Department of Statistics
VI Floor, Library Building
Manipal University
Manipal, Karnataka, INDIA 576 104.

International Conference
on
Linear Algebra and its Applications

December 18–20, 2014

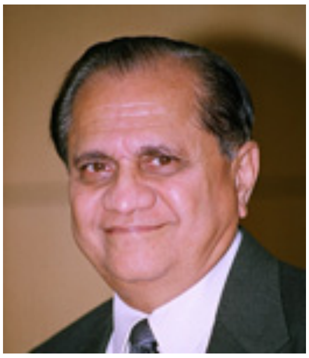
Venue: New Lecture Hall - Academic Block 3, MIT Campus, Manipal University.

MANIPAL
UNIVERSITY

Message

Dr. Ramdas M Pai

PRESIDENT & CHANCELLOR



Among many of their wide-ranging uses, Mathematics & Statistics have a leading role to play in Applied Science such as Engineering, Health, Economic, Management and Administration. I am glad the department of Statistics of our University has organized an International Conference to deliberate at length on the applications of Linear Algebra in different branches of modern science. I am sure, the exchange of thoughts and opinions among the participants will enliven the proceedings. I send my best wishes for the success of the programme.

Dr. H. S. Ballal

PRO CHANCELLOR

Linear Algebra and Matrix Theory have embraced a wide range of scientific advances resulting new innovations. It is good that experts in this field are able to come together at a conference organized by our University Department of Statistics. The discussion and interactions, I have no doubt, will lead to new procedures and formulae to benefit the advancement of the Matrix Theory. My best wishes to all the participants for a successful get together.

Dr. K. Ramnarayan

VICE CHANCELLOR

Mathematics is a puzzle to many yet we all cannot do away with it in our daily life. It is a very important subject and its presence is everywhere from a pin to an aeroplane, from the earth to the space. Indians are known for their knowledge in numbers since ancient days. Our astrological calculations are still very relevant even to this day. Though we have learnt a lot, yet there will be something to learn every day. It is necessary to sharpen ones' knowledge by sharing and discussing it with others now and then. These International Conferences play an important role in bringing together like-minded people to discuss, share and derive new things. I am happy that Department of Statistics is actively supporting the cause of academics in the campus by regularly organizing the conferences such as this. I congratulate the organizers for hosting this International Conference in Manipal and hope that each and every participant will not only benefit from the conference but also from the serene atmosphere of Manipal.

Dr. H. Vinod Bhat

PRO VICE-CHANCELLOR

Department of Statistics, Manipal University is hosting an International Conference on Linear Algebra & its Applications. 'Linear Algebra' being an important branch of Mathematics, embraced a wide range of scientific advances resulting new innovations in different branches of science. This conference, in honor of Prof. Bapat, provide an opportunity to meet and discuss with mathematicians and statisticians of international repute from different parts of globe. I am very glad to know that students and scholars representing different institutions of national repute from different part part of nation will be attending this conference and benefit from the dissemination of knowledge during the conference.
I welcome the delegates of the conference to this knowledge town. I wish everyone the very best.

Dr. G. K. Prabhu

REGISTRAR

I am pleased to note that the Department of Statistics, Manipal University, Manipal, will be organizing International Conference on Linear Algebra & its Applications during 18-19 December, 2014. Linear algebra is vital in multiple areas of science in general. Because linear equations are so easy to solve, practically every area of modern science. Techniques from linear algebra are also used in analytic geometry, engineering, physics, natural sciences, computer science and computer animation.
I am sure, the conference shall provide a platform for leading mathematicians, statisticians, and applied mathematicians working around the globe in the theme area to discuss several research issues on the topic and to introduce new innovations.
My best wishes to all the delegates. I wish the deliberations every success.

Dr. N. Sreekumarn Nair

Professor & Head, Department of Statistics

Dear Participant, we are delighted to have you with us during the international conference on "Linear Algebra & its Applications". During the last eight years of existence, the department of statistics has initiated a Masters programme in Biostatistics and another certificate programme in Epidemiology and Biostatistics. The department faculty regularly conducts workshops and conferences at Regional, National and International level. Department has collaboration with National and International agencies and Industries. You are welcome to the department and please feel free to visit us. We the faculty, staff and students of department of Statistics, Manipal University wish you a very warm welcome, happy stay and fruitful deliberations.

We acknowledge our sincere thanks to

International Linear Algebra Society(http://www.ilasic.math.uregina.ca/iic/) for endorsing this events.

National Board for Higher Mathematics (http://www.nbhm.dae.gov.in) for supporting us by awarding grant-in-aid.

Council of Scientific and Industrial Research (http://www.csir.res.in) for supporting us by awarding grant-in-aid.

SERB, Department of Science and Technology (http://www.serb.gov.in) for supporting us by awarding grant-in-aid.

Science and Engineering Research Board
Established through an Act of Parliament: SERB Act 2008
Department of Science & Technology, Government of India

Indian National Science Academy (http://www.insaindia.org/) for supporting us by awarding grant-in-aid.

Indian National Science Academy

Photograph 1: *Memories from CMTGIM 2012 – Puntanen addressing the participants in the inaugural function*

Preface

Dr. K. Manjunatha Prasad

Department of Statistics, Manipal
Organizing Secretary, ICLAA–2014

On behalf of organizing committee of ICLAA–2014, I welcome all the delegates for the "*International conference on Linear Algebra & its Applications*". ICLAA–2014 is in sequence to the CMTGIM–2012 which was held in January 2012. The theme of conference shall focus on (i) Matrix Methods in Statistics, (ii) Combinatorial Matrix Theory and (iii) Classical Matrix Theory covering different aspects of Linear Algebra. The topic 'Matrix methods in statistics' is a branch of mathematics containing a variety of challenging problems in linear statistical models and statistical inference having applications in various branches of applied statistics such as natural sciences, medicine, economics, electrical engineering, Markov chains, Digital Signal Processing, Pattern Recognition and Neural Network to name a few. Advances in matrices & graphs were motivated by a wide range of subjects such as Networks, Chemistry, Genetics, Bioinformatics, Computer Science, and Information Technology etc. The generalized inverses of matrices such as the incidence matrix and Laplacian matrix are mathematically interesting and have great practical significance.

Conference shall provide a platform for leading mathematicians, statisticians, and applied mathematicians working around the globe in the theme area to discuss several research issues on the topic and to introduce new innovations. The main goal of the conference is to bring experts, researchers, and students together and those to present recent developments in this dynamic and important field. The conference also aims to stimulate research and support the interaction between the scientists by creating an environment for participants to exchange ideas and to initiate collaborations or professional partnerships. More than 150 delegates are expected to present in this ICLAA–2014 and the contingency include about 30 invited speakers and 30 other delegates reading their contributed papers. Three days programme consists of about 10 plenary sessions, 20 invited talks, and several contributed talks in parallel sessions. Poster presentation will be arranged for research scholars to display their research out comes.

The theme of conference is same as the research interest of eminent scientist 'Prof Ravindra B. Bapat' of Indian Statistical Institute, Delhi. The conference will also provide a platform to all his contemporaries to come together to serve the objectives of the conference on the occasion of Prof Bapat's 60th birthday and the conference provides an opportunity to the young scholars around the nation, who will be the asset to the future generation, to join the team of eminent scientists. Manipal University Press will publish 'Linear Algebra with Applications – A volume in honor of Prof Ravindra B Bapat' on this occasion.

I am very glad to share the information that '*International Linear Algebra Society*' (ILAS) has endorsed ICLAA–2014 and '*Electronics Journal of Linear Algebra*'(ELA) will be publishing a special volume for proceedings of ICLAA–2014. All papers will be subject to the usual refereeing procedure for ELA. The deadline for submissions is March 15, 2015. Authors should submit a paper through the ELA portal and clearly indicate that the paper is to be considered for this special volume, or to one of the special editors. Submission guidelines are available at ELA guidelines. Participants are encouraged to submit their articles to special volume. Special editors for the volume are: Rajendra Bhatia (Indian Statistical Institute, Dehli); Steve Kirkland (University of Manitoba, Canada); K. Manjunatha Prasad (Manipal University Manipal); Simo Puntanen (University of Tampere, Finland).

Photograph 2: *Memories from CMTGIM 2012 – Pro-Chancellor with delegates*

Committees for ICLAA 2014

Chief Patrons:

M. Ramdas Pai (Chancellor, Manipal University)
H. S. Ballal (Pro Chancellor, Manipal University

Patrons

K. Ramnarayan (Vice Chancellor, Manipal University)
H. Vinod Bhat (Pro-Vice Chancellor, Manipal University)
G. K. Prabhu (Registrar, Manipal University)

Advisory Committee:

R. Balakrishnan, Bharathidasan University, Tiruchirappalli
N. Eagambaram, Former DDG, CSO, MOSPI
T. E. S. Raghavan, University of Illinois at Chicago, Chicago
Sharad S Sane, Department of Mathematics, IIT Bombay, Mumbai
V. S. Sunder, The Institute of Mathematical Sciences, Chennai
N. Sreekumaran Nair, Professor & Head, Department of Statistics, Manipal University

Scientific Advisory Committee:

Rajendra Bhatia, Indian Statistical Institute, Delhi
Steve Kirkland, University of Manitoba, Canada
K. Manjunatha Prasad, Manipal University- Convener
Simo Puntanen, University of Tampere, Finland

National Organizing Committee:

S. Parameshwara Bhatta, Department of Mathematics, Mangalore University
Sam Johnson, National Institute of Technology, Karnataka
Arbind K Lal, Indian Institute of Technology Kanpur, Kanpur
V. Parameswaran, DDG, NSSO-MOSPI, Pune
Sukanta Pati, Indian Institute of Technology Guwahati, Guwahati
Sivaramakrishnan Sivasubramanian, Indian Institute of Technology Bombay, Mumbai

Local Organizing Committee:

H. Vinod Bhat, Pro-Vice Chancellor, Manipal University - Chairman
N. Sreekumaran Nair, Professor & Head, Department of Statistics, Manipal University
K. Manjunatha Prasad, Department of Statistics, Manipal University- Organizing Secretary
Sreemathi S Mayya, Department of Statistics, Manipal University, Manipal
C. Ramesha, Department of Mathematics, MIT, Manipal
G. Sudhakara, Department of Mathematics, MIT, Manipal
Badri Narayanan, Director, Purchase & General Services, Manipal University, Manipal
Jagadish Nayak, Director, Finance, Manipal University, Manipal
Alexander Chandy, Director, PR & Media Communications, Manipal University, Manipal

Registration & Programme Coordination:

Ashma Monteiro, Department of Statistics, Manipal University - Secretary
Mob.: 9964194513, ashma.monteiro@manipal.edu
Nilima, Department of Statistics
K. V. Soumya, Department of Mathematics, MIT
H. S. Sujatha, Department of Mathematics, MIT
P. Divya Shenoy, Department of Mathematics, MIT

Finance :

C. Ramesha, Department of Mathematics, MIT, Manipal - Secretary
Mobile: 9480402061; email: ramesha.c@manipal.edu
Y. Santhi Sheela, Department of Statistics, Manipal University
K. Arati Bhat, Department of Mathematics, MIT, Manipal
Manjunath Shanbhag, Sr. Finance Executive, Manipal University
Sudhamani N. G., Department of Statistics, Manipal University

Accommodation & Transportation:

G. Sudhakara, Department of Mathematics, MIT, Manipal - Secretary
Mob: 9945670717; sudhakaraadiga@yahoo.co.in
Col. M. G. H. S. Rajan, Chief Warden, KMC Hostel
Col. C. M. S. Kalakoti, Chief Warden, MIT Hostel
Kalesh M Karun, Department of Statistics
Laxman Bangera, Transport Section, MU
N. Babanna, Asst. Director, PR

Souvenir and Book Publication :

Shreemathi S Mayya, Department of Statistics, Manipal University - Secretary
Mob: 9964584055 email: shreemathi.mayya@manipal.edu
Vinay Madhusudhanan, Department of Mathematics, MIT, Manipal
Gracy Samjetsabam, Department of Statistics, Manipal University

Certificates, Media Communication :

V. S. Binu, Department of Statistics - Secretary
Mob.: 9901723031, binuvstvm@gmail.com
Melissa Lewis Glenda, Department of Statistics

Food, Video and Photos:

G. Vasudev, Department of Statistics - Secretary
Mob: 9448823648, email:vasudev.guddattu@manipal.edu
Ravi Shankar, Department of Statistics

International Conference on Linear Algebra and its Applications December 18–20, 2014

List of invited speakers:

1. R. Balakrishnan, *Bharathidasan University, Tiruchirappalli, INDIA.*
2. Abraham Berman, *Technion-Israel Institute of Technology, ISRAEL.*
3. Rajarama Bhat, *Indian Statistical Institute, Bangalore, INDIA.*
4. Rajendra Bhatia, *Indian Statistical Institute, Delhi, INDIA.*
5. Zheng Bing, *School of Mathematics & Statistics, Lanzhou University, PR CHINA.*
6. Arup Bose, *Indian Statistical Institute, Kolkata, INDIA.*
7. Ebrahim Ghorbani, *Institute for Research in Fundamental Sciences, Tehran, IRAN.*
8. Steve Haslett, *Massey University, New Zealand.*
9. Surender Kumar Jain, *Distinguished Emeritus Professor, Ohio University, USA.*
10. Andre Leroy, *Université d'Artois, FRANCE.*
11. Steve Kirkland, *University of Manitoba, CANADA.*
12. Bhaskara Rao Kopparty, *Indiana State University, USA.*
13. S. H. Kulkarni, *Indian Institute of Technology Madras, Chennai, INDIA.*
14. Arbind Kumar Lal, *Indian Institute of Technology Kanpur, INDIA.*
15. S. K. Neogy, *Indian Statistical Institute, Delhi, INDIA.*
16. K. R. Parthasarathy, *Emeritus Distinguished Scientist, ISI, Delhi, INDIA.*
17. Sukanta Pati, *Indian Institute of Technology Guwahati, INDIA.*
18. V. Parameswaran, *DDG, NSSO(MOSPI), Pune, INDIA.*
19. Simo Puntanen, *University of Tampere, FINLAND.*
20. T. E. S. Raghavan, *University of Illinois at Chicago, USA.*
21. Sharad Sane, *Indian Institute of Technology Bombay, INDIA.*
22. Bhaba Kumar Sarma, *Indian Institute of Technology Guwahati, INDIA.*
23. Ajith Iqbal Singh, *INSA Senior Scientist, Delhi, INDIA.*

24. Martin Singull, *Linköping University, SWEDEN.*

25. K. C. Sivakumar, *Indian Institute of Technology Madras, INDIA.*

26. Sivaramakrishnan Sivasubramanian, *Indian Institute of Technology Bombay, INDIA.*

27. Murali K. Srinivasan, *Indian Institute of Technology Bombay, INDIA.*

28. M. S. Srivatsava, *University of Toronto, CANADA.*

29. Wasin So, *San Jose State University, California, USA.*

30. V. S. Sunder, *The Institute of Mathematical Sciences, Chennai, INDIA.*

Photograph 3: *Memories from CMTGIM 2012 – Group photo of conference delegates*

C(O)P matrices and optimization

Abraham Berman

Department of Mathematics
Technion, Israel Institute of Technology
Haifa 32000, Israel.
email: berman@tx.technion.ac.il

Abstract

A real matrix is *completely positive* (CP) if it can be written as $A = BB^T$, where B is an element wise nonnegative matrix. A symmetric real matrix is *co-positive* (COP) if the quadratic form $x^T Ax$ is nonnegative for every nonnegative vector x. The set CP_n of $n \times n$ CP matrices and the set COP_n of $n \times n$ cop matrices are closed convex cones in the space of $n \times n$ symmetric real matrices. The two cones are dual to each other with respect to the inner product $\langle A, B\rangle = \text{trace } AB$. One of the many reasons for the interest in these cones is that many NP-hard optimization problems can be modeled as linear programs over CP_n or over the dual cone COP_n. In the talk I will describe some of the known results on CP matrices and on COP matrices, discuss how the related optimization problems can be relaxed, and mention some open problems.

Keywords

completely positive matrix; co-positive matrix; optimization.

Nilpotent completely positive maps and majorization

B. V. Rajarama Bhat [1] & Nirupam Mallick

Stat-Math Division
Indian Statistical Institute
R.V. College, Post
Bengaluru, 560 059 India.
email: [1]bvrajaramabhat@gmail.com

Abstract

The theory of majorization provides a way of comparing real vectors. This notion appears in a wide variety of fields. Jordan block sizes of nilpotent linear maps obey a bunch of inequalities coming from Littlewood-Richardson rules, including majorization inequalities. In the context of nilpotent completely positive maps, we prove a new type of majorization.

Inertia of Loéwner matrices

Rajendra Bhatia

Indian Statistical Institute, Delhi
7, SJS Sansanwal Marg
New Delhi 110 006
email: rbh@isid.ac.in

Abstract

Given a smooth function f on the positive half-line, the Loéwner kernel associated with f is defined as $\mathcal{L}(x, y) = \frac{(f(x)-f(y))}{(x-y)}$. Several properties of f are studied via this kernel. We consider the special function $f(t) = t^r$ and describe the oscillation of the eigenvalues of matrices associated with this kernel.

Some new perturbation bounds of Moore–Penrose inverse

Zheng Bing [1] **& Ling-Sheng Meng**

School of Mathematics & Statistics
Lanzhou University, PR China
email: [1]*bzheng@lzu.edu.cn*

Abstract

In this talk we summarize some new perturbation bounds of Moore–Penrose inverse under different norms which were recently obtained by Ling-Sheng Meng and Bing Zheng and other researchers. The perturbation analysis involves the classically additive perturbation bounds and multiplicative perturbation bounds. The optimality of some perturbation bounds is also discussed by numerical examples.

Keywords

Moore–Penrose inverse; perturbation bounds.

High dimensional time series models

Arup Bose [1] **& Monika Bhattacharjee**
Statistics and Mathematics Unit
Indian Statistical Institute
203 B.,T. Road
Kolkata 700 108 INDIA.
emails: [1]bosearu@gmail.com; abose@isical.ac.in

Abstract

Consider a high dimensional linear time series model. The corresponding sample autocovariance matrices are crucial objects in any inference procedure. We show that the limiting spectral distribution of symmetrized versions of these matrices exist and also show how to extend this to joint convergence. Ideas from free probability come up naturally in this study and the limits may be described in terms of suitable free independent random elements. Explicit description of the limit is given in some special cases.

Rank and order of graphs

Ebrahim Ghorbani
Department of Mathematics
K. N. Toosi University of Technology
P. O. Box 16315-1618, Tehran, Iran.
School of Mathematics
Institute for Research in Fundamental Sciences (IPM)
P.O. Box 19395-5746, Tehran, Iran
emails: ghorbani1002@yahoo.com; e_ghorbani@ipm.ir

Abstract

The rank of a graph is defined to be the rank of its adjacency matrix. A graph is called reduced if it has no isolated vertices and no two vertices with the same set of neighbors. Akbari, Cameron, and Khosrovshahi conjectured that the order of every reduced graph of rank r is at most $m(r) = 2^{(r+2)/2} - 2$ if r is even and $m(r) = 5 \cdot 2^{(r-3)/2} - 2$ if r is odd. We prove that if the conjecture is not true, then there would be a counterexample of rank at most 47. We also show that the order of every reduced graph of rank r is at most $8m(r) + 14$. This is a joint work with A. Mohammadian and B. Tayfeh-Rezaie.

