies of 11.98 and 15.75 MeV . The partial errors due to different attributes are presented and the present measurements are compared with evaluated data taken from different libraries such as ENDF/BVII.1, JENDL-4.0, JEFF-3.2, ROSFOND-2010, TENDL-2015, CENDL-3.1.


Keywords: nuclear data covariance, uncertainties, evaluated data libraries and correlations
AMS subject classifications. 62P35, 62J12, 62J10

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# Program: ICLAA 2017 

December 11, 2017 (Monday)

09:00-09:10 K. Manjuantha Prasad and Ravindra B. Bapat: Welcome \& Overview of the conference SESSION 1; Chair Person: Ravindra B Bapat

09:10-10:10 Stephen James Kirkland: Markov Chains as Tools for Analysing Graphs I
10:10-10:50 Sivaramakrishnan Sivasubramanian: The arithmetic Tutte polynomial of two matrices associated to trees

10:50-11:10 Tea Break
SESSION 2; Chair Person: Michael Tsatsomeros
11:10-11:50 S K Neogy: On testing matrices with nonnegative principal
11:50-12:30 Rafikul Alam: Fiedler companion pencils for rational matrix functions and the recovery of minimal bases and minimal indices"

12:30-13:10 K C Sivakumar: Nonnegative/nonpositive generalized inverses and applications in LCP
13:10-14:30 Lunch Break
SESSION 3; Chair Person: S. Arumugam
14:30-15:30 Sharad S Sane: Some Linear Algebra related questions in the theory of Block Design I
15:30-16:00 Matjaz Kovse: Distance matrices of partial cubes
16:00-16:20 Tea Break
SESSION 4; Chair Person: Sivaramakrishnan Sivasubramanian
16:20-17:00 S H Kulkarni: Continuity of the pseudospectrum
17:00-17:40 Murali K Srinivasan: Eigenvalues and eigenvectors of the perfect matching association scheme
17:40-18:40 S. Arumugam: Vector spaces associated with graphs
19:15-20:00 Inaugural Day Function of ICLAA 2017
20:00-21:00 DINNER
December 12, 2017 (Tuesday)
SESSION 5; Chair Person: TES Raghavan
09:00-10:00 Sharad S Sane: Some Linear Algebra related questions in the theory of Block Design II
10:00-11:00 Stephen James Kirkland: Markov Chains as Tools for Analysing Graphs II
11:00-11:30 Tea Break
11:30-13:00 Contributory Talks ( CT - 1)
13:00-14:30 Lunch Break

SESSION 6; Chair Person: S K Neogy
14:30-15:30 T E S Raghavan: On completely mixed games
15:30-16:10 B V Rajarama Bhat: Two states
16:10-16:30 Tea Break

16:30-18:30 Contributory Talks ( CT - 2)

## December 13, 2017 (Wednesday)

SESSION 7; Chair Person: Vasudev Guddattu
09:00-09:40 Simo Puntanen: Upper bounds for the Euclidean distances between the BLUPs"
09:40-10:20 Stephen John Haslett: Linear models and sample surveys
10:20-11:00 Ebrahim Ghorbani: Eigenvectors of chain graphs
11:00-11:30 Tea Break
11:30-13:00 Contributory Talks ( CT - 3)
13:00-14:15 Lunch Break
SESSION 8; Chair Person: Muddappa Seetharama Gowda
14:15-15:00 Ajit Iqbal Singh: Fibonacci fervour in linear algebra and quantum information theory
15:10-15:50 Arup Bose: To be announced
16:00-19:00 Cultural Program at Karantha Bhavan, KOTA
19:00-20:00 Dinner at Karantha Bhavan, KOTA
December 14, 2017 (Thursday)
SESSION 9; Chair Person: Helmut Leeb
09:00-09:50 Jeffrey Hunter: Mean first passage times in Markov Chains - How best to compute?
09:50-10:30 Augustyn Markiewicz: Approximation of covariance matrix by banded Toeplitz matrices
10:30-11:10 Martin Singull: The use of antieigenvalues in statistics
11:10-11:30 Tea Break/Photo Session
11:30-13:00 Contributory Talks ( CT - 4)
13:00-14:30 Lunch Break
SESSION 10; Chair Person: Asha Kamath
14:30-15:10 Michael Tsatsomeros: Stability and convex hulls of matrix powers
15:10-15:50 Muddappa Seetharama Gowda: On the solvability of matrix equations over the semidefinite cone

15:50-16:20 Somnath Datta: A combined PLS and negative binomial regression model for inferring association networks from next-generation sequencing count data

16:20-16:40 Tea Break
16:40-18:40 Contributory Talks (CT - 5)
December 15, 2017 (Friday) Session 11; Chair Person: Augustyn Markiewicz
09:00-09:50 Helmut Leeb: R-matrix based solution of Schrödinger equations with complex potentials 09:50-10:30 Zheng Bing: Condition numbers of the multidimensional total least squares problem 10:30-11:10 N Eagambaram: An approach to General Linear Model using hypothetical variables

11:10-11:30 Tea Break

SESSION 12; Chair Person: Steve J Kirkland

11:30-12:10 André Leroy: When singular nonnegative matrices are products of nonnegative idempotent matrices?

12:10-13:00 Sukanta Pati: Inverses of weighted graphs
13:00-14:30 Lunch Break
SESSION 13; Chair Person: Simo Puntanen
14:30-15:30 Bhaskara Rao Kopparty: Generalized inverses of infinite matrices
15:30-16:15 Tea Break
16:15-16:45 VALEDICTORY

## Contributory Talks

## December 12, 2017 (Tuesday)

CT 1-A; Chair Person : Sukanta Pati Venue: Bhargava Hall
11:30-12:00 Fouzul Atik: On the distance and distance signless Laplacian eigenvalues of graphs and the smallest Gersgorin disc

12:00-12:30 Sasmita Barik: On the spectra of bipartite multidigraphs
12:30 - 13:00 Dipti Dubey: On principal pivot transforms of hidden Z matrices
CT 1 - B; Chair Person: Steve J Kirkland
Venue: Shrikhande Hall
11:30-12:00 Sachindranath Jayaraman: Nonsingular subspaces of $M_{n}(F), \mathrm{F}$ a field
12:00-12:30 P Sam Johnson: Hypo-EP operators
12:30-13:00 Vatsalkumar Nandkishor Mer: Semipositivity of matrices over the n-dimensional ice cream cone and some related questions

CT 1 - C; Chair Person : Murali K Srinivasan
Venue: S K Mitra Hall
11:30-12:00 Mukesh Kumar Nagar: Immanants of q-Laplacians of trees
12:00-12:30 Anjan Kumar Bhuniya: A topological proof of Ryser's formula for permanent and a similar formula for determinant of a matrix

12:30 - 13:00 Manami Chatterjee: A relation between Fibonacci numbers and a class of matrices
CT 2 - A; Chair Person : S. Arumugam Venue: Bhargava Hall
16:30-17:00 Debashis Bhowmik: Semi-equivelar maps on the surface of Euler characteristic-2
17:00-17:30 Niranjan Bora: Study of spectrum of certain multi-parameter spectral problems
17:30 - 18:00 Ranjan Kumar Das: Generalized Fiedler pencils with repetition for polynomial eigenproblems and the recovery of eigenvectors, minimal bases and minimal indices

18:00-18:30 Supriyo Dutta: Graph Laplacian quantum states and their properties
CT 2 - B; Chair Person : K. C. Sivakumar
Venue: Shrikhande Hall
16:30-17:00 Projesh Nath Choudhury: Matrix Semipositivity Revisited
17:00 - 17:30 Lavanya Suriyamoorthy: M-operators on partially ordered Banach spaces
17:30 - 18:00 Ramesh G: On absolutely norm attaining paranormal operators
18:00 - 18:30 Kurmayya Tamminana: Comparison results for proper double splittings of rectangular matrices

$$
\text { CT } 2-C \text {; Chair Person : P Sam Johnson }
$$

Venue: S K Mitra Hall
16:30-17:00 Kshittiz Chettri: On spectral relationship of a signed lollipop graph with its underlying cycle 17:00 - 17:30 Balaji V.: Further result on skolem mean labeling

17:30 - 18:00 Shendra Shainy V: Cordial labeling for three star graph
18:00 - 18:30 Ranveer Singh: B-partitions and its application to matrix determinant and permanent
CT 2-D; Chair Person : G Sudhakara
Venue: Berman Hall
16:30-17:00 Mohammad Javad Nikmehr: Nilpotent graphs of algebraic structures
17:00 - 17:30 Somnath Paul: Distance Laplacian spectra of graphs obtained by generalization of join and lexicographic product

17:30 - 18:00 Pankaj Kumar Das: Necessary and sufficient conditions for locating repeated solid burst
18:00 - 18:30 Mahendra Kumar Gupta: Causal detectability for linear descriptor systems
December 13, 2017 (Wednesday) CT 3-A; Chair Person : S. Sivasubramanian Venue: Bhargava Hall

11:30-12:00 Soumitra Das: On Osofsky's 32-elements matrix ring
12:00-12:30 Sriparna Chattopadhyay: Laplacian-energy-like invariant of power graphs on certain finite groups

12:30-13:00 Biswajit Deb: Reachability problem on graphs by a robot with jump: some recent studies CT 3 - B; Chair Person : Martin Singull Venue: Shrikhande Hall

11:30-12:00 Ashma Dorothy Monteiro: Prediction of survival with inverse probability weighted Weibull models when exposure is quantitative

12:00-12:30 Debashish Sharma: Inverse eigenvalue problems for acyclic matrices whose graph is a dense centipede

12:30-13:00 Gokulraj S: Strong Z-tensors and tensor complementarity problems
CT 3 - C; Chair Person : Shreemathi S. Mayya
Venue: S K Mitra Hall
11:30-12:00 M Rajesh Kannan: On distance and Laplacian matrices of a tree with matrix weights
12:00-12:30 Nijara Konch: Further results on AZI of connected and unicyclic graph
12:30 - 13:00 Malathi V.: Nordhaus gaddum type sharp bounds for graphs of diameter two

## December 14, 2017 (Thursday)

CT 4-A; Chair Person : Sharad S Sane Venue: Bhargava Hall
11:30-12:00 B R Rakshith: Some graphs determined by their spectra
12:00 - 12:30 Arindam Ghosh: A note on Jordan derivations over matrix algebras
12:30-13:00 Anu Varghese: Bounds for the distance spectral radius of split graphs
CT 4 - B; Chair Person : Ajit Iqbal Singh Venue: Shrikhande Hall
11:30-12:00 Ranjit Mehatari: On the adjacency matrix of complex unit gain graphs
12:00 - 12:30 Ramesh Prasad Panda: The Laplacian spectra of power graphs of cyclic and dicyclic finite groups

12:30 - 13:00 Abhyendra Prasad: Study of maps on surfaces using face face incident matrix
CT 4-C; Chair Person : Parameshwara Bhat
Venue: S K Mitra Hall
11:30-12:00 T. Anitha: On Laplacian spectrum of reduced power graph of finite cyclic and dihedral groups
12:00-12:30 Jyoti Shetty: Some properties of Steinhaus graphs
12:30-13:00 M A Sriraj: Partition energy of corona of complete graph and its generalized complements CT 5 - A; Chair Person : Pradeep G Bhat Venue: Bhargava Hall

16:40-17:10 Ranveer Singh: B-partitions and its application to matrix determinant and permanent
17:10-17:40 Kurmayya Tamminana: Comparison results for proper double splittings of rectangular matrices

17:40-18:10 Divya P Shenoy: Determinants in the study of Generalized Inverses of Matrices over Commutative Ring

18:10-18:40 Adenike Olusola Adeniji: Spectrum of full transformation semigroup
CT 5-B; Chair Person : André Leroy
Venue: Shrikhande Hall

16:40-17:10 David Raj Micheal: Computational Methods to find Core-EP inverse
17:10-17:40 Vinay Madhusudanan:
17:40-18:10 Nupur Nandini: Jacobi type identities
18:10-18:40 Mojtaba Bakherad: The Jensen-Mercer operator inequality and some its refinements
CT 5-C; Chair Person : Kedukodi Babushri Srinivas Venue: S K Mitra Hall
16:40-17:10 Sonu Rani: On the distance spectra and distance Laplacian spectra of graphs with pockets
17:10-17:40 Jadav Ganesh: Perturbation of minimum attaining operators
17:40-18:10 Dhanajaya Reddy: Minimum matching dominating sets of circular arc graphs
18:10-18:40 Ahmad Jafarian: An alternative approach for solving fully fuzzy linear systems based on FNN CT 5-D; Chair Person : Kuncham Sham Prasad

Venue: Berman Hall
16:40-17:10 P. G. Romeo: On category of R-modules and duals
17:10-17:40 Rajaiah Dasari: Modified triangular and symmetric splitting method for the steady state vector of Markov chains

17:40-18:10 Akash Murthy: Incentive structure reorgnization to maximize healthcare players' payoff while keeping the healthcare service provider's company solvent

# Abstracts: ICLAA 2017 

# Special Lectures \& Plenary Talks <br> Vector spaces associated with a graph ${ }^{1}$ 

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#### Abstract

Let $G=(V, E)$ be a graph of order $n$ and size $m$. The set of all edge-induced subgraphs of $G$ forms a vector space over the field of integers modulo 2 , under the operation symmetric difference and usual scalar multiplication. This vector space is denoted by $\Psi(G)$. A circuit in $G$ is a cycle or edge disjoint union of cycles in $G$. The set $\mathscr{C}(G)$ of all circuits of $G$ is a subspace of $\Psi(G)$ and is called the circuit subspace of $G$. Let $\lambda(G)$ denote the collection of all cutsets and edge disjoint union of cutsets of $G$. The set $\lambda(G)$ is a subspace of $\Psi(G)$ and is called the cutset subspace of $G$. In this talk we present a survey of some of the classical results on these vector spaces, highlighting duality, orthogonality and applications. We also discuss how a graph $\Gamma(V)$ can be associated with a finite vector space $V$ and discuss some properties of $\Gamma(V)$.


Keywords: circuit space, cutsets, orthogonality
AMS subject classifications. $05 \mathrm{C} 12,05 \mathrm{C} 25,05 \mathrm{C} 62$

## References

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[^4]
# Mean first passage times in Markov Chains - How best to compute? 

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#### Abstract

The presentation gives a survey of a variety of computational procedures for finding the mean first passage times in Markov chains. The presenter has recently published a new accurate computational technique [1] similar to that developed by Kohlas [2] based on an extension of the Grassmann, Taksar, Heyman (GTH) algorithm [3] for finding stationary distributions of Markov chains. In addition, the presenter has recently developed a variety of new perturbation techniques for finding key properties of Markov chains including finding the mean first passage times [4]. These procedures are compared with other well known procedures including the standard matrix inversion technique of Kemeny and Snell, [5], some simple generalized matrix inverse techniques developed by the presenter [6] and the FUND technique of Heyman [7] for finding the fundamental matrix of a Markov chain. The accurate procedure of the presenter is favoured following MATLAB comparisons using some test problems that have been used in the literature for comparing computational techniques for stationary distributions. One distinct advantage is that the stationary distribution does not have to be found in advance but is extracted from the computations.


Keywords: Markov chain, stochastic matrix, moments of first passage times, generalized matrix inverses AMS subject classifications. 15A09; 15B51; 60J10

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[1] J. J. Hunter. Accurate calculations of stationary distributions and mean first passage times in Markov renewal processes and Markov chains. Special Matrices, 4:151-175, 2016.
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# Markov chains as tools for analysing graphs 

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#### Abstract

An $n \times n$ entrywise nonnegative matrix $A$ is called stochastic if it has all row sums equal to 1 . Given a nonnegative vector $x_{0} \in \mathbb{R}^{n}$ such that its entries sum to 1 , we form the sequence of iterates $x_{k}^{T}, k \in \mathbb{N}$ via the recurrence $x_{k}^{T}=x_{k-1}^{T} A, k \in \mathbb{N}$. The sequence $x_{k}$ is then a Markov chain associated with the stochastic matrix $A$. The theory of Markov chains has been with us for over a century, and they are used in a wide array of applications, including conformation of biomolecules, vehicle traffic models, and web search.

In this talk we focus on the use of Markov chain techniques as methods for understanding the structure of directed and undirected graphs. We begin with an overview of some of the key ideas and quantities in the study of Markov chains. We then move on to explore the use of Markov chains in analysing graphs. In particular, we will discuss measures of centrality, detection of clustering, and an overall measure of connectedness.


Keywords: Markov chain, stochastic matrix
AMS subject classifications. 60J10

# Generalized inverses of infinite matrices 

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#### Abstract

A formulation for studying generalized inverses of infinite matrices is developed. After proving several results, we shall propose some problems. The results supplement the studies by Sivakumar and Shivakumar[1]. Keywords: infinite matrices AMS subject classifications. 15A09

\section*{References} [1] P. N. Shivakumar, K. C. Sivakumar and Y. Zhang. Infinite Matrices and Their Recent Applications 2016: Springer.


# R-matrix based solution of Schrödinger equations with complex potentials ${ }^{2}$ 

Th. Srdinko ${ }^{1}$, J. Fries ${ }^{2}$, B. Raab ${ }^{3}$ and H. Leeb ${ }^{4}$<br>Atominstitut, TU Wien, Vienna, Austria. ${ }^{1}$ tsrdinko@ati.ac.at<br>${ }^{2}$ e1425904@student.tuwien.ac.at, ${ }^{3}$ e1027403@student.tuwien.ac.at<br>${ }^{4}$ helmut.leeb@tuwien.ac.at


#### Abstract

The description of reaction processes in nuclear and atomic physics requires the solution of Schrödinger equations. Albeit microscopic considerations lead to Schrödinger equations with non-local potentials, most applications make use of equivalent local potentials. In this contribution we present a method for the solution of Schrödinger equations involving complex non-local potentials. Our method is inspired by the R-matrix formalism which divides the configuration space into an internal and an external space, where the solution in the internal part is represented by an appropriate set of basis functions. Thus the representation of the corresponding coupled Bloch-Schrödinger equations leads to a complex symmetric matrix. Using the Tagaki factorization of complex symmetric matrices we extended the R -matrix formalism to complex potentials. The proposed method also allows the solution of Schrödinger equations with complex non-local potentials. In combination with the Lagrange mesh technique the proposed method becomes very appealing for application and has been successfully used. Some examples are given in this presentation.


Keywords: quantum mechanics, Schrödinger equation, Lagrange mesh technique
AMS subject classifications. 81U05, 65L99

## References

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[2] D. Baye. The Lagrange-mesh method. Physics Reports 565:1-107, 2014.

# On completely mixed games 

T.E.S. Raghavan

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#### Abstract

Any non-zero sum two person game in normal form is represented by a pair of real $m \times n$ matrices $A$ and $B$. Player I selects secretly a row " i " and player II selects secretly a column " j " and player I receives $a_{i j}$ while player II receives $b_{i j}$. A mixed strategy for player I is any probability vector $x=\left(x_{1}, x_{2}, \ldots, x_{m}\right)$ where row $i$ is selected with probability $x_{i}$. Independently a mixed strategy $y=\left(y_{1}, y_{2}, \ldots, y_{n}\right)$ can be used to select column $j$ with probability $y_{j}$. Thus players I and II receive respectively $a_{i j}, b_{i j}$ with probability $x_{i} \cdot y_{j}$. The expected payoff to player I is $\sum_{i j} a_{i j} x_{i} y_{j}=(x A y)$. The expected payoff to player II is $(x B y)$. A pair of mixed strategies $\left(x^{*}, y^{*}\right)$ constitute a Nash equilibrium pair if $$
v_{1}=\left(x^{*} A y^{*}\right) \geq\left(x A y^{*}\right) \text { for all mixed strategies } x \text { for player I }
$$ and $$
v_{2}=\left(x^{*} B y^{*}\right) \geq\left(x^{*} B y\right) \text { for all mixed strategies } y \text { for player II. }
$$

The following theorems will be discussed:


[^5]Theorem 1. [2]. Every bimatrix game admits at least one equilibrium pair in mixed strategies.
We call a bimatrix game completely mixed iff in every equilibrium pair the two players' mixed strategies are completely mixed.

Theorem 2. If a bimatrix game is completely mixed then

- The equilibrium pair is unique.
- The matrix is square (i.e. $m=n$ ).
- In case $A+B=\mathbf{O}$ and $v_{1}=0$, the rank of the matrix $A$ is $n-1$.
- In case $A+B=\mathbf{O}$ and $v_{1}=0$, all cofactors of $A$ are different from 0 and are of the same sign.

An N-person game is played as follows: Given finite sets $S_{1}, S_{2}, \ldots, S_{n}$, players $1,2, \ldots n$ choose secretly an element $s_{1} \in S_{1}, s_{2} \in S_{2}, \ldots s_{n} \in S_{n}$ respectively. Let $h_{i}: S_{1} \times S_{2} \times S_{n} \rightarrow R, i=1,2, \ldots n$ be payoffs to players $i=1,2, \ldots n$. Given a set of mixed strategies $x_{1}, x_{2}, \ldots, x_{n}$ for the respective players, let $h_{i}\left(x_{1}, x_{2}, x_{i}, x_{n}\right)$ be the expected payoff to player $i$ when all players stick to their mixed strategies. The set of mixed strategies $x_{1}, x_{2}, \ldots, x_{n}$ constitute a Nash equilibrium for the game if and only if f for each player $i$ and pure choice $s_{i} \in S_{i}$, the expected payoff to player $i$ when he simply chooses an element $s_{i} \in S_{i}$ while all the other players $j \neq i$ stick to their given mixed strategies satisfies

$$
h_{i}\left(x_{1}, x_{2}, s_{i}, x_{n}\right) \leq h_{i}\left(x_{1}, x_{2}, x_{i}, x_{n}\right), \forall s_{i} \in S_{i}, i=1,2, \ldots n .
$$

Thus no player can gain by unilateral deviation to any pure strategies.
Theorem 3. [2] Every n-person game admits at least one Nash equilibrium tuple in mixed strategies.
As soon as we introduce another player with at least 2 pure strategies for the player, uniqueness of the equilibrium is no more true. All we can say is
Theorem 4. If an n-person game is completely mixed then its equilibrium set cannot contain any nondegenerate line segment.
Theorem 5. For a 3 person completely mixed game for the special case where $\left|S_{i}\right|=2, i=1,2,3$ the equilibrium tuple is unique.
Theorem 6. [5]. Any algebraic number can be chosen as the equilibrium payoff for some player of a completely mixed 3 person game.
Theorem 7. [5] We can construct completely mixed n-person games with a continuum of equilibrium strategies.

The so called order field property is valid for bimatrix games and an explicit finite step pivoting algorithm was given by Lemke and Howson.(1964). It finally reduces to algorithmically solving for the so called (Linear Complementarity problem): Given a real square matrix $M$ of order $n$ and given an $n$-vector $q$ check whether

$$
w=M z+q, \text { has a solution } w \geq 0, z \geq 0,(w z)=0
$$

and if so how to locate one such pair $(w, z)$.
Theorem 8. The linear complementarity problem has a unique solution for any given $n$-vector $q$ if and only if the matrix $M$ has all of its principal minors positive. In this case Lemke's algorithm will solve for the unique solution.
Theorem 9. For the special case when the matrix $M$ is a non- singular $M$-matrix the unique solution can be found by a simplex pivoting algorithm.
Keywords: Completely mixed games, linear complementarity
AMS subject classifications. 90C33

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# Some linear algebra related questions in the theory of block designs 

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#### Abstract

This talk will mainly focus on the existence and structural questions concerning the objects mentioned in the title. The full talk is divided in two parts. Beginning with symmetric designs, I will allude to projective planes and biplanes and in particular to biplanes with characteristic 3. Later part of this talk will discuss quasi-symmetric designs that are in a sense combinatorial generalizations of symmetric designs. On the other hand and on the positive side of it, structural study of quasi-symmetric designs is facilitated due to the fact one can associate a simple graph with such a structure which turns out to be a non-trivial and interesting strongly regular graph in many cases of interest. The talk will discuss this connection in some details. The relationship between quasi-symmetric and symmetric designs is not well understood, though it is believed to exist and the existence questions in both the cases are expected to be equally difficult. The second talk will discuss the notorious long standing $\lambda$-design conjecture of Ryser and Woodall and with particular attention to the related linear algebra. The conjecture is widely believed to be true and a number of attempts have been made to prove it. Main interest in this conjecture is because of a bold assertion in the statement that essentially tells us that $\lambda$-designs can only be constructed in a canonically stipulated manner. The talk will discuss all the relevant results including some new ones in this area.