Keywords

Adjacent matrix; rank of graph; order of graph.

Connections between model and design based estimation for linear models for sample survey data

Stephen Haslett

Professor–Statistics
Institute of Fundamental Sciences
Massey University, PO Box 11-222
Palmerston North New Zealand.
email: s.j.haslett@massey.ac.nz

Abstract

Linear models for unit level survey data can be fitted using a model in which the selection probabilities form part of the design matrix. This method is called "model based". The alternative "design based" method uses the inverse of the unit selection probabilities as weights. The estimated variance for parameter estimates under both scenarios will be considered. Connections between the two methods, and conditions for their equivalence will be discussed. The role of the assumed zero correlation in the underlying population, and a possible extension for design-based estimation to non-zero correlation for population level errors will be explored.

Keywords

Bias; correlated model errors; general linear model; joint selection probabilities; model equivalence; regression; selection probabilities; superpopulation models; survey data.

A survey talk on a decomposition of a singular matrix into a product of similarity classes of a given fixed matrix over any field and geometrizations over Euclidean (equivalently, Hermite) domains

S. K. Jain

King Abdulaziz University, Jeddah, SA and
Ohio University, Athens, OH 45701, USA.
email: jain@ohio.edu

Abstract

J. Erdos (Glasgow Math. J. 8 (1967)) proved that every singular matrix over a field F is a product of idempotent matrices . The number of idempotent matrices required in such decompositions has been studied by several authors, including Ballantine and Hannah-O'Meara. Ballantine proved that a matrix $a \in M_n(F)$ is a product of m idempotent matrices if and only if $rank(1-a) \leq m(nullity(a))$ (Linear Algebra Appl.19 (1978)).

Recall a ring R is called von Neumann regular if for each element $a \in R$ there exists an element $x \in R$ such that $axa = a$. A von Neumann regular ring is called unit regular if for each element $a \in R$ there exists a unit $u \in R$ such that $a = aua$. O'Meara and Hannah proved that the number of elements in the factorization of an element of a regular ring is bounded by the index of nilpotency of the ring (Proc. London Math. Soc. (3) 59 (1989)).

Using ring theory methods, for von Neumann regular right self-injective ring R or for unit regular ring R, Hannah-O'Meara (J. Algebra 123 (1989)) gave an ideal-theoretic formulation. They showed that for a unit regular ring an element $x \in R$ is a product of idempotents if and only if x satisfies the condition (*) $\quad R(r.ann_R(x)) = (l.ann_R(x))R = R(1-x)R$, where $r.ann(x)$ stands for the right annihilator (= killer) of the element x. Similar meaning of $l.ann_R(x)$.

We note that in any regular ring R (not necessarily unit regular), all products of idempotents satisfy the condition (*). But we know no example of a von Neumann regular ring in which an element satisfying (*) fails to be a product of idempotents.

A somewhat more general problem is to ask whether a singular matrix can be represented as a product of similarity classes of one fixed matrix instead of different idempotents. It has been shown by Serguis Pazzis (Communications in Algebra (2012)) that any singular matrix is a product of conjugates of a fixed matrix over any field. In a private discussion with Efim Zelmanov, Zelmanov proved independently the same result over any algebraic closed field but his method works over any field. We will also talk, among others, that this decomposition into idempotents is also true for matrices over Bezout domains with stable range 1 and for right quasi-Euclidean domains as shown in (Alahmadi-Jain-Leroy, LAMA, (2013)) and ((Alahmadi-Jain-Lam-Leroy, J. Algebra (2014)), extending known results for commutative Euclidean domains of Laffey (LAMA, 14(4), (1983)). We will talk, if time permits, relationships between quasi-Euclidean rings, regular rings, exchange rings, regular separative rings with rings having the property that each singular element is a product of idempotents modulo units being studied by Jain-Leroy and also some interesting results of O'Meara that have just appeared (J. Algebra and its Appl., November 2014). The results obtained demonstrate the abundance of the class of quasi-Euclidean rings. What is a right quasi-Euclidean ring?

An ordered pair (a, b) with $b \neq 0$ is called a Euclidean pair, if there exits q_i and r_i such that $a = bq_1 + r_1, b = r_1q_2 + r_2, \ldots, r_{i-1} = r_iq_{i+1} + r_{i+1}$ with $i = 1, \ldots, n$ and $r_{n+2} = 0$. If each pair is a Euclidean pair then the ring is called quasi-Euclidean ring. Examples exist of commutative quasi-Euclidean domains that are not Euclidean domains.

What is right Bezout domain (equivalently right Hermite domain of Kaplansky)?

An integral domain (not necessarily commutative) is a right Bezout domain if every finitely generated right ideal is principal.

Question (related to my work on nonnegative matrices with Bapat). Whether a nonnegative matrix can be decomposed into a product of nonnegative idempotent matrices?

Photograph 4: *Memories from CMTGIM 2012 – Prof Jain delivering his talk*

Sensitivity analysis for perfect state transfer in quantum spin networks

Steve Kirkland

Department of Mathematics
University of Manitoba, Canada
email: stephen.kirkland@umanitoba.ca

Abstract

In 2003, Bose proposed the idea of using a network of interacting spins as a quantum wire to transfer information within a quantum computer. Associated with the network is a matrix M whose rows and columns are indexed by the spins, and whose entries represent the strengths of the couplings between the various spins in the network. Once all of the dust settles on the physics side, a key quantity to consider is the so–called *fidelity of transfer*: setting $U(t) = e^{itM}$, and selecting distinct indices s and r, the fidelity of transfer from s to r at time t is given by $p(t) = |u_{s,r}(t)|^2$. It is straightforward to show that $0 \leq p(t) \leq 1$, and if it happens that $p(t_0) = 1$ for some time t_0, then we say that there is *perfect state transfer* (PST) from s to r at time t_0. In Bose's setting, perfect state transfer corresponds to a flawless transfer of information; consequently there has been a great deal of interest in identifying spin networks that exhibit PST.

As one might expect, a number of things – for instance the readout time t_0, the pairs of spins that interact, and the coupling strengths – have to line up just right in order for PST to hold. In view of that observation, it is natural to wonder how sensitive the fidelity of transfer is to small changes in either the readout time or the coupling strengths, and in this talk, we address both of those questions. Using techniques from matrix analysis, we derive formulas for the derivatives of the fidelity of transfer with respect to the readout time and with respect to the coupling strengths.

The results may help to inform the design of spin networks that not only exhibit perfect state transfer but also offer some forgiveness to errors in readout time and/or spin interactions. This talk may also answer another question: can a humble matrix theorist with an infinitesimal knowledge of physics still contribute something to the analysis of PST?

Photograph 5: *We missed you, Steve. Talk over the SKYPE can't be a compensation!*

The null space theorem

S. H. Kulkarni

Department of Mathematics,
Indian Institute of Technology Madras,
Chennai 600036, INDIA.
email: shk@iitm.ac.in

Abstract

The following results are proved.
(The Null Space Theorem) Let X, Y be vector spaces, $P \in L(X), Q \in L(Y)$ be projections and $T \in L(X, Y)$ be invertible. (The restriction of QTP to $R(P)$ can be viewed as a linear operator from $R(P)$ to $R(Q)$. This is called a *section of* T *by* P *and* Q and will be denoted by $T_{P,Q}$.) Then there is a linear bijection between the null space of the section $T_{P,Q}$ of T and the null space of its complementary section $T^{-1}_{I_Y - Q, I_X - P}$ of T^{-1}.
(The Rank Theorem) Let X be a Banach space with a Schauder basis $A = \{a_1, a_2, \ldots\}$. Let T be a bounded(continuous) linear operator on X. Suppose the matrix of T with respect to A is tridiagonal. If T is invertible, then every submatrix of the matrix of T^{-1} with respect to A that lies on or above the main diagonal (or on or below the main diagonal) is of rank ≤ 1.

Keywords

Nullity Theorem; Null Space Theorem; Tridiagonal operator; rank.

References

[1] R. B. Bapat, On generalized inverses of banded matrices, Electron. J. Linear Algebra **16** (2007), 284–290. MR2349923 (2008k:15004)

[2] W. W. Barrett, A theorem on inverses of tridiagonal matrices, Linear Algebra Appl. **27** (1979), 211–217. MR0545734 (80i:15003)

[3] Ben-Israel, Adi and Greville, Thomas N. E., *Generalized inverses*, (3rd edition) Springer-Verlag, New York, 2003.

[4] B. Bollobás, *Linear analysis*, Cambridge Univ. Press, Cambridge, 1990. MR1087297 (92a:46001).

[5] B. V. Limaye, *Functional analysis*, Second edition, New Age, New Delhi, 1996. MR1427262 (97k:46001)

[6] G. Strang, *Linear algebra and its applications*, second edition, Academic Press, New York, 1980. MR0575349 (81f:15003)

[7] G. Strang and T. Nguyen, The interplay of ranks of submatrices, SIAM Rev. **46** (2004), no. 4, 637–646 (electronic). MR2124679 (2005m:15015)

Combinatorial heat and wave equations on certain classes of infinite Cayley and coset graphs

Arbind Kumar Lal & **S. Mohanty**

Department of Mathematics and Statistics
Indian Institute of Technology Kanpur
Kanpur, India - 208 016.
emails: [1]*arlal@iitk.ac.in;* [2]*sumitmth@iitk.ac.in*

Abstract

The combinatorial heat and wave equations on all finite Cayley and coset graphs was solved by Lal *et al.* In this paper, the results of the above paper are extended for infinite Cayley and coset graphs, whenever the associated groups are discrete, abelian and finitely generated. Furthermore, we study the solution of the combinatorial heat and wave equations on a k-regular tree whose associated group is a non-abelian free group on k generators, each of order 2.

Continuant polynomials

A. Facchini [1], & **Andre Leroy** [2],

[1]*Padova University, Italy.*
[2]*Université d'Artois*
Facult Jean Perrin
23 rue J. Souvraz
62300 Lens, France.
email: andreleroy55@gmail.com

Abstract

Continuant polynomials appear as numerators and denominators of the convergent of a continued fractions. P.M. Cohn introduced their noncommutative analogue in order to study the group of invertible matrices $GL_2(R)$ of an arbitrary ring R. Some of these results will be mentioned and extended. Separating the set of continuant polynomials into two families will give interesting graphs emphasizing the leapfrog structure of these polynomials. Relations with Fibonacci sequences and polynomials as well as with tiling will be shown. Generalizations of the continuant polynomials will then appear naturally.

Keywords

Invertible matrices; Continuant polynomials; Fibonacci polynomials; tilings; Permanents.

References

[1] P.M. Cohn (1966). On the structure of the GL2 of a ring, *IHES Publ. Math. 30* 553.

[2] A. Facchni, A. Leroy, Five variations in the theme of leapfrog constructions. *to appear.*

On solving a quadratic programming problem involving Resistance distances in a graph in Polynomial time

S. K. Neogy
Indian Statistical Institute
7, SJS Sansanwal Marg
New Delhi 110 016. INDIA.
email: skn@isid.ac.in

Abstract

Quadratic programming problem involving distances in trees are considered in the literature by Bapat and Neogy[1] and Dankelmann [2]. Bapat and Neogy[1] have shown that indeed for trees this problem can be solved in polynomial time. This solves an open problem posed by Dankelmann [2]. In this talk, we show that this is achieved by converting the problem into a quadratic programming problem with a positive definite matrix.

Then we consider a nonconvex quadratic programming problem involving resistance distance of a connected graph with n vertices. We reformulate this as a strictly convex quadratic programming problem and discuss solving the quadratic programming problem involving resistance distance using polynomial time algorithm. An application to symmetric bimatrix game is also presented.

Keywords

Tree; distance matrix; resistance distance; Laplacian matrix; quadratic programming; polynomial time algorithm; symmetric bimatrix game and Nash equilibrium.

References

[1] R. B. Bapat and S. K. Neogy (2013). On a Quadratic programming problem involving distances in trees. *Submitted for publication in Annals of OR.*

[2] P. Dankelmann (2012). Average distance in weighted graphs. *Discrete Mathematics 312* 12–20.

Symplectic dilations of real matrices of even order, Gaussian states and Gaussian channels

K. R. Parthasarathy

Indian Statistical Institute
7, SJS Sansanwal Marg
New Delhi 110016. INDIA.
e.mail: krp@isid.ac.in

Abstract

By elementary matrix algebra we shall demonstrate how every real $2n \times 2n$ matrix admits a dilation to an element of the symplectic group $\mathrm{Sp}(2(n+m),\ R)$ for some nonnegative integer m. We shall investigate the implications of this result in the study of Gaussian states and channels of quantum information theory.

Keywords

Symplectic matrix and dilation; Gaussian state.

References

[1] K. R. Parthasarathy (2010). What is a Gaussian state? *Commun.Stoch.Anal. 4* 143–160

On the inverse of graphs and the reciprocal eigenvalue properties

Swarup Kumar Panda [1] **&** **<u>Sukanta Pati</u>** [2]

Department of Mathematics
Indian Institute of Technology Guwahati
Guwahati 781 039, India.
[1] *emails: p.swarup@iitg.ernet.in*
[2] *email: pati@iitg.ernet.in*

Abstract

A graph G is nonsingular if its adjacency matrix $A(G)$ is nonsingular. A nonsingular graph G is said to have an inverse if $A(G)^{-1}$ is signature similar to a nonnegative matrix. A class of connected bipartite graphs with a unique perfect matching possessing inverses were given by C. D. Godsil[5]. We supply a larger class of graphs possessing inverses. A nonsingular graph G is said to have the property (R) if for each eigenvalue λ of $A(G)$, the number $1/\lambda$ is also an eigenvalue. If further, the multiplicity of λ and $1/\lambda$ as eigenvalues of $A(G)$ are the same, then G is said to have the property (SR). It is known that for trees, property (R) is equivalent to property (SR), that is, the class of trees with property (R) is the same as that of tree with property (SR). We characterize a larger class graphs than the class of nonsingular trees on which these two properties are equivalent. Furthermore, we supply a class of non-bipartite graphs on which these two properties are not equivalent.

Keywords

Adjacency matrix; Graph inverse; Property (R); Property (SR).

References

[1] S. Akbari and S.J. Kirkland (2007). Unimodular graphs, *Linear Algebra and its Applications 421* 3–15.

[2] R. B. Bapat (2010). *Graphs and Matrices*, TRIM Series-58, Hindustan Book Agency, New Delhi.

[3] S. Barik, M. Nath, S. Pati, B. K. Sarma (2008). Unicyclic graphs with the strong reciprocal eigenvalue property, *Electronic Journal of Linear Algebra 17*, 139–153.

[4] S. Barik, M. Neumann and S. Pati (2006). On nonsingular trees and reciprocal eigenvalue property, *Linear and Multilinear Algebra 54*, 453–465.

[5] S. Barik, S. Pati and B. K. Sarma (2007). The spectrum of the corona of two graphs. *SIAM Journal of Discrete Mathematics 21*, pp. 47–56.

[6] Abraham Berman, Robert J. Plemmons (1994). *Nonnegative Matrices in the Mthematical Sciences*, SIAM.

[7] N. Biggs (1994). *Algebraic graph theory*, Cambridge University Press.

[8] Richard A. Brualdi and Herbert J. Ryser (1991). *Combinatorial Matrix Theory*, Cambridge University Press.

[9] Dragos M. Cvetkovic, Ivan Gutman and Slobodan K. Simic (1978). On self pseudo-inverse graphs, *Univ. Beogard. Publ. Elektrotehn. Fak. 602–633*, 111-117.

[10] D. M. Cvetkovic, M. Doob and H. Sachs (1980). *Spectra of Graphs*, Academic Press, NewYork.

[11] R. Frucht and F. Harary (1970). On the corona of two graphs, *Aequationes Mathematicae 4*, 322–325.

[12] C. D. Godsil and B. D. Mckay (1978). A new graph product and its spectrum, *Bull. Austral. Math. Soc. 18*, 21–28.

[13] C. D. Godsil (1985). Inverses of trees, *Combinatorica 5(1)*, 33–39.

[14] C. Godsil and G. Royal (2001). Algebraic Graph Theory, *Springer-Verlag, New York.*

[15] Roger A. Horn and Charles R. Johnson (1985). Matrix Analysis, *Cambridge University Press.*

[16] D. Kalita and S. Pati (2014). A reciprocal eigenvalue property for unicyclic weighted directed graphs with weights from $\{\pm 1, \pm \mathrm{i}\}$, *Linear Algebra and its Applications 449*, 417–434.

[17] M. Neumann and S. Pati (2013). On reciprocal eigenvalue property of weighted trees, *Linear Algebra and its Applications 438*, 3817–3828.

[18] R. M. Tifenbach, S. J. Kirkland (2009). Directed intervals and dual of a graph, *Linear Algebra and its Applications 431*, 792–807.

[19] R. M. Tifenbach (2011). Strongly self-dual graphs, *Linear Algebra and its Applications 435*, 3151–3167.

Remarks on the connection between the fixed and mixed linear models

Simo Puntanen

University of Tampere
Tampere, Finland.
email: simo.puntanen@uta.fi

Abstract

There is a strong connection between the mixed linear model and the linear model with the fixed effects as has been recently shown by Haslett & Puntanen (2010) and Haslett, Puntanen & Arendacká (2014). It this talk we review further properties of this connection which opens a new viewpoint the relations between the BLUEs and BLUPs in the two models.

Keywords
Best linear unbiased estimator; Best linear unbiased predictor; Linear mixed model; Linear fixed effect model.

References

[1] S. J. Haslett and S. Puntanen (2010). Equality of BLUEs or BLUPs under two linear models using stochastic restrictions. *Statistical Papers, 51*, 465–475.

[2] S. J. Haslett, S. Puntanen and B. Arendacká (2014). The link between the mixed and fixed linear models revisited. *Statistical Papers*, available online.