Keywords: block design, regular graph
AMS subject classifications. 05C50

## Invited Talks

# Fiedler companion pencils for rational matrix functions and the recovery of minimal bases and minimal indices 

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#### Abstract

Linearization is a standard method for computing eigenvalues, eigenvectors, minimal bases and minimal indices of matrix polynomials. Linearization is a process by which a matrix polynomial is transformed to a matrix pencil and has been studied extensively over the years. Frobenius companion pencils are examples of linearizations of matrix polynomials and are well known for almost 140 years. Recently, Fiedler introduced a family of companion pencils known as Fiedler companion pencils which provides an important class of linearizations of matrix polynomials. The poles and zeros of rational matrix functions play an important role in many applications. For computing eigenvalues, eigenvectors, poles, minimal bases and minimal indices of rational matrix functions, we construct Fiedler-like companion pencils for rational matrix functions and show that these pencils are linearizations of the rational matrix functions in an appropriate sense. We describe the recovery of minimal bases and minimal indices of rational matrix function from those of the Fiedler pencils. In fact, we show that the recovery of minimal bases are operation-free, that is, the minimal bases can be recovered from those of the Fiedler pencils without performing any arithmetic operations. Keywords: rational matrix function, Rosenbrock system matrix, matrix polynomial, eigenvalue, eigenvector, minimal realization, matrix pencil, linearization, Fiedler pencil. AMS subject classifications. 65F15, 15A57, 15A18, 65F35

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# Two states 

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#### Abstract

D. Bures defined a metric on states of a $C^{*}$-algebra as the infimum of the distance between associated vectors in common GNS representations. We take a different approach by looking at the completely bounded distance between relevant joint representations. The notion has natural extension to unital completely positive maps. The study yields new understanding of GNS representations of states and in particular provides a new formula for Bures metric. This is a joint work with Mithun Mukherjee (See: https://arxiv.org/abs/1710.00180). Keywords: states, completely positive maps, Hilbert $C^{*}$-modules, Bures distance AMS subject classifications. 46L30, 46L08

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# Condition numbers of the multidimensional total least squares problem 

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#### Abstract

In this talk, we present the Kronecker-product-based formulae for the normwise, mixed and componentwise condition numbers of the multidimensional total least squares (TLS) problem. For easy estimation, we also exhibit Kronecker-product-free upper bounds for these condition numbers. The upper bound for the normwise condition number is proved to be optimal, greatly improve the results by Gratton et al. for the truncated solution of the ill-conditioned basic TLS problem. As a special case, we also provide a lower bound for the normwise condition number of the classic TLS problem when having a unique solution. These bounds are analyzed in detail. Furthermore, we prove that the tight estimates of mixed and componentwise condition numbers recently given by other authors for the basic TLS problem are exact. Some numerical experiments are performed to illustrate our results. Keywords: multidimensional total least squares, truncated total least squares, condition number, singular value decomposition AMS subject classifications. 65F35, 65F20


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# A combined PLS and negative binomial regression model for inferring association networks from next-generation sequencing count data 

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## Abstract

A major challenge of genomics data is to detect interactions displaying functional associations from largescale observations. In this study, a new cPLS-algorithm combining partial least squares approach with negative binomial regression is suggested to reconstruct a genomic association network for high-dimensional next-generation sequencing count data. The suggested approach is applicable to the raw counts data, without requiring any further pre-processing steps. In the settings inves-tigated, the cPLS-algorithm outperformed the two widely used comparative methods, graphical lasso and weighted correlation network analysis. In addition, cPLS is able to estimate the full network for thousands of genes without major computational load. Finally, we demonstrate that cPLS is capable of finding biologically meaningful associations by analysing an example data set from a previously published study to examine the molecular anatomy of the craniofacial development.
Keywords: association networks, network reconstruction, negative binomial regression, next-generation sequencing, partial least-squares regression
AMS subject classifications. 62P10, 62J12

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# An approach to General Linear Model using hypothetical variables 

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#### Abstract

Consider the general linear model, $Y=X \beta+\epsilon$ where $Y$ is a vector of dimension $n, X$ is an $n \times k$ matrix and $\epsilon$ is a n-dimensional random variable with covariance matrix $\sigma^{2} G . X$ and $G$ are known whereas $\beta$ and $\sigma^{2}$ are unknown. Procedures for estimation of functions of $\beta$ and $\sigma^{2}$ are well known in the case of non-singular $G$. Here, similar procedures are explored by adding hypothetical variables to $Y$ so as to have a non-singular covariance matrix in the modified model.


Keywords: linear model, hypothetical random variables, generalized inverse, matrix partial order
AMS subject classifications. 62J12

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# Eigenvectors of chain graphs 

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#### Abstract

A graph is called a chain graph if it is bipartite and the neighborhoods of the vertices in each color class form a chain with respect to inclusion. Let $G$ be a graph and $\lambda$ be an (adjacency) eigenvalues of $G$ with multiplicity $k$. A vertex $v$ of $G$ is called a downer, or neutral, or Parter vertex of $G$ (and $\lambda$ ) depending whether the multiplicity of $\lambda$ in $G-v$ is $k-1$, or $k$, or $k+1$, respectively. We consider vertex types of a vertex $v$ of a chain graph in the above sense which has a close connection with $v$-entries in the eigenvectors corresponding to $\lambda$.


Keywords: chain graph, graph eigenvalue, eigenvector
AMS subject classifications. 05C50

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# On the solvability of matrix equations over the semidefinite cone 

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#### Abstract

In matrix theory, various algebraic, fixed point, and degree theory methods have been used to study the solvability of equations of the form $f(X)=Q$, where $f$ is a transformation (possibly nonlinear), $Q$ is a semidefinite/definite matrix and $X$ varies over the cone of semidefinite matrices. In this talk, we describe a new method based on complementarity ideas. This method gives a unified treatment for transformations studied by Lyapunov, Stein, Lim, Hiller and Johnson, and others. Our method actually works in a more general setting of proper cones and, in particular, on symmetric cones in Euclidean Jordan algebras.


Keywords: solvability, semidefinite cone, complementarity, proper cone, symmetric cone
AMS subject classifications. 15A24, 90C33

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# Linear models and sample surveys 

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#### Abstract

When sample surveys with complex design (which may include stratification, clustering, unequal selection probabilities and weighting) are used as data for linear models then additional complications are introduced into estimation of model parameters and variances. The standard techniques for linear models for sample surveys either model conditional on survey design variables or use design weights based on selection probabilities assuming no covariance between population elements.

When design weights are used, an extension to incorporate joint selection as well as selection probabilities is possible, and when there is correlated error structure this is essential for efficient estimation in linear models and for design unbiased estimation of covariance from the sample.

Sample designs can be either with or without replacement of units when sampling. Although without replacement sampling is more accurate for a given sample size, when sampling with probability proportional to size (pps), with replacement sampling is often used because pps without replacement is difficult to implement due to selection probabilities for the remaining units changing after each draw. However, with replacement sampling complicates fitting linear models and requires generalized inverses for any sample for which any unit is selected more than once. Keywords: linear models, sample surveys, survey design, superpopulation, without replacement, with replacement, ginverse


AMS subject classifications. 15A03; 15A09; 15A24, 15B48; 62D05; 62F12; 62J05; 62J10

# Continuity of the pseudospectrum 

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#### Abstract

Let $A$ be a complex unital Banach algebra with unit 1 . We shall identify a complex scalar $\lambda$ with the element $\lambda 1 \in A$. For $a \in A$, the spectrum $\sigma(a)$ of $a$ is defined by $$
\sigma(a):=\{\lambda \in \mathbb{C}: \lambda-a \text { is not invertible in } A\} .
$$


It is well known that the map $a \mapsto \sigma(a)$ is not continuous. In this talk we show that the pseudospectrum behaves in a better way in many situations. Let $\epsilon>0$. The $\epsilon$ - pseudospectrum $\Lambda_{\epsilon}(a)$ is defined by

$$
\Lambda_{\epsilon}(a):=\left\{\lambda \in \mathbb{C}:\left\|(\lambda-a)^{-1}\right\| \geq \frac{1}{\epsilon}\right\}
$$

with the convention that $\left\|(\lambda-a)^{-1}\right\|=\infty$ if $\lambda-a$ is not invertible. This convention makes the spectrum to be a subset of the $\epsilon$ - pseudospectrum for every $\epsilon>0$. The basic reference for the pseudospectrum is the book [2].

We show that for every fixed $\epsilon>0$ the map $a \mapsto \Lambda_{\epsilon}(a)$ is right continuous and it is continuous if one of the following conditions is satisfied:

1. The resolvent set $\mathbb{C} \backslash \sigma(a)$ is connected.
2. The algebra $A$ is the algebra of all bounded operators on a Banach space $X$ such that $X$ or its dual space $X^{\prime}$ is complex uniformly convex.

These conditions are satisfied when $T$ is a compact operator on a Banach space $X$ or when $T$ is a bounded operator on an $L^{p}$ space, $1 \leq p \leq \infty$.

Some of these results can be found in [1].
Keywords: Banach algebra, spectrum, pseudospectrum
AMS subject classifications. 47A10; 47A12; 46H05

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# When singular nonnegative matrices are products of nonnegative idempotent matrices? 

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#### Abstract

It is well-known that singular matrices over fields, division rings, Euclidean domains, self-injective regular rings can be presented as a product of idempotent matrices (see the works by Erdos, Laffey, O'MearaHanna, Alahmadi-Jain-Lam-Leroy, among others). During the ICLAA 2014 it was asked whether a real nonnegative singular matrix can be represented as a product of real nonnegative idempotent matrices. The answer is negative in general even for nice symmetric stochastic matrices. But we exhibit families of matrices for which the answer is yes. For instance here is list of type of singular nonnegative matrices for which it is known that the decomposition holds. 1. Singular nonnegative matrices of rank 1 or 2. 2. Singular nonnegative matrices having a nonnegative von Neumann inverse. 3. Singular nonnegative quasi-permutation matrices. 4. Singular periodic nonnegative matrices.

It is still an open problem to find necessary and sufficient conditions for the nonnegative decomposition to occur. Keywords: nonnegative matrices, idempotent matrices AMS subject classifications. 15B48


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# Approximation of covariance matrix by banded Toeplitz matrices 

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#### Abstract

The need for estimation of covariance matrix with a given structure arises in various multivariate models. We are studying this problem for banded Toeplitz structure using Frobenius-norm discrepancy. The estimation is made by approximating the unstructured sample covariance matrix by non-negative definite Toeplitz matrices. For this purpose some authors are using the projection on a given space of Toeplitz matrices [1]. We characterize the linear space of matrices for which this method is valid and we show that the space of Toeplitz matrices is not the case. The solution of this problem is the projection on a cone of non-negative definite Toeplitz matrices [2]. We give the methodology and the algorithm of the projection based on the properties of a cone of non-negative definite Toeplitz matrices. The statistical properties of this approximation are studied.


Keywords: covariance estimation, covariance structure, Frobenius norm
AMS subject classifications. 62H20; 65F99

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# On testing matrices with nonnegative principal minors ${ }^{3}$ 

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#### Abstract

In this paper, we revisit various methods proposed in the literature on testing matrices with nonnegative principal Minors and discuss various characterizations useful for testing $P\left(P_{0}\right)$-matrices. We also identify few subclasses of $P_{0}$-matrix for which there is a polynomial time algorithm and review various characterizations of a $P\left(P_{0}\right)$-matrix using linear complementarity. Keywords: $P\left(P_{0}\right)$-matrix, polynomial algorithm, linear complementarity problem AMS subject classifications. 90C33; 15A09; 15A24


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# Inverses of weighted graphs 

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#### Abstract

Consider a connected weighted graph $G$. Let $A(G)$ be its adjacency matrix. Assume that $A(G)$ is nonsingular. Then the matrix $A(G)^{-1}$ may have both positive and negative entries. However, for some $G$, the inverse $A(G)^{-1}$ is similar to a nonnegative matrix, say $B$, via a signature matrix (a diagonal matrix with diagonal entries from $\{1,-1\}$ ). We call the graph of this matrix $B$ as the inverse graph of $G$ and we also say $G$ is invertible.

Recall that a structural characterization of nonsingular graphs is not yet known. Consider a bipartite graph $G$ with a unique perfect matching $\mathscr{M}$ and let $G_{\mathrm{w}}$ be the weighted graph obtained from $G$ by giving weights to its edges using the positive weight function $\mathrm{w}: E(G) \rightarrow(0, \infty)$ such that $\mathrm{w}(e)=1$ for each $e \in \mathscr{M}$. The unweighted graph $G$ may be viewed as a weighted graph with the weight function $\mathrm{w} \equiv \mathbf{1}$, where the weight of each edge is 1 . The matrix $A\left(G_{\mathrm{w}}\right)$ always has determinant $\pm 1$. Hence $G_{\mathrm{w}}$ is nonsingular for each of the above described weight functions w.

Let $G$ be a bipartite graph with a unique perfect matching $\mathscr{M}$. By $G / M$, let us denote the graph obtained from $G$ by contracting each matching to a single vertex. It is known that if $G / M$ is also bipartite, then $G_{\mathrm{w}}$ is invertible for each weight function w.

We discuss the following questions. 1. Is the converse of the above result true? That is, if $G_{\mathrm{w}}$ is invertible for each w , is it necessary that $G / M$ is bipartite? 2. Are there cases, when $G_{\mathrm{w}}$ is invertible for one weight function w but it is not for each w? 3. Are there cases, when ' $G_{\mathrm{w}}$ is invertible for one w' will force that ' $G / M$ is bipartite' (or ' $G_{\mathrm{w}}$ is invertible for each w')?


Keywords: graph inverse, bipartite graphs with unique perfect matching
AMS subject classifications. 05C50

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## Upper bounds for the Euclidean distances between the BLUPs

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#### Abstract

In this paper we consider the linear model $M=\{y, X \beta, V\}$, where $y$ is the observable random vector with expectation $X \beta$ and covariance matrix $V$. Our interest is on predicting the unobservable random vector $y_{*}$, which comes from $y_{*}=X_{*} \beta+\varepsilon_{*}$, where the expectation of $y_{*}$ is $X_{*} \beta$ and the covariance matrix of $y_{*}$ is known as well as the cross-covariance matrix between $y_{*}$ and $y$. We introduce upper bounds for the Euclidean distances between the BLUPs, best linear unbiased predictors, when the prediction is based on the original model and when it is based on the transformed model $T=\left\{F y, F X \beta, F V F^{\prime}\right\}$. We also show how the upper bounds are related to the linear sufficiency of $F y$. The concept of linear sufficiency is strongly connected to the transformed model $T$ : If $F y$ is linearly sufficient for $X \beta$ under $M$, then the BLUEs of $X \beta$ are the same under $M$ and $T$.