Correlated equilibria via minimax theorem

T. E. S. Raghavan

Professor Emeritus
Department of Mathematics, Statistics and Computer Science
851, South Morgan Street # 608
University of Illinois at Chicago
Chicago, IL 60607-7045.
emails: terctu@gmail.com, ter@uic.edu

Abstract

A nonzero sum two person game is played as follows: Player I and II choose secretly a row index $i = 1, \ldots, m$ and column index $j = 1, \ldots n$. the players I and II receive respectively an amount a_{ij}, and b_{ij} as payoff. The matrices $A = (a_{ij})$, $B = (b_{ij})$ are common knowledge to the players. A pair of distributions $x^* = (x_1^*, \ldots x_m^*)$ and $y^* = (y_1^*, \ldots, y_n^*)$ on the row and column indices constitute a Nash equilibium if and only if for any arbitray probability distribution $x = (x_1, \ldots, x_m)$ for players I, and $y = (y_1, \ldots, y_n)$ for player II on their respective indices

$$(x^*,\ Ay^*) \geq (x,\ Ay^*) \quad \text{and} \quad (x^*,\ By^*) \geq (X^*,\ By).$$

Intuitively it says that the expected payoff to player I cannot be strictly improved by unilaterally deviating from x^* to any other x when player II does not deviate from y^* and similarly the expected payoff to player II cannot be strictly improved by unilaterally deviating from y^* to any other y when player I does not deviate from x^*. Nashs seminal contribution to game theory is to prove the existence of such a pair of probability distributions $(x^*;\ y^*)$ for any arbitrary pair of payoff matrices $A = (a_{ij})$ and $B = (b_{ij})$. The proof is topological and relies on the Brouwer's fixed point theorem. The pair $(x^*,\ y^*)$ is called a *Nash equilibrium.* Suppose a referee acts for the two players by choosing a row and column index pair $(i,\ j)$ with probability $p = (p_{ij})$, $i = 1, \ldots, m$; $j = 1, \ldots n$. While this p may be common knowledge to both players, the referee uses a roulette wheel to select a pair $(i,\ j)$ according to p but reveals only the i component of his choice to player I keeping the j component selected as secret and reveals the j component to player II keeping the i component secret and now suggests the players to choose i and j respectively. The players have complete freedom to deviate from the referees advise. Each player can evaluate his conditional expected payoff given the advise by taking the referees advise. If each player finds no strict improvement in their conditional expected gain, in deviating from the referees advise when they assume, the opponent will stick to the referees advise, then the p is said to constitute a correlated equilibrium. In fact a correlated equilibrium $p = (p_{ij})$ is any matrix with nonnegative entries with $\sum_i \sum_j p_{ij} = 1$ satisfying

$$\sum_j (a_{ij} - a_{kj}) p_{ij} \geq 0 \quad \text{for all} \quad i, k = 1, \ldots, m$$

and

$$\sum_i (b_{ij} - s_{is}) p_{ij} \geq 0 \quad \text{for all} \quad j, s = 1, \ldots, m.$$

Aumann introduced the notion of a correlated equilibrium and observed that any Nash equilibrium $(x^*,\ y^*)$ is indeed a correlated equilibrium by taking $p = (p_{ij})$ where $p_{ij} = x_i^* y_j^*$. Thus the problem of existence of a correlated equilibrium is easy once we know the existence of a Nash equilibrium. However it is possible to give an alternative proof of the existence of correlated equilibria by

minimax theorem for matrix games. The proof technique is closely related to Kaplanskis theorem on completely mixed games. The open problem is to prove the existence of Nash equilibria for bimatrix games via the existence of correlated equilibria without resorting to fixed point arguments.

Linear Algebra in the study of combinatorial configurations

Sharad Sane

Department of Mathematics
Indian Institute of Technology Bombay
Powai, Mumbai 400076.
email:sharadsane@gmail.com

Abstract

In this talk, I will mainly concentrate on results on combinatorial configurations that are derived using linear algebraic tools and sometimes those results for which only tools available are linear algebraic. Beginning with Fisher inequality, we study strongly regular graphs, generalized quadrangles and quasi-symmetric designs. We also give an idea of Krein conditions and study of equiangular lines in the the real space.

Keywords

regular graphs; generalized quadrangles; and quasi-symmetric designs; Krein condition.

The completion problems for some classes of Q-matrices

Bhaba Kumar Sarma [1] **& Kalyan Sinha** [2]

Department of Mathematics
Indian Institute of Technology Guwahati
Guwahati 781 039, India
emails: [1]*bks@iitg.ac.in;* [2]*sinha.k@iitg.ac.in.*

Abstract

A *partial matrix* is a square array in which some entries are specified, while others are free to be chosen. A partial matrix M is said to *specify* a digraph D, possibly with loops, if the positions of the specified entries in M correspond to the arcs in D.

A real $n \times n$ matrix A is a P-*matrix* (P_0-*matrix*) if each of the principal minors of A is positive (nonnegative), and is a Q-*matrix* if for every $k = 1, 2, \ldots, n$, the sum $S_k(A)$ of all the $k \times k$ principal minors of A is positive. For a class Π of matrices (e.g., P-, P_0- or Q-matrices) a partial Π-*matrix*

is one whose specified entries satisfy the required properties of a Π-matrix. For example, a partial P-matrix has all fully specified minors positive, and for a partial Q-matrix M, $S_k(M) > 0$ for each k for which all $k \times k$ principal submatrices are fully specified.

A Π-*completion* of a partial Π-matrix is a Π-matrix obtained by some choices of the unspecified entries. In many cases, the positions of the specified entries play a significant role for existence of completions of matrices of a given class. A digraph D is said to have Π-*completion*, if every partial Π-matrix specifying D can be completed to a Π-matrix. The (*combinatorial*) Π-*matrix completion problem* attempts to study the digraphs having Π-completions. For an exposition in matrix completion problems, see the survey article [2]. The study of the Q-matrix completion problem was initiated in [1].

We will present our recent work on the nonnegative and the positive Q-matrix completion problems. For these completion problems, necessary conditions and some sufficient conditions for a digraph to have completion will be discussed, and results on classifications of digraphs of order at most four will be presented.

Keywords

Partial matrix; Matrix completion; Q-matrix; P_0^+-completion; Digraph.

References

[1] L.M. Dealba, L. Hogben, and B.K. Sarma (2009). The Q-matrix completion problem, *Electronic Journal of Linear Algebra, 18*, 176–191.

[2] L. Hogben (2000). Graph theoretic methods for matrix completion problems *Linear Algebra and its Applications, 319*, 83–102.

The passage from two to three and three to infinity in classical to quantum channels

Ajit Iqbal Singh

INSA Honorary Scientist,
Delhi, India. email: aisingh@isid.ac.in

Abstract

The situation changes drastically for matrices, maps on matrix algebras and applications to Quantum Information theory when we go from order two to three or three to infinity. Examples include maximally entangled bases and Quantum Bïrkhoff Theory.

Keywords

Maximally entangled; Quantum channels; Asymptotic Quantum Bïrkhoff Theory.

.

More on explicit estimators of covariance matrices with linear structure

Martin Singull, & F. Berntsson

Linköping University, Sweden

Abstract

In this talk we will discuss different linear covariance structures, e.g., banded, intraclass, Toeplitz and circular Toeplitz, and how to estimate them by minimizing different norms.

One way to estimate the parameters in a linear covariance structure is to use tapering or generalized tapering, which has been shown to be the solution to a universal least squares problem. We know that tapering not always guarantee the positive definite constraints on the estimated covariance matrix and may not be a suitable method. We propose some new methods and try to understand when the universal least squares approach preserves the positive definiteness.

Using this knowledge we will consider the problem of estimating parameters of a multivariate normal p-dimensional random vector for (i) a banded covariance structure reflecting m-dependence. (ii) a banded toeplitz covariance structure, and (iii) a circular toeplitz covariance structure. The estimation procedure for the banded covariance matrix is based on the Cholesky decomposition, while for the circular Toeplitz covariance structure we will use the Fourier matrix for circulant structures.

Keywords

Linear covariance structure; Tapering; Toeplitz Matrix; Cholesky decomposition..

References

[1] M. Ohlson, and D. von Rosen (2010). Explicit estimators in the Growth Curve model with a patterned covariance matrix. *Journal of Multivariate Analysis* 101:1284–1295

[2] M. Ohlson, Z. Andrushchenko, and D. von Rosen (2011). Explicit estimators under m-dependence for a multivariate normal distribution. *Annls of the Institute of Statistical Mathematics,* 63:29–42

Photograph 6: *Memories from CMTGIM 2012 – Baksalary with Trenkler*

On certain positivity classes of operators

M. Rajesh Kannan [1] & **K. C. Sivakumar** [2]
[1] *Department of Mathematics*
Technion, Israel Institute of Technology
Haifa 32000, Israel.
email: rajeshkannan1.m@gmail.com
[2] *Department of Mathematics,*
Indian Institute of Technology Madras
Chennai 600 036, India.
email: kcskumar@iitm.ac.in

Abstract

A real square matrix A is called a P-matrix if all its principal minors are positive. Such a matrix can be characterized by the sign non-reversal property. Taking cue from this, the notion of a P-operator is extended to infinite dimensional spaces, as the first objective. Relationships between invertibility of some subsets of intervals of operators and certain P-operators are then established. These generalize the corresponding results in the matrix case. The inheritance of the property of a P-operator by the Schur complement and the principal pivot transform is also proved. If A is an invertible M-matrix, then there is a positive vector whose image under A is also positive. As the second goal, this and another result on intervals of M-matrices are generalized to operators over Banach spaces. Towards the third objective, the concept of a Q-operator is proposed, generalizing the well known Q-matrix property. An important result, which establishes connections between Q-operators and invertible M-operators, is proved for Hilbert space operators.

Keywords

Interval of operators; M-operators; P-operators; Q-operators.

Photograph 7: *Look, who are here? You may not get them in a team again! Bapat with Team–Tomar*

Squared distance matrix of a tree: Inverse and inertia

R. B. Bapat [1] & **Sivaramakrishnan Sivasubramniam** [2]
[1] *Indian Statistical Institute, Delhi*
7, SJS Sansanwal Marg
New Delhi 110 016
email:rbb@isid.ac.in
[2] *Department of Mathematics*
Indian Institute of Technology Bombay
Powai, Mumbai, India.
email:krishnan@math.iitb.ac.in

Abstract

Let T be a tree with vertices $V(T) = 1, ..., n$. The distance between vertices $i, j \in V(T)$, denoted $d_{i,j}$, is defined to be the length (the number of edges) of the path from i to j. We set $d_{i,i} = 0, i = 1, \ldots, n$. The squared distance matrix Δ of T is the $n \times n$ matrix with (i, j)-element equal to 0 if i = j, and $d_{i,j}^2$ if $i \neq j$. It is known that Δ is nonsingular if and only if the tree has at most one vertex of degree 2. We obtain a formula for Δ^{-1}, if it exists. When the tree has no vertex of degree 2, the formula is particularly simple and depends on a certain "two-step" Laplacian of the tree. We also determine the inertia of Δ.

A combinatorial determinant

Murali Srinivasan
Department of Mathematics
Indian Institute of Technology Bombay
Powai, Mumbai, India.
email: murali.k.srinivasan@gmail.com

Abstract

Don Knuth observed that Delsarte's determination of the eigenvalues of the Johnson and Grassmann schemes can be reformulated as explicit factorization into linear terms of the determinant of a generic matrix in the Bose–Mesner algebras of these schemes. We present a nonabelian analog of this determinant in much the same way as the Frobenius determinant is the nonabelian analog of the Dedekind determinant.

Graph energy change due to a single edge deletion

Wasin So
Department of Mathematics & Statistics
San Jose State University
San Jose, CA 95192, USA
email: wasin.so@sjsu.edu

Abstract

The energy $\mathcal{E}(G)$ of a graph G is defined as the sum of the absolute values of the eigenvalues of its adjacency matrix. This concept is transplanted from chemistry to mathematics. The study of graph energy flourishes recently because it brings together many areas of mathematics. An interesting problem is to study how graph energy changes when a single edge is deleted. The goal is to characterize graphs G and their edges e such that $\mathcal{E}(G-e) \leq \mathcal{E}(G)$. The full characterization is still not known. In this talk, we summarize some existing results, and propose a few related conjectures.

Keywords

graph energy; edge deletion; eigenvalue inequalities.

References

[1] J. Day, and W. So, Graph energy change due to edge deletion, *Linear Algebra Appl.* 428 (2008) 2070–2078.

[2] I. Gutman, The energy of a graph, *Ber. Math-Stat. Sekt. Forschungszent. Graz* 103 (1978) 1–22.

[3] X. Li, Y. Shi, and I. Gutman, *Graph Energy.* New York: Springer, 2012.

Photograph 8: *Memories from CMTGIM 2012 – Team CMTGIM!!*

Tests for covariance matrices in high dimension with less sample size

M. S. Srivastava [1], **H. Yanagihara** [2] & **T. Kubokawa** [3]
[1] *Professor Emeritus*
Department of Statistics
University of Toronto, Canada
email: srivasta@utstat.toronto.edu
[2] *Hiroshima University, Japan*
[3] *University of Tokyo, Japan*

Abstract

In this talk, we propose tests for covariance matrices of high dimension with fewer observations than the dimension for a general class of distributions with positive definite covariance matrices. In one-sample case, tests are proposed for sphericity and for testing the hypothesis that the covariance matrix Σ is an identity matrix, by providing an unbiased estimator of $\mathrm{tr}[\Sigma^2]$ under the general model which requires no more computing time than the one available in the literature for normal model. In the two-sample case, tests for the equality of two covariance matrices are given. The asymptotic distributions of proposed tests in one-sample case are derived under the assumption that the sample size $N = O(p^\delta)$, $1/2 < \delta < 1$, where p is the dimension of the random vector.

Keywords

Asymptotic distributions, covariance matrix, high dimension, non-normal model, sample size smaller than dimension, test statistics.

On Uhlmann's theorem

V. S. Sunder
The Institute of Mathematical Sciences
Chennai, INDIA
email: sunder@imsc.res.in

Abstract

In this brief talk, I shall start with some basic facts concerning majorisation and show how a theorem by Bapat and me leads to an immediate proof of the non-trivial implication in one of the theorems called Uhlmann's theorem in the quantum information science literature.

Keywords

majorisation; Uhlmann's theorem.

International Conference on Linear Algebra and its Applications January 18–20, 2014

List of delegates reading contributed papers:

1. Fouzul Atik, *Distance spectral radius of* 2*-partite distance regular graphs*, Research Scholar, Department of Mathematics, Indian Institute of Technology Kharagpur, Kharagpur, 721302. India.

2. Sasmita Barik, *Algebraic Connectivity and the Characteristic Set of Product Graphs*, School of Basic Science, IIT Bhubaneswar, Bhubaneswar INDIA.

3. Arathi Bhat, *Some matrix equations of graphs*, Manipal Institute of Technology, Manipal INDIA

4. M. B. Bindu, *Some Characterization on Star Partial Ordering Matrices*, Asst. Professor, Department of Mathematics, KCG College of Technology, Karapakkam, Chennai- 600097.

5. Anjan Kumar Bhuniya, *On inner product spaces over idempotent semirings*, Asst. Professor, Viswa-Bharati, Santiniketan, West Bengal, India.

6. Sriparna Chattopadhyay, *On Adjacency and Laplacian Spectrum of Power Graphs of Some Finite Groups*, Research Scholar, Department of Mathematics, Indian Institute of Technology Kharagpur, Kharagpur, 721302. India.

7. Arpita Das, *The normalized Laplacian spectrum of corona, edge corona and neighborhood corona of two regular graphs*, Research Scholar, Department of Mathematics, Indian Institute of Technology Kharagpur, Kharagpur, 721302. India.

8. Rajaiah Dasari, *Loss Behavior of an Internet Router Employing Partial Buffer Sharing Mechanism under Self-Similar Input Traffic-Fractal Point Process*, Department of Mathematics Kakatiya University Warangal, T.S., 506009, India.

9. B. Elavarasan, *Poset properties with respect to semi - ideal - based zero-divisor graph*, Department of Mathematics, School of Science and Humanities, Karunya University, Coimbatore - 641 114, Tamilnadu, India.

10. Yogeshri Gaidhani, *Linear Code using Incidence Matrix of Semigraph*, Asst. Professor, Department of Mathematics, M.E.S. Abasaheb Garware College, Karve Road, Pune, India. 411 004.

11. Mahendra Kumar Gupta, *PD observer design for linear descriptor systems with unknown inputs*, Research Scholar, 111, Science block, IIT Patna Patliputra colony, Patna.

12. K. Hemavathi, *Homomorphism and anti-homomorphism of reverse derivation on prime rings*, Department of Mathematics, S.V. University, Tirupati -517 502.

13. Jadav Ganesh, *On the structure of absolutely minimum attaining operators*, Research Scholar, Dept of Mathematics, Indian Institute of Technology Hyderabad, ODF Estate, Yeddumailaram, Medak-502205.

14. Sachindranath Jayaraman, *On Semipositivity of Matrices*, Asst. Professor, School of Mathematics, IISER Thiruvananthapuram, CET Campus, Engineering College P.O., Thiruvananthapuram - 695016, Kerala, India.

15. Debajit Kalita, *First eigenvectors of nonsingular unicyclic 3-colored digraphs*, Department of Mathematical Science, Tezpur University, Napam, Sonitpur, 784 028.

16. T. Kurmayya, *Nonnegative Moore–Penrose inverses of unbounded gram operators*, Department of Mathematics NIT Warangal, Warangal-506004.

17. Debasisha Mishra, *Some more comparison results of proper nonnegative splittings*, Assistant Professor Dept. of Mathematics NIT Raipur, GE Road Raipur-492 010, CG.

18. K. C. Nandeesh, *Eigenvalues of a digraphs*, Research Scholar, Department of Mathematics, Karnatak University, Dharwad, 580 003.

19. Swarup Kumar Panda, *On the inverse of a graph on Godsil class*, Research Scholar, Indian Institute of Technology Guwahati, Guwahati, India.

20. R. Umamaheshwar Rao, *Applications of Linear Systems in Science and Engineering*, Asso. Professor, Department of Mathematics, Sreenidhi Institute of Science & Technology, Hyderabad, Telangana, India.

21. K. Appi Reddy, *A new characterization of nonnegativity of Moore–Penrose inverses of Gram Matrices in an Indefinite Inner Product Space*, Research Scholar, Department of Mathematics NIT Warangal, Warangal-506004.

22. C. Jaya Subba Reddy, *Left multiplicative generalized derivations acting as homomorphism or anti- homomorphism in prime rings*, Department of Mathematics, S.V. University, Tirupati -517 502.

23. Manideepa Saha, *Characterization of M_v matrices*, National Institute of Technology Meghalaya, Bijni Complex, Laitumkhrah, Shillong-793003, India.

24. Gopinath Sahoo, *On the Signless Laplacian Spectra of Product Graphs*, Room No.507, A2 Block, Toshali Bhawan, IIT Bhubaneswar, Satyanagar, Bhubaneswar, Odisha, Pin-751007, India.

25. P. Santhosh Kumar, *A note on spectral theorem for compact normal operators on a quaternionic Hilbert space*, Room No:101, Boys Hostel, IIT Hyderabad, ODF Estate, Medak - 502205.

26. Ritabrata Sengupta, *Determining quantum entanglement by using positive but not completely positive maps*, Indian Statistical Institute Stat-Math Unit 7, S.J.S. Sansanwal Marg New Delhi 110 016 India.

27. Divya P Shenoy, *Rank additivity in the class of regular matrices*, Manipal University, Manipal India.

28. Kalyan Sinha, *The P+0 -matrix completion problem*, Mathematics Dept, IIT Guwahati, Guwahati, Assam, India.

29. K. V. Soumya, *Matrix product of distance graphs of cycle*, Manipal Institute of Technology, Manipal.

30. Piyush Kumar Tripathi, *Applications of Linear algebra in Engineering*, Department of Mathematics, Amity University Lucknow Campus, India.

31. M. Vinay, *On a Class of Laplacian Integral Graphs*, Manipal Institute of Technology, Manipal, India.

Photograph 9: *Memories from CMTGIM 2012*

Distance spectral radius of 2-partite distance regular graphs

Fouzul Atik [1] **& Pratima Panigrahi**

Department of Mathematics
Indian Institute of Technology Kharagpur
Kharagpur, 721302. India
email: [1]*fouzulatik@gmail.com*

Abstract

The distance matrix of a simple graph G is $D(G) = (d_{i,j})$, where $d_{i,j}$ is the distance between the ith and jth vertices of G. The greatest eigenvalue λ_1 of $D(G)$ is called the distance spectral radius of the graph G and is denoted by $\lambda_1(G)$. A simple connected graph G is called a 2-partite distance regular graph if there exists a partition $V_1 \cup V_2$ of the vertex set of G such that for $i = 1, 2$ and any vertex $x \in V_i$, $\sum_{y \in V_i} d(x, y)$ and $\sum_{y \in V_i^c} d(x, y)$ are constants, where V_i^c is the set complement of V_i. In this paper we find the exact value of the distance spectral radius of 2-partite distance regular graphs. Applying this result we find the distance spectral radius of the wheel graph W_n and the generalized Petersen graphs $P(n, k)$ with $k = 2$ and 3.

Keywords

Distance matrix, Distance eigenvalue, Distance spectral radius, 2-partite distance regular graph, Generalized Petersen graphs.

References

[1] A. T. Balaban, D. Ciubotariu, M. Medeleanu, Topological indices and real number vertex invariants based on graph eigenvalues or eigenvectors, J . Chem. Inf. Comput. Sci. 31 (1991) 517-523.

[2] K. Balasubramanian, Computer generation of distance polynomials of graphs, J. Comput. Chem. 11 (1990) 829-836.

[3] K. Balasubramanian, A topological analysis of the C_{60} buckminsterfullerene and C_{70} based on distance matrices, Chem. Phys. Lett. 239 (1995) 117-123.

[4] R. B. Bapat, Distance matrix and Laplacian of a tree with attached graphs, Linear Algebra Appl. 411 (2005) 295-308.

[5] R. B. Bapat, S. J. Kirkland, M. Neumann, On distance matrices and Laplacians, Linear Algebra Appl. 401 (2005) 193-209.

[6] S. S. Bose, M. Nath, S. Paul, Distance spectral radius of graphs with r pendent vertices, Linear Algebra Appl. 435 (2011) 2828-2836.

[7] V. Consonni, R. Todeschini, New spectral indices for molecule description, MATCH Commun. Math. Comput. Chem. 60 (2008) 3-14.

[8] D. Cvetković, M. Doob, H. Sachs, Spectra of Graphs-Theory and Applications, Academic Press, New York, (1980).

[9] K. C. Das, On the largest eigenvalue of the distance matrix of a bipartite graph, MATCH Commun. Math. Comput. Chem. 62 (2009), 667-672.

[10] F. R. Gantmacher, The Theory of Matrices, Volume Two, Chelsea Publishing Company, New York, N.Y., 1974.

[11] I. Gutman, M. Medeleanu, On the structure-dependence of the largest eigenvalue of the distance matrix of an alkane, Indian J. Chem. A 37 (1998) 569-573.

[12] A. Iliĉ, Distance spectral radius of trees with given matching number, Discrete Appl. Math. 158 (2010) 1799-1806.

[13] G. Indulal, Sharp bounds on the distance spectral radius and the distance energy of graphs, Linear Algebra Appl. 430 (2009) 106-113.

[14] Z. Liu, On spectral radius of the distance matrix, Appl. Anal. Discrete Math. 4 (2010) 269-277.

[15] R. Merris, The distance spectrum of a tree, J. Graph Theory 14 (1990) 365-369.

[16] D. Stevanoviĉ, A . Iliĉ, Distance spectral radius of trees with fixed maximum degree, Electron. J. Linear Algebra 20 (2010) 168-179.

[17] R. Subhi, D. Powers, The distance spectrum of the path P_n and the first distance eigenvector of connected graphs, Linear and Multilinear Algebra 28 (1990) 75-81.

[18] R. Todeschini, V. Consonni, Handbook of Molecular Descriptors, Wiley-VCH, Weinheim, 2000.

[19] B. Zhou, On the largest eigenvalue of the distance matrix of a tree, MATCH Commun. Math. Comput. Chem. 58 (2007) 657-662.

[20] B. Zhou, N. Trinajstiĉ, On the largest eigenvalue of the distance matrix of a connected graph, Chem. P hys. Lett. 447 (2007) 384-387.

Photograph 10: *Memories from CMTGIM 2012 – Prof Bhaskara Rao shaking hand with Pro-Chancellor*

Algebraic connectivity and the characteristic set of product graphs

Sasmita Barik
School of Basic Science
Indian Institute of Technology Bhubaneswar
Bhubaneswar, India.
email: sasmita@iitbbs.ac.in

Abstract

Let G be a simple graph and Y be a Fiedler vector. A vertex u is called a characteristic vertex (with respect to Y) if $Y(u) = 0$ and there is a vertex w adjacent to u satisfying $Y(w) \neq 0$. An edge $\{u,\ v\}$ is called a characteristic edge (with respect to Y) if $Y(u)Y(v) < 0$. The characteristic set $C(G,\ Y)$ is the collection of all characteristic vertices and characteristic edges of G with respect to Y. Graph products and their structural properties have been studied extensively by many researchers. A complete characterization of Laplacian spectrum of the Cartesian product of two graphs has been done by Merris. We give an explicit complete characterization of the Laplacian spectrum of the lexicographic product of two graphs using the Laplacian spectra of the factors. We supply some new results relating to the algebraic connectivity of the product graphs. We describe the characteristic sets for the Cartesian product and for the lexicographic product of two graphs.

Keywords

Product graphs; Laplacian matrix; Laplacian eigenvalues; Algebraic connectivity; Characteristic set.

References

[1] R. B. Bapat, S. Kirkland, and S. Pati (2001). The perturbed Laplacian matrix of a graph. *Linear and Multilinear Algebra, 49*, 219–242.

[2] R. B. Bapat and S. Pati (1998). Algebraic connectivity and the characteristic set of a graph. *Linear and Multilinear Algebra, 45*, 247–273.

[3] S. Barik and S. Pati (2005). On algebraic connectivity and spectral integral variations of graphs. *Linear Algebra and its Applications, 397*, 209–222.

[4] D. M. Cvetkovic, M. Doob, and H. Sachs (1980). *Spectra of Graphs*, Academic Press, New York.

[5] M. Fiedler (1973). Algebraic connectivity of graphs. *Czechoslovak Mathematical Journal, 23*, 298–305.

[6] M. Fiedler (1975). A property of eigenvectors of nonnegative symmetric matrices and its applications to graph theory. *Czechoslovak Mathematical Journal, 25*, 619–633.

[7] W. Imrich and S. Klavžar (2000). *Product graphs*, Wiley-Interscience, New York.

[8] S. Kirkland (1999). Constructions for type I trees with nonisomorphic Perron branches. *Czechoslovak Mathematical Journal, 49*, 617–632.

[9] S. Kirkland, M. Neumann, and B. L. Shader (1996). Characteristic vertices of weighted trees via Perron values. *Linear and Multilinear Algebra, 40*, 311–325.

[10] M. Neumann and S. Pati (2009). The Laplacian spectra of graphs with a tree structure. *Linear and Multilinear Algebra, 57*, 267–291.

[11] R. Merris (1987). Characteristic vertices of trees. *Linear and Multilinear Algebra, 22*, 215–131.

[12] R. Merris (1994). Laplacian matrices of graphs: a survey. *Linear Algebra and its Applications, 197/198*, 143–176.

Some matrix equations of graphs

G. Sudhakara [1] & **K. Arathi Bhat** [2]

Department of Mathematics,
Manipal Institute of Technology, Manipal University, Manipal India.
email: [1]*sudhakaraadiga@yahoo.co.in* [2]*arathi.bhat@manipal.edu*

Abstract

In this paper, we try to find solutions to some equations which involve matrices related to graphs, so are called matrix equation of graphs. They involve either adjacency matrix or incidence matrix or both. We also consider the realizability as graph, of product of adjacency matrices of graph G and G_k^P, where G_k^P is the k-complement of the graph G with reference to a partition P of of the vertex set $V(G)$, of size k.

Keywords

Adjacency matrix; Incidence Matrix; Matrix product; Realizability; k- complement.

References

[1] Akbari.S, Moazami.F, Mohammadian A (2009). Commutativity of the adjacency matrices of graphs, *Discrete Math. 309(3)*, 595-600.

[2] E.Sampath Kuamr and L. Pushpalatha (1998). Complement of a graph a generalization, *Graphs and Combinatorics 14*, 377-392.

[3] K Manjunatha Prasad , G.Sudhakara, H.S.Sujatha, M.Viany (2012). Matrix Product of Graphs, *Combinatorial Matrix Theory and Generalized Inverses of Matrices.* 41-55.

[4] D. B. West, (1996). *Introduction to Graph Theory*, Prentice Hall.

Some characterization on star partial ordering Matrices

M. B. Bindu [1 3], **K. Kamaraj** [2] & **K. Ramanathan** [1]

[1] *Department of Mathematics, KCG College of Technology, India.*
[2] *Department of Mathematics, University College of Engineering, Arni India.*
email: [3]*bindu@kcgcollege.com*

Abstract

Any square matrix $A$is called an EP matrix if it commutes with its Moore-Penrose Inverse, i.e., $AA^{\dagger} = A^{\dagger}A$. The star partial ordering of matrices can be defined by $A \leq^{*} B \Leftrightarrow A^{*}A = A^{*}B$ and $AA^{*} = BA^{*}$ where A and B are any Square Matrices.

In this paper, we present some results on Star Partial ordering involving EP- Matrices.

Keywords

Star Partial Ordering ; EP Matrices; Moore-Penrose Inverse.

On inner product spaces over idempotent semirings

A. K. Bhuniya [1] & **S. Maity** [2]

Department of Mathematics, Visva-Bharati,
Santiniketan-731 235, India.
email: [1]*anjankbhuniya@gmail.com*

Abstract

A semiring I is called an idempotent semiring if for all $a, \in I$, $a + a = a = a^2$. A finite subset B is said to be a basis of an idempotent linear space L if every vector in L can be expressed uniquely as a linear combination of the elements in B. This is more general than [**?**]. Using unique representation of every vector as a linear combination of the basis elements, we have endowed every linear space over an idempotent semiring I with a natural inner product having values in I, and show that this inner product is independent of the choice of basis. Also any two orthonormal bases of an idempotent inner product space have the same number of elements which we call the rank of the idempotent inner product space. This allow us to consider matrix representations of linear mappings on idempotent inner product spaces. Further, we have developed the theory of permanent of a matrix over idempotent semirings and show that permanent of any two matrix representations of a linear mapping is the same and thus we have the idea of permanent of a linear mapping. Also, several characterizations of the permanent of a linear mapping have been done.

Keywords

Idempotent semirings; Inner product spaces; Linear mapping; Permanent.

References

[1] D. Dolžan and P. Oblak (2009). Invertible and nilpotent matrices over antirings, *Linear Algebra Appl. 430*, 271 – 278.

[2] S. Gudder and F. Latremoliere (2009). Boolean inner-product spaces and Boolean matrices, *Linear Algebra Appl. 431*, 274 – 296

[3] S.C. Han and H.X. Li (2004). Invertible incline matrices and Cramer's rule over inclines, *Linear Algebra Appl. 389*, 121 - 138.

[4] V.N. Kolokoltsov and V.P. Maslov (1997). *Idempotent Analysis and Applications*, Kluwer Acad. Publ., Dordrecht.

[5] Shan Zhao and Xue-ping Wang (2010). Invertible matrices and semilinear spaces over commutative semirings, *Information Sciences, 180*, 5115 – 5124.

On adjacency and Laplacian spectrum of power graphs of some finite groups

Sriparna Chattopadhyay [1] **& Pratima Panigrahi**

Department of Mathematics
Indian Institute of Technology Kharagpur
Kharagpur, 721302. India
email: [1]*sriparnamath@gmail.com*

Abstract

The power graph $\mathcal{G}(G)$ of a finite group G is the graph whose vertices are the elements of G and two distinct vertices are adjacent if and only if one is an integral power of the other. In [5, 6] we found adjacency and Laplacian characteristic polynomials and some eigenvalues of power graphs of finite cyclic and dihedral groups. Also we have given bounds for spectral radius of these power graphs. Here we find some more eigenvalues of these two classes of graphs. We determine the adjacency characteristic polynomial and give bounds for the spectral radius of the power graphs of generalized quaternion 2-groups. Also we find the full Laplacian spectrum of this power graph.

Keywords

Finite group; Power graph; Adjacency Spectrum; Laplacian Spectrum; Spectral radius.

References

[1] J. Abawajy, A. Kelarev, and M. Chowdhury (2013). Power graphs: A survey, *Electron. J. Graph Theory and Appl. 1(2)* 125-147.

[2] P. J. Cameron (2010). The power graph of a finite group II, *J. Group Theory 13* 779–783.

[3] P. J. Cameron and S. Ghosh (2011). The power graph of a finite Group, *Discrete Math. 311* 1220–1222.

[4] I. Chakrabarty, S. Ghosh, and M. K. Sen (2009). Undirected power graphs of semigroups, *Semigroup Forum 78* 410–426.

[5] S. Chattopadhyay and P. Panigrahi (2014). On Laplacian spectrum of power graphs of finite cyclic and dihedral groups, *Linear and Multilinear Algebra* DOI 10.1080/03081087.2014.936435 .

[6] S. Chattopadhyay, P. Panigrahi, and F. Atik (2014). On the spectrum of power graphs of finite cyclic and dihedral groups (Submitted revised version) .

[7] D. Cvetković, P. Rowlinson, and S. Simić (2010) *An introduction to the theory of graph spectra*, Cambridge University Press, London.

[8] A. V. Kelarev and S. J. Quinn (2002). Directed graphs and combinatorial properties of semigroups, *J. Algebra 251* 16-26.

[9] H. N. Ramaswamy and C. R. Veena (2009). On the energy of unitary Cayley graphs, *Electron. J. Combin. 16* , N24.

The normalized Laplacian spectrum of corona, edge corona and neighborhood corona of two regular graphs

Arpita Das [1] & Pratima Panigrahi
Department of Mathematics
Indian Institute of Technology Kharagpur
Kharagpur, 721302. India
email: [1]*arpita.das1201@gmail.com*

Abstract

We consider simple graphs G and H with G connected. The corona of G and H is the graph which is consists of the graph G and $|V(G)|$ copies of H such that ith vertex of G is adjacent to all the vertices of the ith copy of H. The edge corona of G and H is the graph which is consists of G and $|E(G)|$ copies of H such that both the end vertices of the ith edge of G are adjacent to all the vertices of the ith copy of H. The neighborhood corona of G and H is the graph that is consists of G and $|V(G)|$ copies of H such that every neighbor of ith vertex of G is adjacent to all the vertices of the ith copy of H. Here we determine the full normalized Laplacian spectrum of the corona, edge corona and neighborhood corona of any two regular graphs in terms of their normalized Laplacian eigenvalues.

Keywords

Normalized Laplacian spectrum, Corona, Edge corona, Neighborhood corona, Kronecker product, Hadamard product.

References

[1] S. Barik, S. Pati, B. K. Sharma (2007). The spectrum of the corona of two graph, *SIAM J. Discrete Math. 24*, 47–56.

[2] F. R. K. Chung (1997). *Spectral Graph Theory*, American Mathematical Society, America.

[3] D. Cvetković, P. Rowlinson, and S. Simić (2010). *An introduction to the theory of graph spectra*, Cambridge University Press, London.

[4] I. Gopalapillai (2011). The spectrum of neighborhood corona of graphs, *Kragujavac J. Math. 35*, 493-500.

[5] F. Harary, (1969). *Graph theory*, Addison-Wilsey.

[6] R. A. Horn, C. R. Johnson (1991). *Topics in matrix Analysis*, Cambridge University Press, Cambridge.

[7] Y. Hou, W. C. Shiu (2010). The spectrum of the edge corona of two graphs, *Electron j. Linear Algebra 20*, 586–594.

[8] X. Liu, S. Zhou (2013). Spectra of the neighborhood corona of two graphs, *Linear and Multilinear Algebra 62*, 1205–1219.

[9] S. Wang, B. Zhou (2012). The signless Laplacian spectra of the corona and edge corona of two graphs, *Linear and Multilinear Algebra 20*, 1–8.

Loss behavior of an internet router employing partial buffer sharing mechanism under self-similar input traffic-fractal point process

Rajaiah Dasari [1], **Malla Reddy Peratil & L. P. Raj Kumar**

Department of Mathematics
Kakatiya University, Warangal
T.S., 506009, India. email: [1]*dsreddy.hari@gmail.com*

Abstract

A number of recent studies have addressed the use of partial buffer sharing mechanism under self-similar input traffic of an Internet router. In all the said papers, Markov modulate Poisson process (MMPP) emulating self-similar traffic over different time scale is taken into consideration. However, the time scale where self-similar nature actually begins is not considered. In this paper, we investigate the Internet router employing partial buffer sharing mechanism under self-similar input traffic by modeling it as $MMPP/D/1/K$ queueing system. We fit MMPP for fractal point process (FPP) by equating the variance while taking FOT into consideration and then compute performance measures, namely, packet loss probabilities of high priority packets and low priority packets against system parameters and traffic parameters are examined by means of matrix geometric solutions. This kind of analysis is useful in dimensioning the Internet router under self-similar input traffic to provide quality of service (QoS) guarantee.

Keywords

Internet router; self-similar; fractal point process; Markov modulated Poisson Process; packet loss probability; partial buffer sharing mechanism.

References

[1] W. E. Leland, M. S. Taqqu, W. Willinger, and D. V. Wilson (1994). On the Self-Similar Nature of Ethernet Traffic (Extended version), *IEEE /ACM Transactions Networking, 2*, 1-15.

[2] V. Paxson, and F. Sally (1995). it Wide Area Traffic: The Failure of Poisson Modeling. *IEEE/ACM Transactions Networking, 3*, 226-244.

[3] M. Crovella and A. Bestavros (1997). Self-Similarity in World Wide Web traffic: evidence and possible causes, *IEEE/ ACM Transactions Networking*, 835-846.

[4] H. Michiel and K. Laevens (1997). Telegraphic engineering in a broad - band era. *Proceedings of the IEEE, 85(12)*, 2007-2033.