The concept of linear sufficiency was essentially introduced in early 1980s by [1, 2]. In this paper we generalize their results in the spirit of [3], [4] and [5].
Keywords: best linear unbiased estimator, best linear unbiased predictor, linear sufficiency, linear mixed model, transformed linear model
AMS subject classifications. 62J05; 62J10

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# Fibonacci fervour in linear algebra and quantum information theory 

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#### Abstract

Fibonacci numbers appear in the context of matrices, resonance valence bond states, Symmetric informationally complete positive operator valued measures and other related matters in Quantum Information theory. We will give a brief account together with adaptation of the recursive process in other set-ups. Keywords: Fibonacci numbers, permutation matrix, resonance valence bond state, quantum entanglement, equiangular lines, symmetrically informationally complete positive operator valued measure (SICPOVM), Zauner's matrix, Fibonacci matrix, Fibonacci-Lucas SIC-POVM, optimal quantum tomography. AMS subject classifications. 11B39, 05A05, 15A69, 15B48, 81D40, 81P50

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# The use of antieigenvalues in statistics 

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#### Abstract

For fifty years ago Karl Gustafson published a series of papers and developed an antieigenvalue theory which has been applied, in a non-statistical manner, to several different areas including, numerical analysis and wavelet analysis, quantum mechanics, finance and optimisation. The first antieigenvector $\mathbf{u}_{1}$ (actually there are two) is the vector which is the one which is the most "turned" by an action of a positive definite matrix $\mathbf{A}$ with a connected antieigenvalue $\mu_{1}$ which indeed is the cosine of the maximal "turning" angle given as $$
\mu_{1}=\frac{2 \sqrt{\lambda_{1} \lambda_{p}}}{\lambda_{1}+\lambda_{p}},
$$ where $\lambda_{1}$ is the largest and $\lambda_{p}$ is the smallest eigenvalue of $\mathbf{A}$, respectively. Antieigenvalues have been introduced in statistics when, for example, analysing sample correlation coefficients, as a measures of efficiency of least squares estimators, and when testing for sphericity, see [1, 2, 3]. In this talk we will consider the distribution for a random antieigenvalue and discuss the use of it.


Keywords: eigenvalue, aniteigenvalue, probability distribution
AMS subject classifications. $62 \mathrm{H} 10,15 \mathrm{~A} 42$, 15A18, 15B52

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# Nonnegative/nonpositive generalized inverses and applications in LCP 

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#### Abstract

Let $A$ be a real square matrix whose off-diagonal entries are nonpositive. A necessary and sufficient condition for $A^{-1}$ to exist and have nonnegative entries is that $A$ is a $P$-matrix (namely, the principal minors of $A$ are positive). This in turn, is equivalent to the statement that the linear complementarity problem $L C P(A, q)$ has a unique solution. Note that $L C P(A, q)$ is to find $x \geq 0$ such that $A x+q \geq 0$ and $x^{T}(A x+q)=0$. In this talk, we shall present a survey of the literature where results that are similar in spirit to the result stated above, are recalled. Quite frequently, these conditions are stated in terms of nonnegativity or nonpositivity of generalized inverses of matrices involving $A$ as a submatrix. Keywords: linear complementarity problem, $M$-matrix, $P$-matrix, $Q$-matrix, inverse positive matrix AMS subject classifications. 15A09, 15B48


## The arithmetic Tutte polynomial of two matrices associated to trees ${ }^{4}$

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#### Abstract

Arithmetic matroids arising from a list $\mathscr{A}$ of integral vectors in $\mathbb{Z}^{n}$ are of recent interest and the arithmetic Tutte polynomial $M_{\mathscr{A}}(x, y)$ of $\mathscr{A}$ is a fundamental invariant with deep connections to several areas. In this work, we consider two lists of vectors coming from the rows of matrices associated to a tree $T$. Let $T=(V, E)$ be a tree with $|V|=n$ and let $\mathscr{L}_{T}$ be the $q$-analogue of its Laplacian $L$ in the variable $q$. Assign $q=r$ for $r \in \mathbb{Z}$ with $r \neq 0, \pm 1$ and treat the $n$ rows of $\mathscr{L}_{T}$ after this assignment as a list containing elements of $\mathbb{Z}^{n}$. We give a formula for the arithmetic Tutte polynomial $M_{\mathscr{L}_{T}}(x, y)$ of this list and show that it depends only on $n, r$ and is independent of the structure of $T$. An analogous result holds for another polynomial matrix associated to $T: \mathrm{ED}_{T}$, the $n \times n$ exponential distance matrix of $T$. More generally, we give formulae


[^7]for the multivariate arithmetic Tutte polynomials associated to the list of row vectors of these two matrices which shows that even the multivariate arithmetic Tutte polynomial is independent of the tree $T$.

As a corollary, we get the Ehrhart polynomials of the following zonotopes:
(i) $Z_{\mathrm{ED}_{T}}$ obtained from the rows of $\mathrm{ED}_{T}$ and (ii) $Z_{\mathscr{L}_{T}}$ obtained from the rows of $\mathscr{L}_{T}$.

Keywords: arithmetic matroids, arithmetic Tutte polynomial, distance matrices, trees
AMS subject classifications. 05E99; 15B36; 52B05

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# Eigenvalues and eigenvectors of the perfect matching association scheme 

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#### Abstract

We revisit the Bose-Mesner algebra of the perfect matching association scheme (= Hecke algebra of the Gelfand pair ( $S_{2 n}, H_{n}$ ), where $H_{n}$ is the hyperoctahedral group). Our main results are: an algorithm to calculate the eigenvalues from symmetric group characters by solving linear equations; universal formulas, as content evaluations of symmetric functions [1, 3], for the eigenvalues of fixed orbitals (generalizing a result of Diaconis and Holmes [2]); and an inductive construction of the eigenvectors (generalizing a result of Godsil and Meagher [4]).


Keywords: perfect matching scheme, content evaluation of symmetric functions
AMS subject classifications. 05E10, 05E05, 05E30

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# Stability and convex hulls of matrix powers 

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#### Abstract

Invertibility of all convex combinations of a matrix $A$ and the identity matrix $I$ is equivalent to the real eigenvalues of $A$, if any, being positive. Invertibility of all matrices whose rows are convex combinations of the respective rows of $A$ and $I$ is equivalent to all of the principal minors of $A$ being positive (i.e., $A$ being a P -matrix). These results are extended to convex combinations of higher powers of $A$ and of their rows. The invertibility of matrices in these convex hulls is associated with the eigenvalues of $A$ lying in open sectors of the right-half plane. The ensuing analysis provides a new context for open problems in the theory of matrices with P-matrix powers.


Keywords: P-matrix, nonsingularity, positive stability, matrix powers, matrix hull
AMS subject classifications. 15A48; 15A15

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## Contributory Talks

# Spectrum of full transformation semigroup 

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#### Abstract

Let $X$ be a set following natural ordering of numbers and let $I D T_{n}$ be the identity difference full transformation semigroup, a subsemigroup of full transformation semigroup, $T_{n}$. The spectral radius of $\alpha$ is 1 for all $\alpha \in I D T_{n}, n \geq 2$. Let $S(\alpha)$ be the shift of $\alpha$. Then $|S(\alpha)|$ sets the boundaries for eigenvalues of $\alpha$. One dimensional linear convolution of the spectrum of $T_{n}$ denoted by $C\left(T_{n: r}\right)$ is obtained using Cayley table and that Symmetric group has complex spectrum and convolution. Keywords: full transformation semigroup, identity difference transformation semigroup, matrix, eigenvalues, spectrum, convolution and Green's relations AMS subject classifications. 20M20


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# On the distance and distance signless Laplacian eigenvalues of graphs and the smallest Geršgorin dise 

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#### Abstract

The distance matrix of a simple connected graph $G$ is $D(G)=\left(d_{i j}\right)$, where $d_{i j}$ is the distance between the $i$ th and $j$ th vertices of $G$. The distance signless Laplacian matrix of the graph $G$ is $D_{Q}(G)=D(G)+\operatorname{Tr}(G)$, where $\operatorname{Tr}(G)$ is a diagonal matrix whose $i$ th diagonal entry is the transmission of the vertex $i$ in $G$. In this work we first give upper and lower bounds for the spectral radius of a nonnegative matrix. Applying this result we find upper and lower bounds for the distance and distance signless Laplacian spectral radius of graphs and obtain the extremal graphs for these bounds. Also we give upper bounds for the modulus of all distance (respectively distance signless Laplacian) eigenvalues other than the distance (respectively distance signless Laplacian) spectral radius of graphs. Finally for some classes of graphs we show that all distance (respectively distance signless Laplacian) eigenvalues other than the distance (respectively distance signless Laplacian) spectral radius lie in the smallest Geršgorin disc of the distance (respectively distance signless Laplacian) matrix.


Keywords: distance matrix, distance eigenvalue, distance spectral radius, distance signless Laplacian matrix, Geršgorin disc.
AMS subject classifications. 05C50

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# On the spectra of bipartite multidigraphs 

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#### Abstract

We define adjacency matrix as well as Laplacian matrix of a multidigraph in a new way and study the spectral properties of some bipartite multidigraphs. It is well known that a simple undirected graph is bipartite if and only if the spectrum of its adjacency matrix is symmetric about the origin (with multiplicity). We show that the result is not true in general for multidigraphs and supply a class of non-bipartite multidigraphs which have this property. We describe the complete spectrum of a multi-directed tree in terms of the spectrum of the corresponding modular tree. In case of the Laplacian matrix of a multidigraph, we obtain a necessary and sufficient condition for which the Laplacian matrix is singular. Finally, it is proved that the absolute values of the components of the eigenvectors corresponding to the second smallest eigenvalue of the Laplacian matrix of a multi-directed tree exhibit monotonicity property similar to the Fiedler vectors of an undirected tree ([3]).


Keywords: multidigrah; bipartite multidigraph; multi-directed tree; weighted digraph; adjacency matrix; spectrum
AMS subject classifications. 05C50; 05C05; 15A18

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# Semi-equivelar maps on the surface of Euler characteristic-2 

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#### Abstract

Semi-equivelar maps are generalization of equivelar maps. We classify some Semi-equivelar maps with 12 vertices on the surface of Euler characteristic $(\chi)=-2$ and calculate their Automorphism Groups.


Keywords: semi-equivelar maps, automorphism group.
AMS subject classifications. 52B70, 57M20, 57N05

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# A topological proof of Ryser's formula for permanent and a similar formula for determinant of a matrix 

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#### Abstract

In this paper we give a topological proof of Ryser's formula for permanents. Also we give a purely combinatorial proof of a Ryser-type formula for determinants. The later argument also includes a combinatorial proof of an interesting identity about Stirling number of second kind. Keywords: permanent, determinant, Stirling number, simplicial complex, Ryser's formula. AMS subject classifications. 05A05; 05A19; 05E45.


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## Study of spectrum of certain multi-parameter spectral problems

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#### Abstract

In this paper, Multi-parameter matrix eigenvalue problems of the form $$
\left(A_{i}-\sum_{j=1}^{k} \lambda_{j} B_{i j}\right) x_{i}=0, i=1,2, \ldots, k
$$ has been considered, where $\lambda_{i} \in C^{k}$ are spectral parameters, $A_{i}, B_{i j}$ are self-adjoint, bounded linear operators, that act on separable Hilbert Spaces $H_{i}$, and $x_{i} \in H_{i}$. The problem is to find k-tuple of values $\lambda=\left(\lambda_{1}, \lambda_{2}, \ldots, \lambda_{k}\right) \in C^{k}$ for non-zero vector $x_{i}$. The k-tuple $\lambda \in C^{k}$ is called an eigenvalue and the corresponding decomposable tensor product $x=x_{1} \otimes x_{2} \otimes x_{3} \cdots \otimes x_{k}$ is called eigenvector (right). Similarly, left eigenvector can also be defined. To study the spectrum, the problem has been identified into three categories from the viewpoint of definiteness conditions adopted by Atkinson. For numerical treatment, the case $k>3$ is considered.


Keywords: multi-parameter matrix eigenvalue problems, Kronecker product, tensor product space AMS subject classifications. 35PXX, 65FXX, 65F15, 35A35

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# A relation between Fibonacci numbers and a class of matrices ${ }^{5}$ 

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#### Abstract

Farber and Berman proved that if $A_{n}$ is the collection of all upper triangular, $\{0,1\}$, invertible matrices, then for any integer $s$ lying between $2-F_{n-1}$ and $2+F_{n-1}$, there exists a matrix $A \in \mathbb{A}_{n}$ such that $S\left(A^{-1}\right)=s$, where $S\left(A^{-1}\right)$ stands for the sum of all entries of $A^{-1}$ and $F_{n}$ is the Fibonacci number defined by $F_{n}=$ $F_{n-1}+F_{n-2}, n \geqslant 3, F_{1}=F_{2}=1$. We will establish the analogue of this result for the collection of all upper triangular, $\{0,1\}$, singular, group invertible matrices.


Keywords: Fibonacci number, group inverse, upper triangular matrix, $\{0,1\}$ matrix, sum of entries AMS subject classifications. 15A09; 15A15; 15B36

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# Laplacian-energy-like invariant of power graphs on certain finite groups ${ }^{6}$ 

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#### Abstract

The power graph $\mathscr{G}(G)$ of a finite group $G$ is the graph whose vertices are the elements of $G$ and two distinct vertices are adjacent if and only if one is an integral power of the other. Here we first find the Laplacian spectrum of the power graph of additive cyclic group $\mathbb{Z}_{n}$ and the dihedral group $D_{n}$ partially. Then we concentrate on Laplacian-energy-like invariant of $\mathscr{G}\left(\mathbb{Z}_{n}\right)$ and $\mathscr{G}\left(D_{n}\right)$. For a nonzero real number $\alpha$, let $s_{\alpha}(\mathbb{G})$ be the sum of $\alpha^{\text {th }}$ power of the nonzero Laplacian eigenvalues of a graph $\mathbb{G}$ and $s_{\frac{1}{2}}(\mathbb{G})$ is known as Laplacian-energy-like invariant (LEL for short) of $\mathbb{G}$. Here we improve lower bound of $s_{\alpha}\left(\frac{\sqrt{G}}{G}\right)$ for $\alpha<0$ or $\alpha>1$ and upper bound of $s_{\alpha}(G)$ for $0<\alpha<1$ given by Zhou [15] for the particular classes of graphs $\mathscr{G}\left(\mathbb{Z}_{n}\right)$ and $\mathscr{G}\left(D_{n}\right)$. Moreover we found lower bounds of $s_{\alpha}\left(\mathscr{G}\left(\mathbb{Z}_{n}\right)\right)$ and $s_{\alpha}\left(\mathscr{G}\left(D_{n}\right)\right)$ for $0<\alpha<1$ in terms of number of vertices and Zagerb index. As a result we get bounds for Laplacian-energy-like invariant of these graphs. Keywords: finite groups, power graphs, Laplacian spectrum, Laplacian-energy-like invariant AMS subject classifications. 05C25; 05C50

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# On spectral relationship of a signed lollipop graph with its underlying cycle 

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#### Abstract

Let $H_{n, k}^{g}$ denote the lollipop graph on $g$ vertices obtained by identifying a vertex of the signed cycle $C_{n}$ of order $n$ and an end vertex of the signless path $P_{k+1}$ of order $k+1$.The sign of the edge connecting the vertex $v$ (say) of the cycle $C_{n}$ to an end vertex of the path is the product of the signs of edges adjacent to $v$ in $C_{n}$. This sign is assigned to remaining edges in $P_{k+1}$. In this work we have deduced a general relationship between the characterstic polynomial of $H_{n, k}^{g}$ and $C_{n}$ for $k=1$, i.e., when the path is of length 1 . Further, we comment on the general case $k$. Also, the relationship between $L$ - spectra and $Q$ - spectra of $C_{n}$ and $H_{n, k}^{g}$ are explored where $L$ and $Q$ stand for Laplacian and signless Laplacian matrix of a signed graph respectively.


Keywords: cycle, lollipop graphs, paths,signed graph, Laplacian, signless Laplacian.
AMS subject classifications. 13C10; 15A09; 15A24; 15B57

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## Matrix semipositivity revisited

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#### Abstract

Semipositive matrices (matrices that map at least one nonnegative vector to a positive vector) and minimally semipositive matrices (semipositive matrices whose no column-deleted submatrix is semipositive) are well studied in matrix theory. In this talk, we present a pot-pourri of results on these matrices. Considerations involving products, difference and the principla pivot transform. We also study the following classes of matrices in relevance to semipositivity and minimal semipositivity: $N$-matricces, almost $N$-matrices and almost $P$-matrices. Keywords: semipositive matrix, minimally semipositive matrix, principal pivot transform, Moore-Penrose inverse, interval of matrices, $N$-matrix, almost $N$-matrix. AMS subject classifications. 15A09,15B48.


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# Generalized Fiedler pencils with repetition for polynomial eigenproblems and the recovery of eigenvectors, minimal bases and minimal indices 

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#### Abstract

A polynomial eigenvalue problem (PEP) is to solve $$
P(\lambda) x:=\left(\sum_{i=0}^{m} A_{i} \lambda^{i}\right) x=0, \text { where } A_{i} \in \mathbb{C}^{n \times n}, i=0,1, \ldots, m,
$$


for $\lambda \in \mathbb{C}$ and a nonzero $x \in \mathbb{C}^{n}$. Linearization is a classical and most widely used method for solving a PEP in which a PEP is transformed to a generalized eigenvalue problem of the form $(A+\lambda B) u=0$ of larger size. Structured (symmetric, anti-symmetric, palindromic, etc.) PEP arises in many applications. For a structured PEP, it is desirable to construct structure-preserving linearizations so as to preserve the spectral symmetry of the PEP which may be important from physical as well as computational view point. In this talk, we consider a special class of structure-preserving linearizations known as generalized Fiedler pencil with repetition (GFPR) and describe the recovery of eigenvectors, minimal bases and minimal indices of PEP from those of the GFPRs.
Keywords: matrix polynomials, matrix pencils, eigenvector, minimal indices, minimal bases, linearization.
AMS subject classifications. 65F15, 15A57, 15A18, 65F35

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# On Osofsky's 32 -elements matrix ring 

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#### Abstract

Let $A=\mathbb{Z} /(4)$ be the ring of integers module 4 and $B=(2) /(4)$ be the ideal in $A$. The ring $R=\left(\begin{array}{cc}A & B \\ 0 & A\end{array}\right)$ is known as Osofsky's 32 -elements matrix ring, and it first appeared in [9] as an example to illustrate the fact that injective hull of a ring may not have a ring structure in general. This paper is an attempt to make an exhaustive study on this matrix ring. Among many things, we found that this ring, along with Example 6.7 in [7], turns out to be another source of example of a semiperfect, CD3-ring for which not every cyclic right $R$-module is quasi-discrete. We observed that the ring has the following properties: Artinian (left/right), $\pi$-regular, $I_{0}$, 2-primal, ACC annihilator (left/right), ACC principal(left/right), Clean, Coherent (left/right), Cohopfian (left/right), Connected, C3, DCC annihilator (left/right), Dedekind finite, essential socle (right/left), exchange, finite, finite uniform dimension (right/left), finitely cogenerated (right/left), finitely generated socle (right/left), Goldie (right/left), IBN, Kasch (right/left), NI (Nilpotents from an ideal), Nil radical, Nilpotent radical, Noetherian (right/left), Non-zero Socle (right/left), Orthogonally finite, Perfect (right/left), Polynomial Identity, Quasi-duo (right/left), Semilocal, Semiperfect, Semiprimary, Semiregular, Stable range 1, Stably finite, Strongly $\pi$-regular, $T$-nilpotent radical (right/left), top regular, Zorn

However, the ring lacks the following properties: Abelian, Armendariz, Baer, Bezout (right/left), Bezout domain (right/left), Cogenerator ring (right/left), C1, C2, distributive (right/ left), division ring, domain, Dual (right/left), duo (right/left), FI-injective (right/left), Finitely pseudo-Frobenius (right/left), Free ideal ring (right/left), Frobenius, Fully prime, Fully semi prime, Hereditary (right/left), Local, Nonsingular (right/left), Ore domain (right/left), Primary, Prime, Primitive (right/left), Principal ideal domain (right/left), Principally injective (right/left), (right/ left), Quasi-Frobenius, Reduced, Reversible, Rickart (right/left), Self injective (right/ left), Semi free ideal ring, Semicommutative (SI condition, Zero-insertive), (right/left), Semiprime, Semiprimitive, Semisimple, Simple, Simple Artinian, Simple Socle (right/left), Simple-injective (right/left), Strongly Connected, Strongly regular, Symmetric, Uniform (left/ right), Unit regular, V ring (right/ left), Valuation ring (right/left), Von Neumann regular, IN (Ikeda-nakayama).


Keywords: matrix ring, injective hull, CD3-ring.
AMS subject classifications. 16D10; 16D40, 16D70,16D60

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# Necessary and sufficient conditions for locating repeated solid burst 

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#### Abstract

Wolf and Elspas [4] introduced a midway concept (known as error location coding) between error detection and error correction. Error locating codes have been found to be efficient in feedback communication systems. Solid burst error is one type of error commonly found in many memory communication channels viz. semiconductor memory data, supercomputer storage system.

In busy communication channels, it is found by Dass, Verma and Berardi [1] that errors repeat themselves. They have initiated the idea of repeated errors and introduced 2-repeated burst. Further, mrepeated burst was introduced by Dass and Verma in [2]. Extending this idea, '2-repeated solid burst of length $b$ ' and ' $m$-repeated solid burst of length $b$ ' are studied by Rohtagi and Sharma [3]. They presented necessary and sufficient conditions for codes correcting such errors. Cyclic codes for the detection of such errors were also studied.

In this paper, we study linear codes that detect and locate such repeated solid burst of length $b$. We provide necessary and sufficient conditions for the existence of linear codes that can locate such errors. An example is also given.


Keywords: parity check matrix, solid burst errors, error pattern-syndromes, EL-codes
AMS subject classifications. 94B05, 94B25, 94B65.

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# Modified triangular and symmetric splitting method for the steady state vector of Markov chains 

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#### Abstract

In this paper, we used a modified triangular and symmetric splitting (MTS) method in order to solve the regularized linear system $A x=b$ associated with stochastic matrices. We proved that there exist $\epsilon \geq 0$ such that the regularized matrix $A=Q^{T}+\epsilon I$ is positive definite, where $I$ is the real identity matrix of designated dimension of $Q^{T}$, and $Q^{T}$ is stochastic rate matrix with positive diagonal and non-positive off-diagonal elements. Theoretical analysis shows that the iterative solution of MTS method converges unconditionally to the unique solution of the regularized linear system.


Keywords: self-similarity, circulant stochastic matrices, steady state probability vector, MTS Method, convergence analysis.
AMS subject classifications. 65F15; 65F35; 65F10; 45C05; 15B51

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# Reachability problem on graphs by a robot with jump: some recent studies 

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#### Abstract

Consider a graph $G$ on $n$ vertices with a robot at one vertex, one empty vertex and obstacles in the remaining $n-2$ vertices. Let $S$ be a set of non-negative integers. A robot can jump from a vertex $u$ to a vertex $v$ provided $v$ is empty and there is $u-v$ path of length $m$ for some $m \in S$. An obstacle can be moved to an adjacent empty vertex only. The graph $G$ is called complete $S$-reachable if the robot can be taken to any vertex of $G$ irrespective of its starting vertex. In this talk we will discuss some recent developments in the characterization of complete $S$-reachable graphs.


Keywords: diameter, reachability, starlike trees, mRJ-moves
AMS subject classifications. 91A43, 68R10, 05C05

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# On principal pivot transforms of hidden $Z$ matrices 

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#### Abstract

In this talk, we demonstrate how the concept of principal pivot transform can be effectively used to extend many existing results on hidden $\mathbf{Z}$ matrices. In fact, we revisit various results obtained for hidden $\mathbf{Z}$ class by Mangasarian [2, 3, 4], Cottle and Pang [1] in context of solving linear complementarity problems as linear programs. We identify hidden $\mathbf{Z}$ matrices of special category and discuss the number of solutions of the associated linear complementarity problems. We also present game theoretic interpretation of various results related to hidden $\mathbf{Z}$ class . Keywords: principal pivot transform, hidden Z-matrix, linear complementarity problem AMS subject classifications. 90C33

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# Graph Laplacian quantum states and their properties 

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#### Abstract

A quantum state can be represented by a density matrix that is a positive semidefinite, Hermitian matrix with unit trace. Given a combinatorial graph $G$ there is a density matrix given by $$
\begin{equation*} \rho(G)=\frac{K(G)}{\operatorname{trace}(K(G))}, \tag{10.1} \end{equation*}
$$ where $K(G)=L(G)$, the Laplacian matrix or $K(G)=Q(G)$, the signless Laplacian matrix. We call the underlined quantum state as graph Laplacian quantum state [1, 2]. A number of important properties of the underlined quantum state can be illustrated by the structure of the graph $G$. In this talk I shall discuss about quantum entanglement, and discord from a graph theoretic perspective [3, 4, 5, 6]. Keywords: combinatorial graphs, Laplacian matrices, quantum states, density matrix, local unitary operators, quantum entanglement, discord. AMS subject classifications. 05C50, 81Q99

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# On absolutely norm attaining paranormal operators 

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#### Abstract

Let $H$ be a complex Hilbert space and $T: H \rightarrow H$ be a bounded linear operator. Then $T$ is said to be norm attaining if there exists a unit vector $x_{0}$ such that $\left\|T x_{0}\right\|=\|T\|$. If for any closed subspace $M$ of $H$, the restriction $T \mid M: M \rightarrow H$ of $T$ to $M$ is norm attaining, then $T$ is called an absolutely norm attaining operator or $\mathscr{A} \mathscr{N}$-operator. These operators are studied in [1, 2, 3]. In this talk, we present the structure of paranormal $\mathscr{A} \mathscr{N}$-operators and give a necessary and sufficient condition under which a paranormal $\mathscr{A} \mathscr{N}$ operator is normal. Keywords: compact operator, norm attaining operator, $\mathscr{A} \mathscr{N}$-operator, Weyl's theorem, paranormal operator, reducing subspace


AMS subject classifications. 47A15, 47B07, 47B20, 47B40

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# Perturbation of minimum attaining operators ${ }^{7}$ 

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#### Abstract

We prove that the minimum attaining property of a bounded linear operator on a Hilbert space $H$ whose minimum modulus lies in the discrete spectrum, is stable under small compact perturbations. We also observe that given a bounded operator with strictly positive essential minimum modulus, the set of compact perturbations which fail to produce a minimum attaining operator is smaller than a nowhere dense set. In fact it is a porous set in the ideal of all compact operators on $H$. Further, we try to extend these stability results to perturbations by all bounded linear operators with small norm and obtain subsequent results.