[5] G. Y. Kim, A. Shiravi and P. S. Min (2006). Congestion Prediction of Self - Similar Network through Parameter Estimation, *Network Operations and Management Symposium, NOMS 2006.* 10th IEEE/IFIP.

[6] B. K. Ryu and S. B. Lowen (1996). Point process approaches to the modeling and analysis of self-similar traffic: Part I - Model construction, In: *Proc. IEEE INFOCOM, San Francisco*, CA.

[7] S. K. Shao, P. Malla Reddy, M. G. Tsai, H. W. Tsao, and J. Wu (2005). Generalized variance-based Markovian

fitting for self-similar traffic modeling, *IEICE Transactions Communications., E88-B(12)*, 4659-4663.

[8] T. Yoshihara, S. Kasahara, and Y. Takahashi (2001). Practical time scale

fitting of self-similar traffic with Markov-modulated Poisson process, *Telecommunication Systems, 17*, 185-211.

[9] B. K. Ryu and S. B. Lowen (1998). Point process models for self-similar network traffic with applications, *Stochastic Models, 14(3)*, 735-761.

[10] W. C. Lau, A. Erramilli, J. Wang, and W. Willinger (1995). Self-similar traffic generation-random midpoint displacement algorithm and its properties, *Proceedings of IEEE ICC'95*, 466-472.

[11] W. Fischer and K. Meier-Hellsterm (1993). The Markov-modulated Poisson process (MMPP) cookbook, *Performance Evacuation, 18(2)*, 149-171.

[12] P. Malla Reddy, Raghvendra,K H.K.Reddy Koppula, Mallikarjuna Reddy Doodipala, Rajaiah Dasari (2012). Self-Similar Behavior of High- way Road Traffic and Performance Analysis at Toll Plazas, *Journal of Trasportation Engineering, 138(10)*, 1233-1238.

[13] P. Malla Reddy , L.P.Raj Kumar, D. Mallikarjuna Reddy and K. Sampath Kumar (2011). Performance Analysis of Internet Router Employing Partial Buffer Shar- ing Mechanism under Markovian Modeled Self- Similar Variable Packet Length Input Traffic, *International Journal of Pure and Applied Mathematics, 67(4)*, 407-421.

[14] P. Malla Reddy , C. H. Chang, S. K. Shao, and J. Wu (2007). An Efficient Approximate Markovian Model for Optical Packet Switches Employ- ing Partial Buffer Sharing Mechanism under Self-Similar Traffic Input, *Proceedings of IEEE HPSR-2007 held at Polytechnic University, Brooklyn , New York, USA*, May 30-June1,ISBN:1-4244-1206-4, IEEE Catalog number: 07EX1775C.

[15] B. Venkataramani, K. B. Sanjay and K.R. Srivathsan (1997). Queuing Analysis of a Non- Preemptive MMPP/D/1 Priority System, Computer Communications, 20, 999-1018.

Poset properties with respect to semi - ideal - based zero-divisor graph

B. Elavarasan [1] & **K. Porselvi** [2]
Department of Mathematics
School of Science and Humanities, Karunya University
Coimbatore - 641 114, Tamilnadu, India.
emails: [1]*belavarasan@gmail.com;* [2]*porselvi94@yahoo.co.in*

Abstract

In this paper, we study some properties of poset P determined by properties of semi-ideal based zero-divisor graph properties $G_I(P)$, for a semi-ideal I of P.

Keywords

Posets; semi-ideals; prime semi-ideals; cycle and neighborhood.

MSC 2000 06D6

References

[1] M. Alizadeh, H. R. Maimani, M. R. Pournaki and S. Yassemi (2013). An ideal theoretic approach to complete partite zero-divisor graphs of posets. *J. Algebra Appl., 12(2)*, DOI: 10.1142/S0219498812501484.

[2] D. F. Anderson and P. S. Livingston (1999). The zero-divisor graph of a commutative ring. *J. Algebra, 217*, 434 - 447.

[3] I. Beck (1988). Coloring of commutative rings. *J. Algebra*, 116, 208 - 226.

[4] J. A. Bondy and U. S. R. Murty (1976). Graph theory with applications. North-Holland, Amsterdam.

[5] Frank DeMeyer and Lisa DeMeyer (2005). Zero divisor graphs of semigroups, *J. Algebra, 283*, 190-198.

[6] F. R. DeMeyer, T. McKenzie and K. Schneider (2002). The zero-divisor graph of a commutative semigroups. *Semigroup Forum, 65*, 206 - 214.

[7] P. Dheena and B. Elavarasan (2009). An ideal based-zero-divisor graph of 2-primal near-rings. *Bull. Korean Math. Soc., 46(6)*, 1051 - 1060.

[8] B. Elavarasan and K. Porselvi (2013). An ideal - based zero-divisor graph of posets. *Commun. Korean Math. Soc.*, 28, No. 1, 79 - 85.

[9] D. Lu and T. Wu (2010). The zero-divisor graphs of posets and an application to semigroups. *Graphs Combin., 26*, 793 - 804.

[10] K. Porselvi and B. Elavarasan (2014). Poset properties determined by the ideal - based zero-divisor graph. *Kyungpook Math. J. 54*, 197-201.

[11] V. Joshi. Zero divisor graph of a poset with respect to an ideal, Order: DOI 10.1007/s11083011-9216-2.

[12] Li Chen and Tongsuo Wu. On a class of semigroup graphs, (Reprint).

[13] Radomr Hala and Marek Jukl (2009). On becks coloring of posets. *Discrete Mathematics, 309*, 4584 - 4589.

[14] Radomr Hala (2008). On extensions of ideals in posets. *Discrete Mathematics, 308*, 4972-4977.

[15] S. P. Redmond (2003). An ideal-based zero-divisor graph of a commutative ring. *Comm. Algebra, 31(9)*, 4425 - 4443.

[16] Tongsuo Wu and Dancheng Lu (2008). Sub-semigroups determined by the zero-divisor graph. *Discrete Mathematics, 308*, 5122-5135.

Linear code using incidence matrix of semigraph

Deshpande C. M. [1] **& Yogeshri Gaidhani** [2]

[1] *College of Engineering, Pune, India*
[2] *M.E.S. Abasaheb Garware College, Pune, India*
emails: [1] *dcm.maths@coep.ac.in* [2]*yogeshrigaidhani@gmail.com*

Abstract

Semigraph was defined by Sampathkumar as a generalization of graph. In this paper incidence matrix which represents semigraph uniquely and characterization of such a matrix is obtained. Some properties of incidence matrix of semigraph are studied. Linear code is generated using this matrix and its properties are studied.

Keywords
Semigraph; incidence matrix of semigraph; *i*-semigraphical matrix; linear code.

References

[1] R. B. Bapat(2012). *Graphs and Matrices*, First Edition, Hindustan Book Agency, New Delhi.

[2] Biggs Norman(1974). *Algebraic Graph Theory*, Cambridge Universty Press, First Edition.

[3] N. S. Bhave, C. M. Deshpande and B. Y. Bam (2010). Characterization of Potentially Hamiltonian graph in terms of Dual Semigraph, *Proceedings of 3rd National Conference, Mathematical Techniques: Emerging Paradigms for Electronics and IT Industries.* (MATEIT-2010) ppPS10.1-PS10.4

[4] P. J. Cameron and J. H. Van Lint(1991). *Designs, Graphs, Codes and their Links*, London Mathematical Society.

[5] F. Harary (1988). *Graph Theory*, Narosa Publishing House, New Delhi.

[6] San Ling, Chaoping Xing (2008), Cambridge University Press.

[7] E. Sampathkumar, *Semigraphs and their applications*, Report on the DST project, submitted to Department of Science and Technology (DST), India, May-2000.

PD observer design for linear descriptor systems with unknown inputs

Mahendra Kumar Gupta [1], **Nutan Kumar Tomar** [2], **& Shovan Bhaumik** [3]

[1 2] *Department of Mathematics*
Indian Institute of Technology Patna, Patna, India
[3] *Department of Electrical Engineering,*
Indian Institute of Technology Patna, Patna, India
email: [1]*mahendra14389@gmail.com*

Abstract

In the last few decades, many researchers have given a lot of attention on the analysis and design of descriptor systems as these are general enough to provide a solid understanding of the inner dynamics of any physical system [1, 2]. Observer is a mathematical realization which uses input and output of a system and provides information about the states of the original system [3, 4]. In this paper, a method is given to design a proportional derivative (PD) observer for irregular descriptor systems with unknown inputs. Results from basic matrix theory is used to design an observer. In the present work, we consider the linear time invariant descriptor system: $Ex = Ax + Bu + Fv$, $y = Cx$, where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^k$, and $v \in \mathbb{R}^q$ are the state vector, the input vector, the unknown input vector and the output vector, respectively. $E, A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times k}$, $F \in \mathbb{R}^{n\times q}$ and $C \in \mathbb{R}^{p\times n}$ are known constant matrices, and the rank of $E = n_0 < n$.

Keywords

Observer design; Descriptor systems; Unknown inputs; Proportional derivative.

References

[1] S. L. Campbell (1980). *Singular systems of differential equations.* Pitman.

[2] G. R. Duan (2010). *Analysis and design of descriptor linear systems.* Springer.

[3] M. Darouach and M. Boutayeb (1995). Design of observers for descriptor systems. *IEEE Trans. Autom. Control. 40*, 1323–1327.

[4] M. K. Gupta, N. K. Tomar, and S. Bhaumik (2014). Detectability and observer design for linear descriptor system. In: *22nd Mediterranean Conference on Control and Automation, IEEE, Palermo, Italy*, 1094–1098.

Homomorphism and anti-homomorphism of reverse derivations on prime rings

C. Jaya Subba Reddy [1] & **K. Hemavathi** [2]
Department of Mathematics, S.V.University,
Tirupati 517 502, Andhra Pradesh, India.
emails: [1]*cjsreddysvu@gmail.com* [2]*hemavathiphd@gmail.com*

Abstract

In this paper we show that if a reverse derivation d acts as a homomorphism or an anti-homomorphism on a non-zero right ideal U of a prime ring R, then $d = 0$.

Keywords
Derivation; Reverse derivation; Prime ring; Center.

References

[1] I. N. Herstein (1957). Jordan derivations of prime rings, *Proc. Amer. Math. Soc. 8*, 1104–1110.

[2] I. D. Macdonald(1979). Some group elements defined by commutators, *Math. Scientist 4*, 129–131.

[3] H. E. Bell,and L. C. Kappe (1989). Rings in which derivations satisfy certain algebraic conditions, *Acta. Math. Hungar. 53*, 339–346.

[4] M. Bresar, J. Vukman (1989). On some additive mappings in rings with involution, *Aequationes Math. 38*, 178-185.

[5] M. Samman and N. Alyamani (2007). Derivations and reverse derivations in semi prime rings, *International Mathematical Forum* 2, No. 39 1895–1902.

On the structure of absolutely minimum attaining operators

J. Ganesh [1], **G. Ramesh** [2] **& D. Sukumar** [3]

Indian Institute of Technology Hyderabad, Hyderabad, India.
emails: [1]*MA12P1003@iith.ac.in* [2]*rameshg@iith.ac.in* [3]*suku@iith.ac.in*

Abstract

We discuss about a class of operators that are diagonalizable, namely positive absolutely minimum attaining operators of *first type* and prove a characterization theorem for those operators. Using this we derive a representation theorem for general absolutely minimum attaining operators of *first type*, which is similar to that of singular value decomposition for compact operators. We also study their spectrum and discuss about several properties of such operators.

Keywords

Diagonalizable operator; Minimum modulus; Absolutely minimum attaining operator; Compact operator; Spectrum.

References

[1] R. Bhatia (2009). Notes on functional analysis,*Texts and Readings in Mathematics, 50*, Hindustan Book Agency, New Delhi.

[2] D. Bakić (1998). Compact operators, the essential spectrum and the essential numerical range, *Math. Commun. 3 no. 1*, 103–108.

[3] S. J. Bernau (1965). The spectral theorem for normal operators, *J. London Math. Soc. 40*, 478–486.

[4] X. Carvajal and W. Neves (2012). Operators that achieve the norm, *Integral Equations and Operator Theory 72*, 179-195.

[5] X. Carvajal and W. Neves (2014). Operators that achieve their minimum , *Bull Braz Math Soc, Newseries 45(2)*, 293-312.

[6] J. B. Conway (1990). A course in functional analysis, *second edition, Graduate Texts in Mathematics, 96*, Springer, New York.

[7] I. Gohberg, S. Goldberg and M. A. Kaashoek (2003). Basic classes of linear operators, Birkhäuser, Basel.

[8] P. R. Halmos (1982). A Hilbert space problem book,*second edition, Graduate Texts in Mathematics, 19*, Springer, New York.

[9] T. Kato (1995). Perturbation theory for linear operators, *reprint of the 1980 edition, Classics in Mathematics,* Springer, Berlin.

[10] C. S. Kubrusly (2011). The elements of operator theory, *second edition*, Birkhäuser/Springer, New York.

[11] G. Ramesh (2014). Structure theorem for $\mathcal{AN}$ operators, *Journal of the Aust. Math. Soc. 96(3)*, 384-395.

[12] K. Schmüdgen (2012). Unbounded self-adjoint operators on Hilbert space,*Graduate Texts in Mathematics, 265,* Springer, Dordrecht.

On semipositivity of matrices

Sachindranath Jayaraman
School of Mathematics
IISER Thiruvananthapuram, CET Campus
Engineering College P.O.
Thiruvananthapuram - 695016, Kerala, India
email: sachindranathj@gmail.com

Abstract

An $m \times n$ matrix A with real entries is said to be semipositive if there exists an $x \geq 0$ such that $Ax > 0$, where the inequalities are understood componentwise. A is said to be minimally semipositive if it is semipositive and no proper $m \times p$ submatrix of A is semipositive. The class of matrices that are semipositive consists of two mutually disjoint subclasses : ones that are minimally semipositive and those that are redundantly semipositive (these are ones that are semipositive but not minimally semipositive) [1, 4]. It is known that an $m \times n$ matrix A is minimally semipositive if and only if it has a nonnegative left inverse. My objective is to study linear preservers of semipositivity and I intend to present a few results on this.

Keywords
Minimal semipositivity; Inverse positivity; Linear preserver problems; Full rank preservers.

References

[1] C. R. Johnson, M. K. Kerr and D. P. Stanford (1994). Semipositivity of matrices, *Linear Multilinear Algebra. 37(4)*, 265-271.

[2] L. Rodman and P. Semrl (2010). A localization technique for linear preserver problems, *Linear Algebra Appl. 433*, 2257-2268.

[3] C. de Seguins Pazzis (2010). The singular linear preservers of nonsingular matrices, *Linear Algebra Appl. 433*, 483-490.

[4] H. J. Werner (1904). Characterizations of minimal semipositivity, *Linear Multilinear Algebra. 37(4)*, 273-278.

First eigenvectors of nonsingular unicyclic 3-Colored digraphs

Debajit Kalita
Department of Mathematical Science
Tezpur University, Napam
Sonitpur, 784 028 India
email: k.debajit@gmail.com

Abstract

In this talk we discuss the smallest Laplacian eigenvalue and the corresponding eigenvectors of 3-colored digraphs containing exactly one nonsingular cycle. We show that the smallest Laplacian eigenvalue of such graphs can be realized as the algebraic connectivity, the second smallest Laplacian eigenvalue of a suitable undirected graph. We describe the monotonicity property on the real and imaginary parts of the eigenvectors corresponding to the smallest Laplacian eigenvalue of such graphs, which is analogous to Fiedler's monotonicity Theorem.

Keywords
Laplacian matrix; 3-Colored digraph; First eigenvector.

References

[1] R.B. Bapat, D.Kalita and S.Pati (2012). On Weighted Directed graphs, *Linear Algebra Appl. 436*, 99–111.

[2] R. B. Bapat, J. W. Grossman, and D. M. Kulkarni (1999). Generalized matrix tree theorem for mixed graphs, *Linear and Multilinear Algebra 46*, 299–312.

[3] R. B. Bapat, S. Pati (1998). Algebraic connectivity and characteristic set of a graph, *Linear and Multilinear Algebra 45*, 247–273.

[4] Y. Z. Fan (2007). On eigenvectors of mixed graphs with exactly one nonsingular cycle, *Czechoslovak Mathematical Journal 57*, 1215–1222.

[5] M. Fiedler(1975). A property of eigenvectors of nonnegative symmetric matrices and its application to graph theory, *Czechoslovak Mathematical Journal 25*, 619–633.

[6] J. M. Guo, W. C. Shiu and J. Li (2011). The algebraic connectivity of lollipop graphs, *Linear Algebra and its Applications 434*, 2204–2210.

[7] D. Kalita (2012). On 3-colored digraphs with exactly one nonsingular cycle, *Electronic journal of Linear Algebra 23*, 397–421.

[8] D. Kalita and S. Pati (2012). On the spectrum of 3-colored digraphs, *Linear and Multilinear Algebra 60*, 743–756.

Nonnegative Moore–Penrose inverses of unbounded Gram operators

T. Kurmayya [1] & **G. Ramesh** [2]

[1] *National Institute of Technology Warangal, Warangal, India.*
[2] *Indian Institute of Technology Hyderabad, Hyderabad, India*
email: [1] *kurmayya@nitw.ac.in*

Abstract

In this paper we derive necessary and sufficient conditions for the nonnegativity of Moore–Penrose inverses of unbounded Gram operators between real Hilbert spaces. These conditions include statements on acuteness of certain closed convex cones. The main result generalizes the existing result for bounded operators [**?**, Theorem 3.6].

Keywords

Closed operator, Cone, Gram operator

References

[1] A. Ben-Israel and T. N. E. Greville (2003). *Generalized Inverses : Theory and Applications*, 2nd edition, Springer Verlag, New York.

[2] A. Berman and R. J. Plemmons(1994). *Nonnegative Matrices in the Mathematical Sciences*, Classics in Applied Mathematics, SIAM.

[3] S. Goldberg(1966). *Unbounded linear operators: Theory and applications*, McGraw-Hill, New York. MR0200692 (34 #580)

[4] T. Kurmayya and K. C. Sivakumar (2007), Nonnegative Moore–Penrose inverses of Gram operators, *Linear Algebra Appl. 422*, no. 2-3, 471–476. MR2305132 (2008g:15037)

Comparison results for proper nonnegative splittings of matrices

Alekha Kumar Baliarsingh & Debasisha Mishra [1]
Department of of Mathematics
National Institute of Technology Raipur
GE Road Raipur-492 010, CG. India.
email: [1]*dmishra@nitrr.ac.in*

Abstract

The theory of splittings of matrices is a useful tool in the analysis of iterative methods for solving systems of linear equations. When two splittings are given, it is of interest to compare the spectral radii of the corresponding iteration matrices. The aim of this paper is to bring out few comparison results for the recent matrix splitting called proper nonnegative splitting introduced by Mishra, D. [Computers and Mathematics with Applications 67 (2014) 136-144; MR3141710]. Comparison results for double proper nonnegative splitting are also discussed.