Keywords: minimum modulus, spectrum, essential spectrum, porous set
AMS subject classifications. Primary 47B07, 47A10, 47A75, 47A55, 47B65

[^9]
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# A note on Jordan derivations over matrix algebras ${ }^{8}$ 

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#### Abstract

In 2006, Zhang and Yu [4] have shown that every Jordan derivation from triangular algebra $U$ over 2torsionfree commutative ring into itself is a derivation. Let $C$ be a commutative ring with identity $1 \neq 0$. We prove that every Jordan derivation over an upper triangular matrix algebra $\mathscr{T}_{n}(C)$ is a derivation. We also prove the result for Jordan derivation on $\mathscr{T}_{n}(F)$, where $F=\{0,1\}$ and further we characterize Jordan derivation on full matrix algebras $\mathscr{M}_{n}(C)$. Keywords: Jordan derivations, derivations, upper triangular matrix algebra, full matrix algebra AMS subject classifications. 47B47; 47L35

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[^10]
# Causal detectability for linear descriptor systems 

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#### Abstract

Consider the linear descriptor systems of the form $$
\begin{align*} E \dot{x} & =A x+B u,  \tag{10.2a}\\ y & =C x, \tag{10.2b} \end{align*}
$$


where $x \in \mathbb{R}^{n}, u \in \mathbb{R}^{k}, y \in \mathbb{R}^{p}$ are the state vector, the input vector, and the output vector, respectively. $E, A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{m \times k}, C \in \mathbb{R}^{p \times n}$ are known constant matrices. During past few decades, a lot of work has been done on various types of observer design for the systems of the form (10.2), see [1, 2] and the references therein. Among all the observers, Luenberger observers were paid the most attention due to its explicit nature. Several techniques have been developed to design Luenberger observer for the descriptor system (10.2) and sufficient conditions on system operators have been provided for the existence of the Luenberger observer. Hou and Müller [3] have proved that a rectangular descriptor system (10.2) can be observed by a Luenberger observer if and only if it is causally detectable. But these authors have given the condition of causal detectability of the system on a transformed system that can only be obtained by applying a finite number of orthogonal transformations on the original system. Thus without getting the transformed system, it is not possible to know that for a given descriptor system a Luenberger observer can be designed or not. In this work, the causal observability has been established in terms of system coefficient matrices. Therefore, necessary and sufficient conditions for the existence of Luenberger observers are provided in terms of system matrices.
Keywords: observer design, descriptor systems, Luenberger observer, causal detectability
AMS subject classifications. 47N70; 93B07; 93B30; 93B11; 93B10; 93C05

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# An alternative approach for solving fully fuzzy linear systems based on FNN 

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#### Abstract

Artificial neural networks have the advantages such as learning, adaptation, fault-tolerance, parallelism and generalization. The focus of this paper is to introduce an efficient computational method which can be applied to approximate solution of a fuzzy linear equations system with fuzzy square coefficients matrix and fuzzy right hand vector. Supposedly the given fuzzy system has an unique fuzzy solution, an architecture of fuzzy feed-forward neural networks (FFNN) is presented in order to find the approximate solution. The proposed FFNN can adjust the fuzzy connection weights by using a learning algorithm that is based on the gradient descent method. The proposed method is illustrated by several examples. Also results are compared with the exact solutions by using computer simulations.


Keywords: fully fuzzy linear system, fuzzy neural network(FNN), learning algorithm, cost function

# Nonsingular subspaces of $M_{n}(\mathbb{F}), \mathbb{F}$ a field 

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#### Abstract

For a field $\mathbb{F}$, a subspace $\sqrt[V]{ }$ of $M_{n}(\mathbb{F})$ is said to be nonsingular if every nonzero element of $V$ is nonsingular. When $\mathbb{F}=\mathbb{C}$, any such subspace has dimension at most 1 and when $\mathbb{F}=\mathbb{R}$, a nonsingular subspace of dimension $n$ in $M_{n}(\mathbb{R})$ will exist if and only if $n=2,4,8$. Our objective is to understand the structure of nonsingular subspaces of dimension in $n$ in $M_{n}(\mathbb{R})$. Connections with a specific linear preserver problem will be pointed out.


Keywords: nonsingular subspace, invertibility (full-rank) preservers, linear preservers of minimal semipositivity
AMS subject classifications. 15A86, 15B48, 15A09

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# Hypo-EP Operators ${ }^{9}$ 

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#### Abstract

An analytic characterization of hypo- $E P$ operator is given. Sum, product, restriction and factorization of hypo- $E P$ operators are discussed. Keywords: hypo- $E P$ operator, $E P$ operator, Moore-Penrose inverse AMS subject classifications. 47A05, 47B20


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# On distance and Laplacian matrices of a tree with matrix weights 

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#### Abstract

The distance matrix of a simple connected graph $G$ is $D(G)=\left(d_{i j}\right)$, where $d_{i j}$ is the distance between the vertices of $i$ and $j$ in $G$. We consider a weighted tree $T$ on $n$ vertices with each of the edge weight is a square matrix of order $s$. The distance $d_{i j}$ between the vertices $i$ and $j$ is the sum of the weight matrices of the edges in the unique path from $i$ to $j$. Then the distance matrix $D$ of $T$ is a block matrix of order $n s \times n s$. In this paper we establish a necessary and sufficient condition for the distance matrix $D$ to be invertible and the formula for the inverse of $D$, if it exists. This generalizes the existing result for the distance matrix of a weighted tree, when the weights are positive numbers. Some more results which are true for unweighted tree and tree with scaler weights are extended here in case of tree with matrix weights. We also extend some result which involves relation between the eigenvalues of distance and Laplacian matrices of trees.


Keywords: trees, distance matrix, Laplacian matrix, matrix weights, inverse.
AMS subject classifications. $05 \mathrm{C} 50,05 \mathrm{C} 22$.

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# Further results on AZI of connected and unicyclic graph ${ }^{11}$ 

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#### Abstract

The Augmented Zagreb index ( $A Z I$ ) of a graph $G$, initially refers as a molecular descriptor of certain hydrocarbons is defined as $$
A Z I(G)=\sum_{u v \in E(G)}\left(\frac{d_{u} d_{v}}{d_{u}+d_{v}-2}\right)^{3}
$$ where $E(G)$ is the edge set of G and $d_{u}$ and $d_{v}$ are respectively degrees of end vertices $u$ and $v$ of the edge $u v$. This topological index introduced by Furtula et al.[6], has characterized as a useful measure in the study of the heat and formation in heptane and octanes. In this paper, we obtain further results on $A Z I$ for connected complement of a graph, and $n$ - vertex unicyclic chemical graph with some improvement as well as extremal cases. We also obtain some standard $A Z I$ results for known graphs. Keywords: augmented Zagreb index, chemical graph, unicyclic graph AMS subject classifications. 05 C 10 ; 05C35; 05C75

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[^12]
# Distance matrices of partial cubes ${ }^{12}$ 

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#### Abstract

Partial cubes are isometric subgraphs of hypercubes. Median graph is a graph in which every three vertices $u, v$, and $w$ have a unique median: a vertex $m$ that belongs to shortest paths between each pair of $u, v$, and $w$. Median graphs present one of the most studied subclasses of partial cubes. We determine the Smith normal form of the distance matrices of partial cubes and the factorisation of Varchenko determinant of product distance matrices of median graphs.


Keywords: distance matrix, Smith normal form, hypercube, isometric embedding, partial cube, median graph
AMS subject classifications. 05C12; 05C50

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# On the spectrum of the linear dependence graph of finite dimensional vector spaces ${ }^{13}$ 

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#### Abstract

In this paper, we introduce a graph structure called linear dependence graph of a finite dimensional vector space over a finite field. Some basic properties of the graph like connectedness, completeness, planarity, clique number, chromatic number etc. have been studied. Also, adjacency spectrum, Laplacian spectrum and distance spectrum of the linear dependence graph have been studied. Keywords: graph, linear dependence, Laplacian, distance, spectrum AMS subject classifications. 05C25; 05C50; 05C69

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# On the adjacency matrix of complex unit gain graphs 

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#### Abstract

A complex unit gain graph is a graph in which each orientation of an edge is given a complex number with modulus 1 and it's inverse is assigned to the opposite orientation of the edge. The adjacency matrix of a complex unit gain graph [5] is a Hermitian matrix. Interestingly the spectral theory of complex unit gain graphs generalizes the spectral theory of undirected graphs [1, 2] and some weighted graphs [4]. Here, we establish some useful properties of the adjacency matrix of complex unit gain graph. We provide bounds for the eigenvalues of the complex unit gain graphs. Then we establish some of the properties of the adjacency matrix of complex unit gain graph in connection with the characteristic [3] and the permanental polynomials. Then we derive spectral properties of the adjacency matrices of complex unit gain bipartite graphs. Finally, for trees and unicyclic graphs, we establish relationships between the characteristic and permanental polynomials of adjacency matrix of complex unit gain graph with the usual characteristic and permanental polynomials of the $(0,1)$ adjacency matrix of the underlying graph.


Keywords: gain graphs, characteristics polynomial of graphs, permanental polynomials of graphs, eigenvalues, unicyclic graphs, bipartite graphs.
AMS subject classifications. $05 \mathrm{C} 50,05 \mathrm{C} 22$

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# Semipositivity of matrices over the $n$-dimensional ice cream cone and some related questions ${ }^{14}$ 

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#### Abstract

An $m \times n$ matrix $A$ with real entries is said to be semipositive if there exists $x \geq 0$ such that $A x>0$, where the inequalities are understood componentwise. Our objective is to characterize semipositivity over the Lorentz or ice cream cone in $\mathbb{R}^{n}$, defined by $\mathscr{L}_{+}^{n}=\left\{x=\left(x_{1}, \ldots, x_{n}\right) \in \mathbb{R}^{n} \mid x_{n} \geq 0, \sum_{i=1}^{n-1} x_{i}^{2} \leq x_{n}^{2}\right\}$. We also investigate products of the form $A_{1} A_{2}^{-1}$, where $A_{1}$ is either positive or semipositive and $A_{2}$ is positive and


 invertible. Time permitting, preservers of semipositivity with respect to $\mathscr{L}_{+}^{n}$ will be pointed out.Keywords: semipositive matrices, Lorentz cone, linear preservers
AMS subject classifications. 15B48,15A99

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# Computational methods to find core-EP inverse ${ }^{15}$ 

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#### Abstract

Core-EP inverse $G$ of a square matrix $A$ is an outer inverse such that Column space $(A)=$ Row space $(A)=$ Column space ( $A^{k}$ ) for some $k \geq$ index ( $A$ ). Core-EP inverse has been firstly defined and obtained an explicit expression by Prasad [1] in 2015. In this work, we describe the bordering method and iterative method to find the core-EP inverse and core-EP generelized inverse. Keywords: core-EP inverse, core-EP generalized inverse, bordering, g-inverse, iterative method AMS subject classifications. 15A09, 15A29, 15A36

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# Prediction of survival with inverse probability weighted Weibull models when exposure is quantitative 

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#### Abstract

Survival analysis, based on propensity scores (PS), is a promising methodology to conduct causal inference. Propensity score method for analyzing time-to-event outcomes in the categorical exposure case is perceived to be very efficient in the estimation of effect measures such as marginal survival curves and marginal hazard ratio in the cohort studies. These methods include techniques such as matching, covariate adjustment, stratification and inverse probability of weighting (IPW) to adjust for confounding factors between exposure groups.

But in several practical situations, the exposure/s could be continuous variable/s. For example in the study of risk factors for diabetic foot, plantar foot pressures may be considered as exposures, which are continuous variables in nature. Also, we come across distribution of the survival time that is different from exponential distribution. The generalization of the exponential distribution to include the shape parameter is the Weibull distribution.

The objective of this presentation is to describe and compare propensity score weighted model Weibull survival model with basic Weibull survival model for different shape parameters of survival distribution. Also, we present a methodology to compare PS based Weibull models for predicting survival (hazard rate) when the exposure is quantitative and continuous.


Keywords: propensity score, Weibull survival, inverse probability weights, causal inference
AMS subject classifications. 62 N 99

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# Incentive structure reorgnization to maximize healthcare players' payoff while keeping the healthcare service provider's company solvent 

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#### Abstract

This study focuses on modeling an incentive structure of stakeholders (Doctors, Patients, Service Providers) in healthcare sector and optimize the stakeholders' Payoff with the use of solution concepts of Game Theory and Decision eMaking to arrive at an optimal solution which puts a downward pressure on the cost of healthcare for all the players. This is done by considering the Ruin probability problem to determine the risk or surplus process to keep the average cost burden on the consumers floating at the community health level. These models are of the type non-cooperative extensive games which determines the tractability in healthcare from the point of view of the utility function of stakeholders.


Keywords: Game theory, ruin probability, healthcare, extensive games
AMS subject classifications. 13C10; 15A09; 15A24; 15B57

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# Immanants of $q$-Laplacians of trees ${ }^{16}$ 

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#### Abstract

Let $T$ be a tree on $n$ vertices with Laplacian matrix $L_{T}$ and $q$-Laplacian $\mathscr{L}_{T}$. Let $\chi_{\lambda}$ be the character of the irreducible representation of the symmetric group $\mathfrak{S}_{n}$ indexed by the partition $\lambda$ of $n$. Let denote $d_{\lambda}\left(L_{T}\right)$ and $d_{\lambda}\left(\mathscr{L}_{T}\right)$ as the immanant of $L_{T}$ and $\mathscr{L}_{T}$ respectively, indexed by $\lambda$. The immanantal polynomial of $L_{T}$ indexed by partition $\lambda \vdash n$ is defined as $f_{\lambda}^{L_{T}}(x)=d_{\lambda}\left(x I-L_{T}\right)$. Let $f_{\lambda}^{L_{T}}(x)=\sum_{r=0}^{n}(-1)^{r} c_{\lambda, r}^{L_{T}} x^{n-r}$. Let $\bar{d}_{\lambda}\left(L_{T}\right)=\frac{c_{\lambda, n}^{L_{T}}}{\chi_{\lambda} \text { (id) }}$ be the normalized immanant of $L_{T}$ indexed by $\lambda$, where id is the identity permutation in $\mathfrak{S}_{n}$.

When $\lambda=k, 1^{n-k}$, inequalities are known for $\bar{d}_{k, 1^{n-k}}\left(L_{T}\right)$ as $k$ increases (see [1, 4, 5]). By using matchings and assigning statistics to vertex orientations, we generalize these inequalities to the matrix $\mathscr{L}_{T}$, for all $q \in \mathbb{R}$ and to the bivariate $q, t$-Laplacian $\mathscr{L}_{T}^{q, t}$ for a specific set of values $q, t$, where both $q, t \in \mathbb{R}$ or both $q, t \in \mathbb{C}$. Our statistic based approach also gives generalization of inequalities given in [2] for a Hadamard inequality changing index $k\left(L_{T}\right)$ of $L_{T}$, to the matrices $\mathscr{L}_{T}$ and $\mathscr{L}_{T}^{q, t}$ for trees.

Csikvári [3] defined a poset on the set of unlabelled trees on $n$ vertices. We proved that when we go up in this poset, $\left|c_{\lambda, r}^{\mathscr{L}_{T}}\right|$ (the coefficient of $(-1)^{r} x^{n-r}$ in $f_{\lambda}^{\mathscr{L}_{T}}(x)$ in absolute value) decreases for all $q \in \mathbb{R}$ and for $0 \leq r \leq n$.


Keywords: normalized hook immanants, $q$-Laplacian, trees, Hadamard inequality
AMS subject classifications. 15A15; 05C05

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## Jacobi type identities

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#### Abstract

Jacobi identity relates any minor of $A^{-1}$, the inverse of a matrix $A$, with determinant $|A|$ and the complementary minor in the transpose of $A$. Several extensions have been attempted by Stanimirović et al. [1] and Bapat [2], where the given matrix over a real or complex field is singular and rectangular. In this paper, we consider the matrices over a commutative ring and characterize the class of outer inverses for which Jacobi type identities could be extended. Keywords: matrices over commutative ring, determinantal rank, generalized inverse, outer inverse, Jacobi identity, Rao-regular matrix AMS subject classifications. 15A09


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# Determinants in the study of Generalized Inverses of Matrices over Commutative Ring 

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#### Abstract

Determinantal rank serves as an alternative notion for the column rank of a matrix, when the matrices are with entries from a commutative ring. The notion of minors defined with the help of determinant, also helps in characterizing the matrices having generalized inverses, and in providing determinantal formula for generalized inverses, whenever they exist. The Jacobi identity provides an expression for the minors of a nonsingular matrix in terms of the determinant of a given matrix. We were successful in extending the Jacobi identity for the outer inverses of a matrix over a commutative ring. In the process, we attempted to characterize the existence of an outer inverse in terms of minors of a given matrix and provide a determinantal formula for the same. As a special case, a determinantal formula for a Raoregular outer inverse has been provided. Also, the minus partial order on the class of regular matrices over a commutative ring has been characterized and an extension of rank-additivity, whenever a matrix is dominated by the other matrix with respect to the minus partial order has been explored.


Keywords: matrices over commutative ring, determinantal rank, generalized inverse, Drazin inverse, Jacobi identity, Rao-regular matrix
AMS subject classifications. 15A09

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# The Laplacian spectra of power graphs of cyclic and dicyclic finite groups ${ }^{17}$ 

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#### Abstract

The power graph of a group $G$ is the graph whose vertex set is $G$ and two distinct vertices are adjacent if one is a power of the other. In this article, the Laplacian spectra of power graphs of certain finite groups is studied. Firstly, certain upper and lower bounds of algebraic connectivity of power graphs of finite cyclic groups are obtained. Then the Laplacian spectra of power graphs of dicyclic groups is investigated and the complete Laplacian spectra of power graphs of some class of dicyclic groups are determined.


Keywords: power graph, Laplacian spectrum, algebraic connectivity, cyclic group, dicyclic group
AMS subject classifications. 05C50; 05C25

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[^15]
# Distance Laplacian spectra of graphs obtained by generalization of join and lexicographic product 

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#### Abstract

The distance Laplacian matrix of a simple connected graph $G$ is defined as $D^{L}(G)=\operatorname{Tr}(G)-D(G)$, where $D(G)$ is the distance matrix of $G$ and $\operatorname{Tr}(G)$ is the diagonal matrix whose main diagonal entries are the vertex transmissions in $G$. In this article, we determine the distance Laplacian spectra of the graphs obtained by generalization of the join and lexicographic product of graphs (namely joined union). It is shown that the distance Laplacian spectra of these graphs not only depend on the distance Laplacian spectra of the participating graphs but also depend on the spectrum of another matrix of vertex-weighted Laplacian kind (analogous to the definition given by Chung and Langlands [6]). Keywords: distance Laplacian matrix, join, lexicographic product, joined union AMS subject classifications. 05C50; 05C12; 15A18.

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# Study of maps on surfaces using face face incident matrix 

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#### Abstract

We introduce face face (FF) incidence matrix associated to maps on surfaces. Eigenvalues of this matrix correponds to many topological properties. We present some observations in this direction.


Keywords: maps on surfaces
AMS subject classifications. 05E45; 05C50

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# On Laplacian spectrum of reduced power graph of finite cyclic and dihedral groups ${ }^{18}$ 

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#### Abstract

The reduced power graph $\mathscr{P}(G)$ of a group $G$ is the graph having all the elements of $G$ as its vertex set and two vertices $u$ and $v$ are adjacent in $\mathscr{P}(G)$ if and only if $u \neq v$ and $\langle u\rangle \subset\langle v\rangle$ or $\langle v\rangle \subset\langle u\rangle$. In this paper, we study the Laplacian spectrum of the reduced power graph of additive cyclic group $\mathbb{Z}_{n}$ and dihedral group $D_{n}$. We determine the algebraic connectivity of $\mathscr{P}\left(\mathbb{Z}_{n}\right)$ and $\mathscr{P}\left(D_{n}\right)$. Moreover, we give a lower bound for the Laplacian energy of $\mathscr{P}\left(\mathbb{Z}_{n}\right)$. Keywords: finite group, reduced power graph, Laplacian eigenvalues, algebraic connectivity, Laplacian energy AMS subject classifications. 05C50; 05C25

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# Some graphs determined by their spectra 

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#### Abstract

The graph $K_{n} \backslash K_{l, m}$ is obtained from the complete graph $K_{n}$ by removing all the edges of a complete bipartite subgraph $K_{l, m}$. In [2], Cámara and Haemers proved that the graph $K_{n} \backslash K_{l, m}$ is determined by its spectrum. In this paper, we show that the graph $K_{n} \backslash K_{1, m}$ with $m \geq 4$ is determined by its signless Laplacian spectrum and also we prove that the graph $K_{n} \backslash K_{l, m}$ is determined by its distance spectrum. In addition, we show that the join graph $m K_{2} \vee K_{n}$ is determined by its signless Laplacian spectrum. This result extends earlier studies on signless Laplacian spectral determination of $m K_{2} \vee K_{n}$, when $n=1,2$ see [1, 5].


Keywords: cospectral graphs, signless Laplacian spectrum, distance spectrum.
AMS subject classifications. 05C50

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# On the distance spectra and distance Laplacian spectra of graphs with pockets ${ }^{19}$ 

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#### Abstract

Let $F, H_{v}$ be simple connected graphs. Let $v$ be a specified vertex of $H_{v}$ and $u_{1}, \ldots, u_{k} \in F$. Then the graph $G=G\left[F, u_{1}, \ldots, u_{k}, H_{v}\right]$ obtained by taking one copy of $F$ and $k$ copies of $H_{v}$, and then attaching the $i$-th copy of $H_{v}$ to the vertex $u_{i}, i=1, \ldots, k$, at the vertex $v$ of $H_{v}$ (identify $u_{i}$ with the vertex $v$ of the $i$-th copy) is called a graph with $k$ pockets. We give some results describing the distance spectrum of $G$ using the distance spectrum of $F$ and the adjacency spectrum of $H_{v}$. Consequently, a class of distance singular graphs is obtained. Further, the distance Laplacian spectrum of $G$ is also described using the distance Laplacian spectrum of $F$ and the Laplacian spectrum $H_{v}$. In a particular case, distance and distance Laplacian spectra of generalized stars are discussed. Keywords: graphs, eigenvalues, spectrum, distance matrix, distance Laplacian matrix AMS subject classifications. 05C50, 05C12, 15A18

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# Strong $\mathcal{Z}$-tensors and tensor complementarity problems ${ }^{20}$ 

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#### Abstract

For an m-order n-dimensional real tensor $\mathscr{A}$ (hypermatrix) and $q \in \mathbb{R}^{n}$, the tensor complementarity problem denoted by $T C P(\mathscr{A}, q)$ is to find an $x \in \mathbb{R}^{n}$ such that $$
x \geq 0, y=\mathscr{A} x^{m-1}+q \geq 0 \text { and }\langle x, y\rangle=0 .
$$

Motivated by the study on strong Z-matrices[1] in standard linear complementarity problems, we define strong $\mathcal{Z}$-tensors as a subclass of $\mathfrak{Z}$-tensors. In this talk, we present some of the properties of strong $\mathcal{Z}$-tensors in tensor complementarity problems. Keywords: tensor complementarity problem, strong $\mathcal{Z}$-tensor. AMS subject classifications. 90C33; 65K05; 15A69; 15B48

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[^18]
# Inverse eigenvalue problems for acyclic matrices whose graph is a dense centipede 

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#### Abstract

The reconstruction of a matrix having a pre-defined structure from given spectral data is known as an inverse eigenvalue problem (IEP) [1]. The objective of an IEP is to construct matrices of a certain predefined structure which also satisfy the given restrictions on eigenvalues and eigenvectors of the matrix or its submatrices. The level of difficulty of an IEP depends on the structure of the matrices which are to be reconstructed and on the type of eigen information available. Whereas eigenvalue problems for matrices described by graphs have been studied by several authors[2, 3, 4, 5, 6], IEPs for matrices described by graphs have received little attention [7, 8]. In this paper, we consider two IEPs involving the reconstruction of matrices whose graph is a special type of tree called a centipede. We introduce a special type of centipede called dense centipede. We study two IEPs concerning the reconstruction of matrices whose graph is a dense centipede from given partial eigen data. In order to solve these IEPs, a new system of nomenclature of dense centipedes is developed and a new scheme is adopted for labelling the vertices of a dense centipede as per this nomenclature. Using this scheme of labelling, any matrix of a dense centipede can be represented in a special form which we define as a connected arrow matrix. For such a matrix, we derive the recurrence relations among the characteristic polynomials of the leading principal submatrices and use them to solve the above problems. Some numerical results are also provided to illustrate the applicability of the solutions obtained in the paper.


Keywords: dense centipede, inverse eigenvalue problem, acyclic matrix, leading principal submatrices
AMS subject classifications. $05 \mathrm{C} 50,65 \mathrm{~F} 18$

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# Some properties of Steinhaus graphs 

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#### Abstract

A Steinhaus graph is a simple graph whose adjacency matrix is a Steinhaus matrix. Steinhaus matrix is a matrix obtained by Steinhaus triangle, Steinhaus triangle were first studied by Harboth[1] and later by Chang[2]. Mullunzzo in 1978 made graphs from Steinhaus trianle by extending the Steinhaus triangle in to an adjacency matrix of a Graph. In this paper we introduced Steinhaus complement of a graph and Steinhaus self complementary graph. We characterize Steinhaus complementary graph $G$ using two complement of graph $G$.