References

[1] A. Ben-Israel, T. N. E. Greville (2003). *Generalized Inverses : Theory and applications*, Springer-Verlag, New York.

[2] L. Jena, D. Mishra, S.,Pani (2014). Convergence and comparison theorems for double splittings of matrices, *Calcolo 67*, 141-149.

[3] D. Mishra (2014). Nonnegative splittings for rectangular matrices, *Computers and Mathematics with Applications 397*, 136-144.

[4] D. Mishra, K.,C, Sivakumar,(2012). Comparison theorems for a subclass of proper splittings of matrices, *Appli. Math Lett. 25*, 2339-2343.

Photograph 11: *Memories from CMTGIM 2012 – Inauguration*

Eigenvalues of a digraphs

H. S. Ramane [1] & **Nandeesh K. C.** [2]

Department of Mathematics, Karnatak University
Dharwada, India.email: [1]*hsramane@yahoo.com*

Abstract

The skew-adjacency matrix of a directed graph G of order n is the $n \times n$ matrix $S(G) = [s_{ij}]$, where $s_{ij} = 1$, whenever edge from vertex v_i to v_j, $s_{ij} = -1$, whenever edge from v_j to v_i and $s_{ij} = 0$, otherwise. Hence $S(G)$ is a skew symmetric matrix of order n and all its eigenvalues are of the form $i\lambda$ where $i = \sqrt{-1}$ and $\lambda \in \mathbb{R}$. The skew energy of G is the sum of the absolute values of the eigenvalues of $S(G)$. In this paper we obtain the eigenvalues of the some class of directed graphs and study the skew energy.

References

[1] C. Adiga, R. Balakrishnan and Wasin So (2010). The skew energy of a digraph, *Linear algebra and its Appl. 432*, 1825-1835.

[2] I. Gutman (1978). The energy of a graph, *Ber. Math.-Stat. Sekt. Forschungszentrum Graz, 103*, 1-22.

[3] I. Gutman, O. E. Polansky (1986). *Mathematical Concepts in Organic Chemistry*, Springer-Verlag, Berlin.

[4] G. Indulal and A. Vijaykumar (2006). On a pair of equienergetic graphs, *MATCH Commun. Math. Comput. Chem. 55*, 83-90.

[5] H. S. Ramane, I. Gutman, H. B. Walikar and S. B. Halkarni (2005). Equienergetic complement graphs, *Kragujevac J. Sci, 27*, 67-74.

[6] H. S. Ramane and H. B. Walikar (2007). Construction of equienergetic graphs, MATCH Commun. Math. Comput. Chem. 57, 203-210.

[7] H. S. Ramane, H. B. Walikar, S. B. Rao, B. D. Acharya, P. R. Hampiholi, S. R. Jog and I. Gutman (2005). Spectra and energies of iterated line graphs of regular graphs, *Appl. Math. Lett. 18*, 679-682.

On the inverse of a graph on Godsil class

Swarup Kumar Panda [1] **& Sukanta Pati** [2]
Department of Mathematics
Indian Institute of Technology Guwahati
Guwahati 781 039, India.
[1] *emails: p.swarup@iitg.ernet.in*
[2] *email: pati@iitg.ernet.in*

Abstract

A weighted (undirected) graph G is said to be nonsingular if its adjacency matrix $A(G)$ is nonsingular. In [5], Godsil has given a class $\mathcal{G}$ of connected, unweighted, bipartite, nonsingular graphs G with a unique perfect matching, such that $A(G)^{-1}$ is signature similar to a nonnegative matrix, that is, there exists a diagonal matrix D with diagonal entries ± 1 (called a signature matrix) such that $DA(G)^{-1}D$ is nonnegative. The graph associated to the matrix $DA(G)^{-1}D$ is called the inverse of G and it is denoted by G^+. The graph G^+ is an undirected, weighted, bipartite graph with a unique perfect matching. Nonsingular unweighted trees are inside the class $\mathcal{G}$. We first give a constructive characterization of the class of weighted graphs H which can occur as the inverse of some graph $G \in \mathcal{G}$. This generalizes Theorem 2.6 of Neumann and Pati [7], where the authors have characterized graphs that occur as inverses of nonsingular unweighted trees.

References

[1] S. Akbari and S.J. Kirkland. *On Unimodular graphs.* Linear Algebra and its Applications, 421 (2007), pp. 3–15.

[2] R. B. Bapat. *Graphs and Matrices.* TRIM Series-58, Hindustan Book Agency, New Delhi, 2010.

[3] D. M. Cvetkovic, M. Doob and H. Sachs. *Spectra of Graphs.* Academic Press, NewYork, 1980.

[4] R. Frucht and F. Harary. *On the corona of two graphs.* Aequationes Mathematicae, 4 (1970), pp. 322–325.

[5] C. D. Godsil. *Inverses of trees.* Combinatorica, 5(1) (1985), pp. 33–39.

[6] Roger A. Horn and Charles R. Johnson. *Matrix Analysis.* Cambridge University Press, 1990.

[7] M. Neumann and S. Pati. *On reciprocal eigenvalue property of weighted trees.* Linear Algebra and its Applications, 438 (2013), pp. 3817–3828.

[8] R. Simion and D. S. Cao. *Solution to a problem of C. D. Godsil regarding bipartite graphs with unique perfect matching.* Combinatorica, 9(1) (1989), pp. 85–89.

Applications of linear systems in science and engineering

R. Umamaheshwar Rao

Department of Mathematics
Sreenidhi Institute of Science & Technology
Hyderabad, Telangana, India.
email: ummrao@sreenidhi.edu.in

Abstract

The aim of this article is to discuss the applications of systems of linear equations in science and various branches of engineering.

Keywords

spring mass system; temperature distribution; mechanics; chemical equations; electrical circuits–loop circuit analysis; nodal voltage analysis.

References

[1] Devid C Lay. *Linear Algebra and its Applications.*

[2] Mousa Hussein. *Linear Algebra and Engineering Applications.*

[3] Vinod K. Sharma. *Matrix Methods and Vector Spaces in Physics.*

A new characterization of nonnegativity of Moore–Penrose Inverses of Gram matrices in an indefinite inner product Space

K. Appi Reddy [1] **& T. Kurmayya** [2]

Department of Mathematics
National Institute of Technology Warangal
Warangal 506004. India
email: [2]*kurmayya@nitw.ac.in*

Abstract

In this paper we obtain a new characterization for the nonnegativity of Moore–Penrose inverses of Gram Matrices defined in an indefinite inner product space using indefinite matrix multiplication. These conditions include the acuteness of certain closed convex cones.

Keywords

Gram Matrices; Moore–Penroseinverse; acute cones.

References

[1] A. Ben-Israel and T. N. E. Greville (2003). *Generalized Inverses : Theory and Applications*, 2nd edition, Springer Verlag, New York.

[2] J. Bognar (1974). *Indefinite inner product spaces*, Springer Verlag.

[3] K. Ramanathan and K. C. Sivakumar (2009). Nonnegative Moore–Penrose inverse of Gram matrices in an indefinite inner product space, *J Optim Theory Appl. 140*, 189-196.

[4] K. C. Sivakumar (2009). A new chaterization of Nonnegativity of Moore–Penrose inverses of Gram operators, *Positivity, 13*, 277–286.

Left multiplicative generalized derivations acting as homomorphism or anti-homomorphism in prime rings

C. Jaya Subba Reddy [1] **& S. Mallikarjuna Rao** [2]

Department of Mathematics, S.V.University,
Tirupati 517 502, Andhra Pradesh, India.
emails: [1]*cjsreddysvu@gmail.com* [2]*s.mallikarjunarao123@gmail.com*

Abstract

Let R be a ring. A map $F : R \to R$ is called a left multiplicative generalized derivation, if $F(xy) = g(x)y + xF(y)$ is fulfilled for all $x,\ y$ in R, where $g : R \to R$ is any map (not necessarily derivation or additive map). The main purpose of this paper is to study the following conditions: Let R be a 2-torsion free prime ring and U be a nonzero square closed Lie ideal of R. If $F : R \to R$ is a left multiplicative generalized derivation associated with the map $g : R \to R$ such that

1. $F(uv) = F(u)F(v)$, then $[F(u),\ u] = 0$ for all $u \in U$,
2. $F(uv) = F(v)F(u)$, then $[F(u),\ u] = 0$ for all $u \in U$

Keywords

Prime ring; Derivation; Generalized derivation; Multiplicative generalized derivation; Left multiplicative generalized derivation; Homomorphism; and Anti-homomorphism.

References

[1] H. E. Bell, L. C. Kappe (1989). Rings in which derivations satisfy certain algebraic conditions, *Acta. Math. Hung. 53*, 339–346.

[2] A. Ali, S. Ali and N. Rehman (2003). On Lie ideals with derivations as homomorphisms and anti-homomorphisms. *Acta Math. Hung. 101(1-2)*, 59–62.

[3] N. Rehman(2004). On generalized derivations as homomorphisms and anti-homomorphisms, *Glass Mat. 39(59)*, 27–30.

[4] L. Gusic (2005). A note on generalized derivations of prime rings, *Glass Mat. 40(60)*, 47–49.

[5] Shakir Ali, Basudeb DharaNadeem, Ahamad Dar and Abdul Nadim Khan (2013). On lie ideals with multiplicative(generalized)-derivations in prime and semiprime rings. *Beitra algebra geom.*

Characterization of $M_\vee$-matrices

Manideepa Saha [1] **& Sriparna Bandhopadhay** [2]

[1] *National Institute of Technology Meghalaya,*
Shillong, India. email: sahamanideepa@gmail.com
[2] *Department of Mathematics*
Indian Institute of Technology Guwahati,
Assam, 781039, India.

Abstract

In this paper we consider generalized M-matrices known as $M_\vee$- matrices, which was introduced in [3]. We show that inverse positivity property of M-matrices does not carry over to the entire class of $M_\vee$-matrices, but to a subclass. We extend this property of nonsingular $M_\vee$-matrices to singular $M_\vee$-matrices. Motivated by interesting characterizations of singular M-matrices, the concepts of eventually monotonicity and eventually nonnegativity have been introduced. Definitions of eventually nonnegative and eventually monotone matrices on a set are given below.
Definition: Let $A \in \mathbb{R}^{n,\ n}$ and $S \subseteq \mathbb{R}^n$. Then we say that A is eventually nonnegative on S, if $x \in S$ and $X \geq 0$ imply that there exists a positive integer k_0, such that $A^k x \geq 0$, for all $k \geq k_0$.
Definition: Let $A \in \mathbb{R}^{n,\ n}$ and $S \subseteq \mathbb{R}^n$. Then we say that A is eventually monotone on S, if there exists a positive integer k_0, such that for any $x \in S$; $A^k x \geq 0$, for all $k \geq k_0$, implies $x \geq 0$.

Keywords
Preferred basis; quasi-preferred basis; height characteristic; level characteristic.

References

[1] A. Berman and R. Plemmons (1994). *Nonnegative matrices in the Mathematical Sciences*, SIAM.

[2] R. A. Horn and C. Johnson (1991). *Topics in Matrix Analysis*, Cambridge University Press, New York.

[3] D. D. Olesky. M. J. Tsatsomeros and P. Van Den Driessche (2009). $M_{\vee}$- matrices: a generalization of M-matrices based on eventually nonnegative matrices, *ELA 18*, 339-351.

[4] M. Neumann and R. J. Plemmons (1978). Generalized Inverse-positivity and splitting of M-matrice, *Lin. Algebra Appl. 17*, 208-234.

On the signless Laplacian spectra of product graphs

Gopinath Sahoo

Indian Institute of Technology, Bhubaneswar, India.
email: sahoo.gopi@gmail.com

Abstract

Graph products and their structural properties have been studied extensively by many researchers. In this article we investigate the signless Laplacian eigenvalues and eigenvectors of product graphs for the four standard products, namely: cartesian product, direct product strong product and lexicographic product. We provide a complete characterization of signless Laplacian spectrum of cartesian product of two graphs. For the other three products, we describe the complete spectrum of product graphs in some particular cases. Using graph products we construct new classes of signless Laplacian integral graphs.

Keywords

Signless Laplacian; Product graphs.

A note on the spectral theorem for compact normal operators on a quaternionic Hilbert space.

G. Ramesh [1] & P. Santhosh Kumar [2]

Indian Institute of Technology Hyderabad, India.
email: [1]*rameshg@iith.ac.in*

Abstract

In this talk we present the spectral theorem for compact normal operators on quaternionic Hilbert spaces. Though the version of spectral theorem for such operators in quaternionic Hilbert space is appeared in recent literature, we present a different approach, which is similar to the classical setup. It is observed that the whole spherical spectrum of a compact normal operator is determined by the standard eigenvalues and deduce that the spherical spectrum of any $n \times n$ quaternion matrix has exactly $n-$ complex eigenvalues. We illustrate our method with an example and compare it with that of the method given by Ghiloni etal [7, Theorem 1.2].

Keywords

Standard eigenvalue; Slice complex plane; Minimum modulus; Generalized standard eigenvalue.

References

[1] A. Baker (1999). Right eigenvalues for quaternionic matrices: a topological approach, *Linear Algebra Appl. 1-3*, 303–309.

[2] J. L. Brenner(1951). Matrices of quaternions, *Pacific J. Math. 1* 329–335.

[3] D. R. Farenick and B. A. F. Pidkowich (2003). The spectral theorem in quaternions, *Linear Algebra Appl. 371* , 75–102.

[4] M. Fashandi (2013). Compact operators on quaternionic Hilbert spaces, *Facta Univ. Ser. Math. Inform. 28* 3, 249–256. MR3152932

[5] M. Fashandi (2014). Some properties of Bounded linear operators on Quaternionic Hilbert spaces, *Kochi J. Math. 9*, 127-135.

[6] R. Ghiloni, V. Moretti, and A. Perotti (2013). Continuous slice functional calculus in quaternionic Hilbert spaces, *Rev. Math. Phys. 25.*

[7] R. Ghiloni, V. Moretti, and A. Perotti, Spectral properties of compact normal quaternionic operators, (pre-print), arXiv:1207.0666.

[8] I. Gohberg, S. Goldberg, and M. A. Kaashoek (2003). Basic classes of linear operators, Birkhäuser, Basel.

[9] H. C. Lee(1949), Eigenvalues and canonical forms of matrices with quaternion coefficients, *Proc. Roy. Irish Acad. Sect. A. 52*, 253–260.

[10] Zhang, Fuzhen (1997). Quaternions and matrices of quaternions, *Linear Algebra Appl. 251* , 21–57.

Determining quantum entanglement by using positive but not completely positive maps

Ritabrata Sengupta
Theoretical Statistics and Mathematics Unit
Indian Statistical Institute
New Delhi, 110 0016. India.
email: rb@isid.ac.in

Abstract

We give a matrix theoretic approach for detection of quantum entanglement. Our method uses positive maps which are not completely positive and are not decomposable. Unfortunately there are only a few examples of such maps available in literature and few such states where these maps are applicable. We give new methods for construction of such maps and states to be detected.

We extend the above concept, which is inherently bipartite, to a multipartite settings. In this case, we give a method to generate new witnesses for detecting entanglement. We also give new examples of states detectable by such maps.

Keywords

Quantum entanglement; positive maps; matrix analysis.

References

[1] Sung Je Cho, Seung-Hyeok Kye, and Sa Ge Lee (1992). Generalized Choi maps in three-dimensional matrix algebra, *Linear Algebra Appl., 171*, 213-224.

[2] Man Duen Choi (1975). Positive semidefinite biquadratic forms, *Linear Algebra and Appl. 12(2)*, 95-100.

[3] Man Duen Choi and Tsit Yuen Lam (1977/78). Extremal positive semidefinite forms, *Math. Ann. 231(1)*, 1-18.

[4] Dariusz Chruśsciński and Andrzej Kossakowski (2007). On the structure of entanglement witnesses and new class of positive indecomposable maps. *Open Syst. Inf. Dyn. 14(3)*, 275-294.

[5] Dariusz Chruśsciński and Andrzej Kossakowski (2008). A class of positive atomic maps, *J. Phys. A, 41(21)*, 215201, 13, 2008.

[6] A. Einstein, B. Podolsky, and N. Rosen (May 1935). Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev. 47*, 777-780.

[7] K. Artur Ekert (Aug 1991). Quantum cryptography based on bell's theorem. Phys. Rev. Lett. 67, 661-663.

[8] Leonid Gurvits (2003). Classical deterministic complexity of Edmonds' problem and quantum entanglement. In: Proceedings of the thirty-fifth annual ACM symposium on Theory of computing, STOC '03, New York, NY, USA, ACM, 10-19.

[9] Kil-Chan Ha and Seung-Hyeok Kye (2012). Entanglement witnesses arising from choi type positive linear maps. Journal of Physics A: Mathematical and Theoretical, 45(41), 415305.

[10] Ryszard Horodecki, Pawe l Horodecki, Micha l Horodecki, and Karol Horodecki (Jun 2009). Quantum entanglement. Rev. Mod. Phys. 81(2), 865-942.

[11] Marcus Huber and Ritabrata Sengupta (Sep 2014). Witnessing genuine multipartite entanglement with positive maps. Phys. Rev. Lett. 113, 100501.

[12] Andrzej Kossakowski (2003). A class of linear positive maps in matrix algebras. *Open Syst. Inf. Dyn. 10(3)*, 213-220.

[13] Michael A. Nielsen and Isaac L. Chuang (2010). *Quantum Computation and Quantum Information.* Cambridge University Press, 10th anniversary edition, Cambridge Books Online.

[14] Hiroyuki Osaka (1992). A class of extremal positive maps in 3×3 matrix algebras. *Publ. Res. Inst. Math. Sci. 28(5)*, 747-756.

[15] K. R. Parthasarathy (2013). *Coding theorems of classical and quantum information theory*, volume 45 of Texts and Readings in Mathematics. Hindustan Book Agency, New Delhi, second edition.

[16] Vern Paulsen(2002). *Completely bounded maps and operator algebras*, volume 78 of Cambridge Studies in Advanced Mathematics. Cambridge University Press, Cambridge.

[17] E. Schröodinger. Discussion of probability relations between separated systems, Mathematical Proceedings of the Cambridge Philosophical Society, 31 555-563, 10 1935.

[18] E. Schröodinger Probability relations between separated systems. *Mathematical Proceedings of the Cambridge Philosophical Society, 32* 446-452, 10 1936.

[19] R. Sengupta and Arvind (Sep 2011). Extremal extensions of entanglement witnesses: Finding new bound entangled states. *Phys. Rev. A, 84:032328.*

[20] R. Sengupta and Arvind (Jan 2013). Extremal extensions of entanglement witnesses and their connection with unextendable product bases. *Phys. Rev. A, 87:012318.*

[21] Peter W. Shor (1994). Algorithms for quantum computation: discrete logarithms and factoring. In: 35th Annual Symposium on Foundations of Computer Science (Santa Fe, NM, 1994), IEEE Comput. Soc. Press, Los Alamitos, CA, 124-134.