Keywords: adjacency matrix, Steinhaus complement, K-complement.
AMS subject classifications. 05C07; 05C50; 05C60

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# $\mathscr{B}$-partitions and its application to matrix determinant and permanent 

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#### Abstract

There is a digraph corresponding to every square matrix over $\mathbb{C}$. We generate a recurrence relation using the Laplace expansion to calculate the determinant and the permanent of a square matrix. Solving this recurrence relation, we found that the determinant and the permanent can be calculated in terms of the determinant and the permanent of some specific induced subdigraphs of blocks in the digraph, respectively. Interestingly, these induced subdigraphs are vertex-disjoint and they partition the digraph. We call such a combination of subdigraphs as $\mathscr{B}$-partition. Let $G$ be a graph (directed or undirected) having $k$ number of blocks $B_{1}, B_{2}, \ldots, B_{k}$. A $\mathscr{B}$-partition of $G$ is a partition into $k$ vertex-disjoint subgraph ( $\hat{B_{1}}, \hat{B_{1}}, \ldots, \hat{B_{k}}$ ) such that $\hat{B}_{i}$ is induced subgraph of $B_{i}$ for $i=1,2, \ldots, k$. The terms $\prod_{i=1}^{k} \operatorname{det}\left(\hat{B}_{i}\right), \prod_{i=1}^{k} \operatorname{per}\left(\hat{B}_{i}\right)$ are the detsummands and the per-summands, respectively, corresponding to the $\mathscr{B}$-partition ( $\hat{B_{1}}, \hat{B_{1}}, \ldots, \hat{B_{k}}$ ). The procedure to calculate the determinant and the permanent of a square matrix using the $\mathscr{B}$-partitions is given in [1]. In particular, the determinant (permanent) of a graph having no loops on its cut-vertices is equal to the summation of the det-summands (per-summands), corresponding to all the possible $\mathscr{B}$ partitions. Thus, we calculate the determinant and the permanent of some graphs, which include block graph, block graph with negatives cliques, bi-block graph, signed unicyclic graph, mixed complete graph, negative mixed complete graph, and star mixed block graphs.


Keywords: $\mathscr{B}$-partitions, blocks (2-connected components), determinant, permanent.
AMS subject classifications. 15A15; 05C20; 68R10.

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# Partition energy of corona of complete graph and its generalized complements 

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## Abstract

Let $G$ be a graph and $P_{k}=V_{1}, V_{2}, \ldots, V_{k}$ be a partition of its vertex set $V$. Recently E. Sampathkumar and M. A. Sriraj in [3] have introduced $L$-matrix of $G=(V, E)$ of order $n$ with respect to a partition $P_{k}=$ $\left\{V_{1}, V_{2}, \ldots, V_{k}\right\}$ of the vertex set $V$. It is a unique square symmetric matrix $P_{k}(G)=\left[a_{i j}\right]$ whose entries $a_{i j}$ are defined as follows:

$$
a_{i j}= \begin{cases}2 & \text { if } v_{i} \text { and } v_{j} \text { are adjacent where } v_{i}, v_{j} \in V_{r}, \\
-1 & \text { if } v_{i} \text { and } v_{j} \text { are non-adjacent where } v_{i}, v_{j} \in V_{r}, \\
1 & \text { if } v_{i} \text { and } v_{j} \text { are adjacent between the sets } \\
& \begin{array}{l}
V_{r} \text { and } V_{s} \text { for } r \neq s \text { where } v_{i} \in V_{r} \text { and } v_{j} \in V_{s} \\
0
\end{array} \\
\text { otherwise. }\end{cases}
$$

This $L$-matrix determines the partition of vertex set of graph $G$ uniquely. We determine the partition energy using its $L$-matrix. The eigenvalues of the partition matrix $P_{V_{1} \cup V_{2} \cup \ldots \cup V_{k}}(G)=P_{k}(G)$ are called $k$ partition eigenvalues. We define the energy of a graph with respect to a given partition as the sum of the absolute values of the $k$-partition eigenvalues of $G$ called $k$-partition energy or partition energy of a graph and is denoted by $E_{P_{k}}(G)$.

In this paper we obtain partition energy of Corona of $K_{n}$ and $K_{n-1}$ and also its generalized complements with respect to uniform partition.

Uniform graph partition is a type of graph partitioning problem that consists of dividing a graph into components, such that the components are of about the same size and there are few connections between the components. Important applications of graph partitioning include scientific computing, partitioning various stages of a VLSI design circuit and task scheduling in multi-processor systems. Recently, the graph partition problem has gained importance due to its application for clustering and detection of cliques in social, pathological and biological networks. Hence we have considered Uniform graph partition in this paper to find the partition energy of some large graphs.
Keywords: corona, $n$-complement, $n(i)$-complement, $n$-partition energy
AMS subject classifications. 15A18, 05C50

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# $M$-operators on partially ordered Banach spaces 

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#### Abstract

For a matrix $A \in \mathbb{R}^{n \times n}$ whose off-diagonal entries are nonpositive, there are several well-known properties that are equivalent to $A$ being an invertible $M$-matrix. One of them is the positive stability of $A$. A generalization of this characterization to partially ordered Banach spaces is considered in this article. Relationships with certain other equivalent conditions are derived. An important result on singular irreducible $M$-matrices is generalized using the concept of $M$-operators and irreducibility. Certain other invertibility conditions of $M$-operators are also investigated.


Keywords: $M$-operators, positive stability, irreducibility, invertibility
AMS subject classifications. [msc2010]15B48, 46B40, 47B65, 47B99

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# Comparison results for proper double splittings of rectangular matrices 

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#### Abstract

In this article, we consider two proper double splittings satisfying certain conditions, of a semi-monotone rectangular matrix $A$ and derive new comparison results for the spectral radii of the corresponding iteration matrices. These comparison results are useful to analyse the rate of convergence of the iterative methods (formulated from the double splittings) for solving rectangular linear system $A x=b$.


Keywords: double splittings, semi-monotone matrix, spectral radius, Moore-Penrose inverse, group inverse.
AMS subject classifications. 15A09; 65F15

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# Cordial labeling for three star graph ${ }^{21}$ 

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#### Abstract

Cordial labelingis used to label the vertices and edges of a graph with $\{0,1\}$ under constraint, such that the number of vertices with label 0 and 1 differ by atmost 1 and the number of edges with label 1 and 0 differ by atmost 1 . In this paper we prove that the three star graph $K_{1, p} \wedge K_{1, q} \wedge K_{1, r}$ is a cordial graph for all $p \geq 1, q \geq 1$ and $r \geq 1$.


Keywords: cordial graph and star
AMS subject classifications. 05C78.

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# Further result on skolem mean labeling ${ }^{22}$ 

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#### Abstract

In this paper, we prove if $a \leq b<c$, the seven star $K_{1, a} \cup K_{1, a} \cup K_{1, a} \cup K_{1, a} \cup K_{1, a} \cup K_{1, b} \cup K_{1, c}$ is a skolem


 mean graph if $|b-c|<4+5 a$ for $a=2,3,4, \ldots ; b=2,3,4, \ldots$ and $5 a+b-3 \leq c \leq 5 a+b+3$.Keywords: Skolem mean graph and star
AMS subject classifications. $05 C 768$.

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# Bounds for the distance spectral radius of split graphs ${ }^{23}$ 

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#### Abstract

A graph $G$ is a split graph, if its vertex set can be partitioned into an independent set and a clique. It is known that the diameter of a split graph is atmost 3 . We obtain sharp bounds for the distance spectral radius of split graphs. We also find the distance spectral radius of biregular split graphs of diameter 2 and that of biregular split graphs in which the distance between any two vertices in the independent set is 3 .


Keywords: split graphs, distance matrix, distance spectral radius.
AMS subject classifications. 05C50

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# Nordhaus-Gaddum type sharp bounds for graphs of diameter two 

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#### Abstract

The spectral radius of a graph is the largest eigenvalue of its adjacency matrix and its Laplacian spectral radius is the largest eigenvalue of its Laplacian matrix. Here we try to find Nordhaus-Gaddum type bounds for spectral radius of adjacency matrix, Laplacian spectral radius of the graph G. We here by establish sharp bounds for $\lambda(G)+\lambda\left(G^{c}\right), \mu(G)+\mu\left(G^{c}\right), \lambda(G) \cdot \lambda\left(G^{c}\right), \mu(G) \cdot \mu\left(G^{c}\right)$ for star graph and Friendship graphs which possess the following unique properties like (a) It is of diameter -2 , every vertex is connected to the common vertex O . (b) $\mu(G)+\mu\left(G^{c}\right)=2 n-1$ and (c) Its complement is a disjoint union of edge-disconnected components of a connected regular graph and an isolated vertex. In this paper we restrict our discussion to odd values of n , in particular for $n=7,9,11,13, \ldots, 2 k+1$ for $k=3,4,5$.


Keywords: Adjacency matrix, Laplacian matrix, Nordhaus-Gaddum type bounds, star graph, friendship graph, complement of a graph
AMS subject classifications. 05C50; 15A42

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## Posters

# Gaussian prime labeling of some cycle related graphs 

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#### Abstract

A graph $G$ on n vertices is said to have prime labelling if there exists a labelling from the vertices of $G$ to the first n natural numbers such that any two adjacent vertices have relatively prime labels. Gaussian integers are the complex numbers whose real and imaginary parts are both integers. A Gaussian prime labelling on $G$ is a bijection $l: V(G) \rightarrow\left[\gamma_{n}\right]$, the set of the first n Gaussian integers in the spiral ordering such that if $u v \in E(G)$, then $l(u)$ and $l(v)$ are relatively prime. Using the order on the Gaussian integers, we investigate the Gaussian prime labelling of some cycle related graphs and unicyclic graphs.


Keywords: Gaussian Prime labelling, Gaussian integers, unicyclic graphs
AMS subject classifications. 05C78

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# Skolem mean labeling of parallel transformation of a class of trees 

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#### Abstract

A graph $G=(V, E)$ with p vertices and q edges is said to be Skolem mean labeling of a graph for $q \geq p+1$, if there exists a function $f: V(G) \rightarrow\{1,2,3, \ldots, p\}$ such that the induced map $f^{*}: E(G) \rightarrow\{2,3,4, \ldots, p\}$ defined by $$
f^{*}(u v)=\left\{\begin{array}{cc} \frac{f(u)+f(v)}{2} & \text { if } f(u)+f(v) \text { is even } \\ \frac{f(u)+f^{2}(v)+1}{2} & \text { if } f(u)+f(v) \text { is odd } \end{array}\right.
$$

Then the resulting edges get distinct labels from the set $\{2,3, \ldots, p\}$. In this paper we investigate the Skolem Mean Labeling of parallel transformation of a class of trees.


Keywords: Skolem mean labeling, trees
AMS subject classifications. 05C78

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# Finite-direct-injective modules and column finite matrix rings 

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#### Abstract

In this paper we generalize the concept of direct injective (or C2) modules to finite direct injective modules. Some properties of finite direct injective modules with respect to column finite matrix rings are investigated. We show that direct summand of finite direct injective modules inherits the property, while direct sum need not. Some well known classes of rings are characterize in terms of finite direct injective modules. Keywords: C2-module, C3-module, finite-direct-injective module, regular ring AMS subject classifications. 16D50, 16E50


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# Minimum matching dominaitng sets of circular-arc graphs 

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#### Abstract

A graph $G$ is called a circular-arc graph if there is a one-to-one correspondence between $V$ and $A$ such that two vertices in $V$ are adjacent in $G$ if and only if their corresponding arcs in $A$ intersect. A dominating set for a graph $G=(V, E)$ is a subset $D$ of $V$ such that every vertex not in $D$ is adjacent to at least one member of $D$. The theory of domination in graphs introduced by [1] and [3] is an emerging area of research in graph theory today. A matching in $G$ is a subset $M$ of edges of $E$ such that no two edges in $M$ are adjacent. A matching $M$ in $G$ is called a perfect matching if every vertex of $G$ is incident to some edge in $M$. A dominating set $D$ of $G$ is said to be a matching dominating set if the induced subgraph $\langle D\rangle$ admits a perfect matching. The cardinality of the smallest matching dominating set is called matching domination number. In this paper presents an algorithm for finding minimum matching dominating sets in circular arc graphs.


Keywords: circular arc graphs, dominating set, domatic number, matching dominating sets AMS subject classifications. $05 \mathrm{C}, 65 \mathrm{~S}$

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# On category of $R$-modules and duals 

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#### Abstract

In [5] K.S.S.Nambooripad describe categories with subobjects in which every inclusion splits and every morphism has factorization as a category $\mathscr{C}$ with factorization property. A cones in such categories $\mathscr{C}$ is certain map from $v \mathscr{C}$ to $\mathscr{C}$ and a cone $\gamma$ in $\mathscr{C}$ is a proper cone if there is at least one component of $\gamma$ an epimorphism. Here it is shown that the category of $R$-modules where $R$ is any commutative ring -a well known abelian category- is a proper category. Further we discuss the semigroup of cones in this category and the dual category.

\section*{Keywords:}

AMS subject classifications.


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(from left) Prof Sharad S Sane, Prof Murali K. Srinivasan, Prof N. Eagambaram, Prof R. B. Bapat and Prof T. E. S. Raghavan at Malpe Beach

Prof Steve Kirkland chairing a session

(from left) K. P.S. Bhaskara
Rao, Sharad S. Sane and
Seetharama Gowda on discussion during Tea Break

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