Rank additivity in the class of regular matrices

K. Manjunatha Prasad [1], **K.S. Mohana** [2] & **Divya P Shenoy** [3]

[1 3] *Department of Statistics*
Manipal University, Manipal
Karnataka 576 104 India
[2] *Department of Mathematics*
Mangalore University
Mangalagangothri, Karnataka, India
[3] *Department of Mathematics*
Manipal Institute of Technology
Manipal University Manipal
Karnataka, 576 104 India
emails: [1] *km.prasad@manipal.edu;* [2] *- krish305@gmail.com;* [3] *divya.shenoy@manipal.edu*

Abstract

The theory of 'Minus Partial Order' on the class of matrices over a field is well studied in the literature, and it is known that the rank additive property '$\rho(A) = \rho(B) + \rho(A-B)$' holds whenever B is lesser than A under minus partial order. Though, several characterizations of minus partial order relation remain hold for the class of regular matrices over a commutative ring, rank additive property fails. So, the rank additive property in the class of regular matrices has been further investigated. In the process, Rao–Mitra's theorem on invariance of $BA^{-}C$ has been further probed and a general condition for such invariance for the matrices over a commutative ring is obtained.

Keywords

regular matrix, generalized inverse, minus partial order, rank additivity

References

[1] K.P.S. Bhaskara Rao (1983). Generalized Inverses of Matices over Integral Domain, *Linear Algebra Appl.*, 49 : 179–189.

[2] M.P Drazin (1986). A partial order in Completly Regular Semigroups, *Journal of Algebra 98(2):* 181–196.

[3] R.E. Hartwig (1980). How to Order Regular Elements ?, *Math. Japon., 25*, 1–13.

[4] N. Jacobson (1974). *Basic Algebra I & II*, W.H. Freeman & Co.

[5] K. Manjunatha Prasad (1994). Generalized Inverse of Matrices Over Commutative Rings, *Linear Algebra Appl., 211* 35–52.

[6] S. K. Mitra (1986). The Minus Partial Order and Shorted Matrix, *Linear Algebra Appl., 81*: 207–236.

[7] S.K Mitra, P. Bhimasankarm and Saroj B Malik (2010). *Matrix Partial Orders, Shorted Operators and Applications*, World Scientific.

[8] K.S.S. Nambooripad (1980). The Natural Partial order on a Regular Semigroup, *Proc. Edinburgh Math. Soc. 23:* 249–260.

[9] C.R. Rao and S.K. Mitra (1971). *Generalized Inverse of Matrices and Its Applications*, Wiley, New York.

The P_0^+-matrix completion problem

Bhaba Kumar Sarma [1] & **Kalyan Sinha** [2]

Department of Mathematics
Indian Institute of Technology Guwahati,
Guwahati, India. email: kalyansinha90@gmail.com

Abstract

A real $n \times n$ matrix A is a P_0^+-matrix, if for each $k \in \{1, 2, \ldots, n\}$, every $k \times k$ principal minor of A is nonnegative and at least one $k \times k$ principal minor is positive. The matrix A is a *Q-matrix* if for every $k \in \{1, 2, \ldots, n\}$, the sum $S_k(A)$ of all the $k \times k$ principal minors of A is positive.

A *partial matrix* is a rectangular array of numbers in which some entries are specified while others are free to be chosen. For a class Π of matrices (e.g., P, P_0 or Q-matrices) a *partial* Π-*matrix* is one whose specified entries satisfy the required properties of a Π-matrix. Thus, a *partial* P_0^+-*matrix* M is a partial matrix in which all fully specified principal submatrices are nonnegative and $S_k(M) > 0$ for every $k \in \{1, 2, \ldots, n\}$, whenever all $k \times k$ principal submatrices are fully specified.

A Π-*completion* of a partial Π-matrix is a Π-matrix obtained by some choices of the unspecified entries. The (*combinatorial*) Π-*matrix completion problem* attempts to study the digraphs D having the property that any partial Π-matrix specifying D has a Π-completions. For an exposition in matrix completion problems, one may see the survey articles [1] and [2].

A digraph D is said to have P_0^+-completion if every partial P_0^+-matrix specifying D can be completed to a P_0^+-matrix. In this presentation, some necessary conditions and sufficient conditions for a digraph to have P_0^+-completion are discussed and those digraphs of order at most four that have P_0^+-completion have been characterized.

Keywords

Partial matrix; Matrix completion; P_0^+-matrix; P_0^+-completion; Digraph.

References

[1] L. Hogben and A. Wangsness (2007). *Matrix completion problems.*. Handbook of Linear Algebra, L. Hogben, Editor. Chap-man and Hall/CRC Press, Boca Raton.

[2] L. Hogben (2000). Graph theoretic methods for matrix completion problems. *Linear Algebra and its Applications 319*, 83–102.

Matrix product of distance graphs of cycle

G. Sudhakara [1] & **K. V. Soumya** [2]
Department of Mathematics,
Manipal Institute of Technology, Manipal University
Manipal, 576 104 India.
email: [1]*sudhakaraadiga@yahoo.co.in;* [2]*sumi822003@yahoo.com*

Abstract

In this paper, we characterize circulant graphs G for which there exist a graph H such that $A(G)A(H)$ is graphical. In particular, we consider the class of circulant graphs which are distance graphs of cycles.

Keywords
Matrix Product; distance graphs; circulant graph

Photograph 12: *Look, who is playing flute?*

Applications of Linear algebra in Engineering

Piyush Kumar Tripathi [1], **Praveen Kumar Misra** [2], **Apurva Anand** [3] & **Vivek Verma** [4]

[1 2] *Department of Mathematics, Amity University Lucknow Campus, India.*
[3 4] *Department of Mechanical Engineering, Amity University Lucknow Campus, India.*
emails: [1] *Piyush.tripathi2007@gmail.com;* [2] *praveenkumarmisra@gmail.com;* [3] *apurva2050@yahoo.co.in;* [4] *iitrviv@gmail.com*

Abstract

Linear Algebra is a branch of mathematics that deals with the study of vectors, vector spaces and linear equations. In addition to science, engineering and mathematical sciences, linear algebra has extensive applications in the natural as well as the social sciences. Linear algebra today has been extended to consider n-dimensional space. In linear algebra one studies sets of linear equations and their transformation properties. It is possible to consider the analysis of rotations in space, selected curve fitting techniques, differential equation solutions, as well as many other problems in science and engineering using techniques of linear algebra. Two tools are extensively used in linear algebra are : The Matrix and The Determinant. In this paper we are going to discuss various applications of linear algebra in different engineering branches such as in mechanical engineering electronics and communications, computer science and applications, civil engineering also in social sciences with examples.

On a class of Laplacian integral graphs

L. Rüttimann [1], **G. Sudhakara** [2] & **M. Vinay** [3]

[1] *ETH Zürich, 8092 Zürich, Switzerland*
[2 3] *Manipal Institute of Technology, Manipal, India.*
emails: [2] *sudhakaraadiga@yahoo.co.in;* [3] *vinay.m2000@gmail.com*

Abstract

A graph whose adjacency (Laplacian) matrix has a spectrum consisting only of integers is called (Laplacian) integral. We observe that a certain class of digraphs is integral as well as Laplacian integral. We then construct a class of Laplacian integral undirected graphs, and discuss the relation between their spectrum and structure.

Keywords

Laplacian matrix; Integral graph; Laplacian integral graph.

References

[1] J. Bang-Jensen and G.Z. Gutin (2007). Digraphs: Theory, Algorithms and Applications 2nd ed., Springer.

[2] S. Kirkland (2005). Completion of Laplacian integral graphs via edge addition, *Discrete Mathematics 295(1–3)* 75–90.

[3] F. Yizheng (2002). On Spectral Integral Variations of Graphs, *Linear and Multilinear Algebra 50(2)* 133–142.

Photograph 13: *Memories from CMTGIM 2012 – RB Bapat with TES Raghavan*

Ravindra B Bapat

Prof Ravindra B Bapat was born on December 20, 1954 in Mumbai, to the couple Bhalchandra S Bapat, Usha B Bapat. He has no siblings but was brought up in a joint family with his parents, uncle, aunt, and two cousin sisters. His family was somewhat orthodox, as he says, and came from a place known as 'Ganapatipule' in Konkan of Maharastra, which is now a tourist place. His grandfather moved to Mumbai as he got a job in Indian Railways. His father was also employed in Indian Railways.

Young Ravi had spent his childhood in Girgaum area of Mumbai which used to be culturally strong, middle class, and Marathi neighbourhood. He had his primary education in a local Municipal school and continued his higher primary and secondary education in Wilson High School, Mumbai. First two year of his undergraduate study was done at Jai Hind College and the later two years in K C College, Mumbai (affiliated to University of Mumbai) for his BSc degree with Statistics as principal subject and Mathematics as secondary subject.

Ravindra joined Indian Statistical Institute, Delhi in 1976 for his MStat degree. Like we see in many great mathematicians, Bapat also had very strong interest in extracurricular activities of his choice. His interests were chess and music. He played very good chess and plays flute. He learnt flute from Pandit Malhar Kulkarni. In fact, this common interest of music between Prof Raghavan and Bapat, made them close to each other. During his MStat graduation, Bapat met Prof Raghavan, who visited ISI Delhi on sabbatical leave from University of Illinois, Chicago. The relationship that began here, led R B Bapat to choose University of Illinois at Chicago and obtain his PhD under the guidance of Prof T E S Raghavan in 1981.

Dr Bapat married 'Aruna (renamed Ragini as per traditional custom) Vartak' from Kelshi, near Dapoli, Maharashtra in 1983. Their son 'Sudeep' is presently doing his graduate study in the US.

After spending a year at Northern Illinois University in DeKalb, Illinois and two years in the Department of Statistics, University of Mumbai, Prof Bapat joined the Indian Statistical Institute, New Delhi, in 1983, where he presently holds the position of Prof, Stat-Math Unit. He was also the Head of Indian Statistical Institute, New Delhi from 2007 to 2011. He held visiting faculty positions at various Universities in the US and visited several Institutes abroad in countries including Canada, China, France, Holland and Taiwan for collaborative research and seminars.

The main research interests of Prof Bapat are nonnegative matrices, matrix inequalities, matrices in graph theory, and generalized inverses. He has published about 130 research papers in the above mentioned areas in reputed national and international journals. He has written books titled "Nonnegative Matrices and Applications" with Prof T E S Raghavan in the Cambridge University Mathematical Series Encyclopedia of Mathematics and its Applications, "Linear Algebra and Linear Models" and "Graphs and Matrices" in TRIM Series of Hindustan Book Agency, New Delhi, India which is co–published by Springer as its UTX Series. He also wrote a book on Mathematics for the general reader, in Marathi (his mother tongue), which won the state government award for best literature in Science in 2004.

He was PhD advisor for Arbind K Lal (presently at IIT, Kanpur), Sukanta Pati (presently at IIT Guwahati), and Zheng Bing (presently in Lanzhou University, PR CHINA).

The following is the list of his collaborators: T E S Raghavan; Y S Sathe; V S Sunder; M K Kwong; M I Beg; K P S Bhaskara Rao; K Manjunatha Prasad; A K Lal; Alok Dey; K Balasubramanian; G Constantine; D W Robinson; S C Kochar; S K Jain; Dave Stanford; P van den Driessche; Dale Olesky; Adi Ben-Israel; Sukanta Pati; L Snyder; Stef Tijs; J W Grossman; D M Kulkarni; Seok-Zun Song;

S Kirkland; M Akian; S Gaubert; Bing Zheng; Ivan Gutman; Wenjun Xiao; John Tynan; Michael Neumann; M Catral; Kinkar Chandra Das; R Balaji; Somit Gupta; Pritha Rekhi; K Reji Kumar; Gary MacGillivray; S Sivasubramanian; Debjit Kalita; P Chebotarev; Souvik Roy; Ebrahim Ghorbani; R E Hartwig; Marianne Akian; Stéphane Gaubert; S K Neogy; A K Das; T Parthasarathy; Simo Puntanen.

Prof Bapat serves on the editorial boards of Linear and Multilinear Algebra, Electronic Journal of Linear Algebra, Indian Journal of Pure and Applied Mathematics, Journal of Mathematical Science of Delhi University and Kerala Mathematical Association Bulletin. He was a guest editor for several special issues of the journal "Linear Algebra and its Applications", e.g., g-inverse conference in Delhi, C R Rao's 75th birthday volume, ILAS conference volume etc. He is an elected Fellow of the Indian Academy of Sciences, Bangalore and the Indian National Science Academy, Delhi. He was awarded the J C Bose Fellowship of DST in 2009. He is also on the Advisory Board of the journal "Mathematical Forum" of Dibrugarh University.

Prof Bapat served as the President of the Indian Mathematical Society during its centennial year 2007–2008. He is also the current President of 'Academy of Discrete Mathematics and Applications'. For the past several years, he has been involved as the National Coordinator of the International Mathematics Olympiad Training Program in India.

Research Papers Published

[1] A generalization of a theorem of Ky Fan on simplicial maps, *Journal of Combinatorial Theory, Series A* 29(1): 32–38 (1980)

[2] (with T E S Raghavan) On diagonal products of doubly stochastic matrices, *Linear Algebra and Its Applications* 31: 71–75 (1980)

[3] D_1AD_2 theorems for multidimensional matrices, *Linear Algebra and Its Applications* 48: 437–442 (1982)

[4] Doubly stochastic matrices with equal subpermanents, *Linear Algebra and Its Applications* 51: 1–8 (1983)

[5] A matrix of permanents, *Linear Algebra and Its Applications* 56: 57–62 (1984)

[6] A stronger form of the Egorychev-Falikman theorem on permanents, *Linear Algebra and Its Applications* 63: 95–100 (1984)

[7] On permanents of positive semidefinite matrices, *Linear Algebra and Its Applications* 65: 83–90 (1985)

[8] (with Y S Sathe) On the E-optimality of truncated BIBD, *Bull. Calcutta Stat. Assoc.* 34: 117 (1985)

[9] (with V S Sunder) On majorization and Schur products, *Linear Algebra and Its Applications* 72: 107–117 (1985)

[10] A bound for the permanent of the Laplacian matrix, *Linear Algebra and Its Applications* 74: 219–223 (1986)

[11] (with V S Sunder) An extremal property of the permanent and the determinant, *Linear Algebra and Its Applications* 76: 153–163 (1986)

[12] Applications of an inequality in information theory to matrices, *Linear Algebra and Its Applications* 78: 107–117 (1986)

[13] Inequalities for mixed Schur functions, *Linear Algebra and Its Applications* 83: 143–149 (1986)

[14] Two inequalities for the Perron root, *Linear Algebra and Its Applications* 85: 241–248 (1987)

[15] A refinement of Oppenheim's inequality, *Current Trends in Matrix Theory*, Bob Grone, Frank Uhlig Eds. 29–32, Elsevier (1987)

[16] (with M K Kwong) A generalization of $A \circ A^{-1} \geq I$, *Linear Algebra and Its Applications* 93: 107–112 (1987)

[17] Majorization and singular values, *Linear and Multilinear Algebra*, 21: 211–214 (1987)

[18] Refining Schur's inequality using Schur complements, *Linear and Multilinear Algebra* 23: 55–62 (1988)

[19] Multinomial probabilities, permanents and a conjecture of Karlin and Rinott, *Proceedings Amer. Math. Society* 102(3): 467–472 (1988)

[20] Discrete multivariate distributions and generalized log-concavity, *Sankhya, Series A* 50: 98–110 (1988)

[21] Infinite divisibility of multivariate gamma distributions and M-matrices, *Sankhya, Series A* 51: 73–78 (1989)

[22] (with M I Beg) Order statistics for nonidentically distributed variables and permanents, *Sankhya, Series A* 51: 79–93 (1989)

[23] Comparing the spectral radii of two nonnegative matrices, *American Mathematical Monthly* 96(2): 137–139 (1989)

[24] (with T E S Raghavan) An extension of a theorem of Darroch and Ratcliff in log-linear models and its applications to scaling multidimensional matrices, *Linear Algebra and Its Applications* 114–115: 705–715 (1989)

[25] A constructive proof of a permutation based generalization of Sperner's lemma, *Mathematical Programming* 44: 113–120 (1989)

[26] (with M I Beg) Identities and recurrence relations for order statistics corresponding to nonidentical variables, *Communications in Statistics, Theory and Methods* 18(5): 1993–2004 (1989)

[27] Majorization and singular values II, *SIAM Journal of Matrix Theory and Applications* 10(4): 429–434 (1989)

[28] Mixed discriminants of positive semidefinite matrices, *Linear Algebra and Its Applications* 126: 107–124 (1989)

[29] Permanents in Probability and Statistics, *Linear Algebra and Its Applications* 127: 3–25 (1990)

[30] (with K P S Bhaskara Rao and K Manjunatha Prasad) Generalized inverses over integral domains, *Linear Algebra and Its Applications* 140: 181–196 (1990)

[31] Majorization and singular values III, *Linear Algebra and Its Applications* 145: 59–70 (1991)

[32] (with K P S Bhaskara Rao and K Manjunatha Prasad) Generalized inverses over integral domains II : Group inverses and Drazin inverses, *Linear Algebra and Its Applications* 146: 31–47 (1991)

[33] (with A K Lal) Path-positive graphs, *Linear Algebra and Its Applications* 149: 125–149 (1991)

[34] An interlacing theorem for tridiagonal matrices, *Linear Algebra and Its Applications* 150: 331–340 (1991)

[35] (with A Dey) Optimal block designs with minimal number of observations, *Statistics and Probability Letters* 11: 399–402 (1991)

[36] (with K Balasubramanian) Identities for order statistics and a theorem of Renyi, *Statistics and Probability Letters* 12: 141–143 (1991)

[37] (with V S Sunder) On hypergroups of matrices, *Linear and Multilinear Algebra* 29: 125–140 (1991)

[38] A note on the permanental Hadamard inequality, *Linear and Multilinear Algebra* 30: 205–208 (1991)

[39] (with K Balasubramanian and M I Beg) On families of distributions closed under extrema, *Sankhya, Series A* 53(3): 375–388 (1991)

[40] (with K Manjunatha Prasad) The generalized Moore–Penrose inverse, *Linear Algebra and Its Applications* 165: 59–69 (1992)

[41] (with K Manjunatha Prasad) A note on the Khatri–inverse, *Sankhya, Series A* 54: 291–295 (1992)

[42] (with G Constantine) An enumerating function for spanning forests with color restrictions, *Linear Algebra and Its Applications* 173: 231–237 (1992)

[43] (with D W Robinson) The Moore-Penrose inverse over a commutative ring, *Linear Algebra and Its Applications* 177: 89–103 (1992)

[44] Mixed discriminants and spanning trees, *Sankhya, Special volume in memory of Prof R C Bose* 54: 49–55 (1992)

[45] Symmetric function means and permanents, *Linear Algebra and Its Applications* 182: 101–108 (1993)

[46] (with S C Kochar) Characterizations of identically distributed independent random variables using order statistics, *Statistics and Probability Letters* 17: 225–230 (1993)

[47] (with A K Lal) Path–positivity and infinite Coxeter groups, *Linear Algebra and Its Applications* 196: 19–35 (1994)

[48] (with A K Lal) Inequalities for the $q-$permanent, *Linear Algebra and Its Applications* 198: 397–409 (1994)

[49] (with S C Kochar) On likelihood–ratio ordering of order statistics, *Linear Algebra and Its Applications* 199: 281–291 (1994)

[50] Generalized inverses with proportional minors, *Linear Algebra and Its Applications* 211: 27–34 (1994)

[51] (with S K Jain) Roots of symmetric idempotent boolean matrices, *Applied Linear Algebra, Proc. Fifth SIAM Conference*: 489–492 (1994)

[52] König's theorem and bimatroids, *Linear Algebra and Its Applications* 212/213: 353–365 (1994)

[53] (with Dave Stanford and P van den Driessche) Pattern properties and spectral inequalities in max algebra, *SIAM Journal of Matrix Analysis and Applications* 16(3): 964–976 (1995)

[54] Permanents, max algebra and optimal assignment, *Linear Algebra and Its Applications* 226/228: 73–86 (1995)

[55] (with Dale Olesky and P van den Driessche) Perron–Frobenius theory for a generalized eigenproblem, *Linear and Multilinear Algebra* 40: 141–152 (1995)

[56] (with Adi Ben-Israel) Singular values and maximum rank minors of generalized inverses, *Linear and Multilinear Algebra* 40: 153–161 (1995)

[57] The Laplacian matrix of a graph, *The Mathematics Student* 65: 214–223 (1996)

[58] (with K Balasubramanian and M I Beg) An identity for the joint distribution of order statistics and its applications, *Journal of Statistical Planning and Inference* 55: 13–21 (1996)

[59] (with S K Jain and Sukanta Pati) Weighted Moore–Penrose inverse of a boolean matrix, *Linear Algebra and Its Applications* 255: 267–279 (1997)

[60] (with S K Jain and L Snyder) Nonnegative idempotent matrices and the minus partial order, *Linear Algebra and Its Applications* 261: 143–154 (1997)

[61] Moore-Penrose inverse of the incidence matrix of a tree, *Linear and Multilinear Algebra* 42(2): 159–167 (1997)

[62] (with Stef Tijs) Incidence matrix games, in *Game Theoretical Applications to Economics and Operations Research,* T Parthasarathy et al Eds., Kluwer Academic Publishers, 1997, 9–16.

[63] Structure of a nonnegative regular matrix and its generalized inverses, *Linear Algebra and Its Applications* 268: 31–39 (1998)

[64] A max version of Perron-Frobenius theory, *Linear Algebra and Its Applications* 275–276: 3–18 (1998)

[65] (with S Pati) Algebraic connectivity and the characteristic set of a graph, *Linear and Multilinear Algebra* 45: 247–273 (1998)

[66] A bound for the spectral radius of a unicyclic graph, *Bona Mathematica* 9(4): 75–79 (1998)

[67] (with K Manjunatha Prasad) Cochran's Theorem and related results on matrix rank over a commutative ring, *Statistical Inference and Design of Experiments*, U J Dixit and M R Satam Eds., Narosa Publishing House, 125–133 (1999)

[68] Resistance distance in graphs, *Mathematics Student* 68(1–4): 87–98 (1999)

[69] (with S K Jain and K Manjunatha Prasad) Generalized power symmetric stochastic matrices, *Proceedings of the American Mathematical Society* 127(7): 1987–1994 (1999)

[70] (with J W Grossman and D M Kulkarni) Generalized matrix tree theorem for mixed graphs, *Linear and Multilinear Algebra* 46: 299–312 (1999)

[71] Linear estimation in models based on a graph, *Linear Algebra and Its Applications* 302/303: 223–230 (1999)

[72] (with J W Grossman and D M Kulkarni) Edge version of matrix tree theorem for trees, *Linear and Multilinear Algebra* 47: 217–229 (2000)

[73] Moore-Penrose inverse of set inclusion matrices, *Linear Algebra and Its Applications* 318: 35–44 (2000)

[74] (with Sukanta Pati and Seok-Zun Song) Rank preservers of matrices over max algebra, *Linear and Multilinear Algebra* 48: 149–164 (2000)

[75] (with D M Kulkarni) Minors of some matrices associated with a tree, *Algebra and Its Applications,* D V Huynh, S K Jain, S R Lopez-Permouth Eds., Contemporary Mathematics 259, American Math Society, 2000, 45–66

[76] (with S Kirkland and S Pati) The perturbed Laplacian matrix of a graph, *Linear and Multilinear Algebra* 49: 219–242 (2001)

[77] (with M Akian and S Gaubert) Generic asymptotics of eigenvalues using min-plus algebra, *Proceedings of the Satellite Workshop on Max-plus Algebras,* IFAC SSSC'01, Elsevier, Praha, (2001)

[78] (with Sukanta Pati) Path matrices of a tree, *The Journal of Mathematical Sciences,* New Series 1: 46–52 (2002)

[79] Outer inverses: Jacobi type identities and nullities of submatrices, *Linear Algebra and Its Applications* 361: 107–120 (2003)

[80] (with Bing Zheng) Generalized inverses of bordered matrices, *Electronic Journal of Linear Algebra* 10: 16–30 (2003)

[81] Perron eigenvector of the Tsetlin matrix, *Linear Algebra and Its Applications* 363: 3–16 (2003)

[82] (with Ivan Gutman and Wenjun Xiao) A simple method for computing resistance distance, *Zeitschrift fur Naturforschung* 58a: 494–498 (2003)

[83] (with S K Jain and John Tynan) Decomposition of generalized polynomial symmetric matrices, *Linear Algebra and Its Applications* 376: 69–83 (2004)

[84] Resistance matrix of a weighted graph, *MATCH Commun. Math. Comput. Chem.* 50: 73–82 (2004)

[85] (with Bing Zheng) Characterization of generalized inverses by a rank equation, *Applied Mathematics and Computation* 151(1): 53–67 (2004)

[86] (with Michael Neumann) Inequalities for permanents involving Perron complements, *Linear Algebra and Its Applications* 385: 95–104 (2004)

[87] (with Bing Zheng) Generalized inverse $A^{(2)}_{T,S}$ and a rank equation, *Applied Mathematics and Computation* 155(2): 407–415 (2004)

[88] (with Sukanta Pati) Energy of a graph is never an odd integer, *Bulletin of Kerala Mathematics Association* 1(2): 129–132 (2004)

[89] On some matrices related to a tree with attached graphs, *Operations Research with Economic and Industrial Applications: Emerging trends,* S R Mohan and S K Neogy Eds., 53–64, Anamaya Publishers, New Delhi (2005)

[90] (with S Kirkland and M Neumann) On distance matrices and Laplacians, *Linear Algebra and Its Applications* 401: 193–209 (2005)

[91] (with M Catral and Michael Neumann) On functions that preserve M-matrices and inverse M-matrices, *Linear and Multilinear Algebra* 53(3): 193–201 (2005)

[92] (with Kinkar Chandra Das) A sharp upper bound on the largest Laplacian eigenvalue of weighted graphs, *Linear Algebra and Its Applications* 409: 153–165 (2005)

[93] Distance matrix and Laplacian of a tree with attached graphs, *Linear Algebra and Its Applications* 411: 295–308 (2005)

[94] (with K Somasundaram) Cramer's rule for generalized matrix functions, *Bulletin of Kerala Mathematical Association* 2(2): 1–7 (2005)

[95] Determinant of the distance matrix of a tree with matrix weights, *Linear Algebra and Its Applications* 416: 2–7 (2006)

[96] (with A K Lal and Sukanta Pati) A q-analogue of the distance matrix of a tree, *Linear Algebra and Its Applications* 416: 799–814 (2006)

[97] (with R Balaji) On Euclidean distance matrices, *Linear Algebra and Its Applications* 424: 108–117 (2007)

[98] On generalized inverses of banded matrices, *Electronic Journal of Linear Algebra* 16: 284–290 (2007)

[99] (with R Balaji) Block distance matrices, *Electronic Journal of Linear Algebra* 16: 435–443 (2007)

[100] Recent developments and open problems in the theory of permanents, *The Mathematics Student* 76: 55–69 (2007)

[101] Determinantal divisor rank of an integral matrix, *International Journal of Information and System Sciences* 4(1): 8–14 (2008)

[102] (with Somit Gupta) Resistance matrices of blocks in a graph, *AKCE International Journal of Graphs and Combinatorics* 5(1): 35–45 (2008)

[103] On some classes of balanced games, *Mathematical Programming and Game Theory for Decision Making,* Statistical Science and Interdisciplinary Research, Vol. 1, Ed. S K Neogy et al, World Scientific, Singapore (2008) 391–399.

[104] (with Kinkar Chandra Das) A sharp upper bound on the spectral radius of weighted graphs, *Discrete Mathematics* 308(15): 3180–3186 (2008)

[105] (with A K Lal and Sukanta Pati) Laplacian spectrum of weakly quasi–threshold graphs, *Graphs and Combinatorics* 24(4): 273–290 (2008)

[106] Resistance matrix and q-Laplacian of a unicyclic graph, *Discrete Mathematics,* Ramanujan Mathematical Society Lecture Notes Series, Number 7, Proceedings of ICDM 2006, R Balakrishnan, E Veni Madhavan Eds., 63–72, 2008

[107] Sperner's lemma with multiple labels, *Modeling, Computation and Optimization,* Statistical Science and Interdisciplinary research, Vol.6, S K Neogy et al Eds., World Scientific, Singapore (2009) 257–262

[108] (with A K Lal and Sukanta Pati) The distance matrix of a bidirected tree, *Electronic Journal of Linear Algebra* 18: 233–245 (2009)

[109] (with Pritha Rekhi) Inverses of q-distance matrices of a tree, *Linear Algebra and Its Applications* 431: 1932–1939 (2009)

[110] (with Somit Gupta) Resistance distance in wheels and fans, *Indian Journal of Pure and Applied Mathematics,* INSA Platinum Jubilee Special Issue 14(1): 1–13 (2010)

[111] On a conjecture concerning spanning tree invariants and loop systems, *Linear Algebra and Its Applications* 433: 1642–1645 (2010)

[112] Least-squares approximation by a tree distance, *Contributions to Discrete Mathematics* 5(2): 185–194 (2010)

[113] (with K Reji Kumar and Gary MacGillivray) Topological properties of the set of all minimal total dominating functions of a graph, *J. Combin. Math. Combin. Comput.* 76: 213–223 (2011)

[114] (with S Sivasubramanian) Identities for minors of the Laplacian, resistance and distance matrices, *Linear Algebra and Its Applications* 435: 1479–1489 (2011)

[115] On the first passage time of a simple random walk on a tree, *Statistics and Probability Letters* 81: 1552–1558 (2011)

[116] (with S Sivasubramanian) Inverse of the distance matrix of a block graph, *Linear and Multilinear Algebra* 59(12): 1393–1397 (2011)

[117] A note on singular line graphs, *Bulletin of Kerala Mathematics Association* 8(2) 207–209 (2011)

[118] (with D Kalita and S Pati) On weighted directed graphs, *Linear Algebra and Its Applications* 436: 99–111 (2012)

[119] (with S Sivasubramanian) The product distance matrix of a tree and a bivariate zeta function of a graph, *Electronic Journal of Linear Algebra* 23: 275–286 (2012)

[120] (with A K Lal and S Pati) On algebraic connectivity of graphs with at most two points of articulation in each block, *Linear and Multilinear Algebra* 60(4): 415–432 (2012)

[121] (with S Sivasubramanian) The third immanant of q-Laplacian matrices of trees and Laplacians of regular graphs, *Combinatorial Matrix Theory and Generalized Inverses of matrices*, R B Bapat, S Kirkland, K M Prasad and S Puntanen Eds., Springer (2013) 33–40.

[122] (with P Chebotarev and R Balaji) Simple expressions for the long walk distance, *Linear Algebra and Its Applications* 439: 893–898 (2013)

[123] On the adjacency matrix of a threshold graph, *Linear Algebra and Its Applications* 439: 3008–3015 (2013)

[124] On minors of the compound matrix of a Laplacian, *Linear Algebra and Its Applications* 439: 3378–3386 (2013)

[125] (with S Sivasubramanian) Product distance matrix of a graph and squared distance matrix of a tree, *Applicable Analysis and Discrete Mathematics* 7: 285–301 (2013)

[126] (with Souvik Roy) On the adjacency matrix of a block graph, *Linear and Multilinear Algebra* 62(3): 406–418 (2014)

[127] (with Ebrahim Ghorbani) Inverses of triangular matrices and bipartite graphs, *Linear Algebra and Its Applications*, to appear

[128] (with S Sivasubramanian) Product distance matrix of a tree with matrix weights, *Linear Algebra and Its Applications*, to appear

Other Publications

[1] Interpolating the determinantal and permanental Hadamard inequality (Research Problem), *Linear and Multilinear Algebra* 32: 335–337 (1991)

[2] (with R E Hartwig) A master of the row space and the column space, The mathematical work of Sujit Kumar Mitra, *Linear Algebra and Its Applications* 211: 5–14 (1994)

[3] Matrices, matchings and matroids, *Applied Abstract Algebra,* 70–77, S K Jain Ed., Centre for Professional Development in Higher Education, University of Delhi, Delhi, (1996)

[4] (with Marianne Akian and Stéphane Gaubert) Asymptotics of the Perron eigenvalue and eigenvector using max-algebra, *C. R. Acad. Sci. Paris*, t 327, Série 1: 927–932 (1998)

[5] Largest two entries in a row and column, *Classroom* Section, *Resonance*, 5(2): 68–71 (2000)

[6] Eigenanalysis, Combinatorial Matrix Theory in *Handbook of Discrete and Combinatorial Mathematics*, Kenneth H Rosen Ed., CRC Press, (2000) 405–426

[7] Existence of generalized inverse: Ten proofs and some remarks, *Resonance* 6(12): 19–28 (2001)

[8] Extensions of König-Egerváry theorem to matroids and bimatroids, *Combinatorial Optimization*, M M Shikare and B N Waphare Eds., Narosa (2004) 207–225

[9] (with Marianne Akian and Stéphane Gaubert) Max-plus algebra, *Handbook of Linear Algebra,* Leslie Hogben Ed., Chapman and Hall/CRC, New York (2007) 25-1–25-7

[10] Exploring mathematical ideas with a deck of cards, *Resonance* 12(3): 77–90 (2007)

[11] *Mathematical Programming and Game Theory for Decision Making,* Statistical Science and Interdisciplinary research, Vol.1, S K Neogy, R B Bapat, A K Das Eds., T Parthasarathy, World Scientific, Singapore, 2008

[12] *Modeling, Computation and Optimization,* Statistical Science and Interdisciplinary research, Vol.6, Ed. S K Neogy, A K Das and R B Bapat, World Scientific, Singapore, 2009

[13] Challenges and frustrations of being a mathematician, Presidential Address, Indian Mathematical Society Annual Conference, Pune, *The Mathematics Student* 76: 49–54 (2007)

[14] Permanents in Probability Theory, *International Encyclopedia of Statistical Science*, Miodrag Lovric, Ed., Springer, Part 16: 1058–1060, 2011

[15] (with R Balaji and P Chebotarev) Simple expressions for the long walk distance, http://arxiv.org/pdf/1112.0088.pdf

[16] *Lectures on matrix and graph methods*, Ed. R B Bapat, S Kirkland, K M Prasad and S Puntanen, Manipal University Press, 2012

[17] *Combinatorial Matrix Theory and Generalized Inverses of matrices*, R B Bapat, S Kirkland, K M Prasad and S Puntanen Eds., Springer, 2013

Books

[1] Linear Algebra and Linear Models, Third Edition, Hindustan Book Agency, Delhi, *TRIM* series and Springer, *Universitext*, 2012 (First Edition, Hindustan Book Agency, 1993.)

[2] (with T E S Raghavan) Nonnegative Matrices and Applications, Encyclopedia of Mathematical Sciences, Vol. 64, Cambridge University Press, 1997.

[3] Aisi Prameye Rasike (in Marathi), Rajhans Prakashan, Pune, 2004.

[4] Graphs and Matrices, Second Edition, Hindustan Book Agency, Delhi, *TRIM* series and Springer, *Universitext*, 2014.

Challenges and Frustration of being Mathematician[1]

Ravindra B Bapat

India Statistical Institute
7, SJS Sansanwal Marg
New Delhi 110 016 email: rbb@isid.ac.in

Abstract

The article is the contents of Professor Bapat's presidential address (General) in the '*Indian Mathematical Society Annual Conference*' held at 'University of Pune' during December, 2007.

It is a great honor and pleasure for me to have had this opportunity of addressing this distinguished gathering. I express my gratitude to the members of the Indian Mathematical Society for having elected me to the office of the President, in the centenary year of the Society. I am also thankful to the local organizers for undertaking a task of such magnitude towards a successful and productive conference.

As mathematicians, we belong to a minority. Most people are turned off by mathematics. Very few have some liking for the subject and even a smaller number choose it as a profession. In this talk I have tried to give way to some feelings about the nature of our profession and the problems that we face. We all agree that it is a pleasure to do mathematics and so I dwell mostly on things that tend to obstruct that pleasure. The views expressed here have evolved largely out of my personal experiences and perspectives. It is hoped that they might induce a discussion which might result in further inputs and exchange of ideas. The tone of the writing is definitive, but since it is meant to generate a debate, it is justified.

We are mathematicians by choice. We chose the profession because we love the subject. Reading and assimilating deep results of masters and then solving some of our own small problems brings us pleasure to which nothing else compares much. Yet we live in a world populated mostly be nonmathematicians. We must survive and thrive in their midst. This brings forth its own challenges and frustrations.

Our professional activity is divided mainly into teaching and research, except for some administrative duties. A lot has been said about mathematics education and I will confine myself to a few comments. It is a fact that most of us like to do our own thing. We enjoy teaching if it is a course of our choice and the class consists of a few eager, motivated, well-behaved students. That is only a dream. Often we must teach large classes of uninterested students who are there only for completing the requirements. But in spite of all this we must strive to teach, giving it our best and at the same time maintaining the standard of our subject. Compromises have no place here. We believe in teaching in a certain way and

[1]Reproduced from 'The Mathematics Student, 76(1–4) 49–54 (2007) with permission from IMS'

it can be fine-tuned depending upon the reactions of the students. But it should not prevent us from communicating the basic spirit of mathematics, especially the importance of logical enquiry. All aspects of mathematics, including history, biographies, motivation, definitions, lemmas, theorems, corollaries, proofs, examples, counterexamples, conjectures, construction, computation and applications can and should find a place in the classroom.

The views expressed in this talk pertain to mathematics in general and not particularly to the Indian context. But with special reference to the situation in India it must be remarked that the bifurcation of undergraduate teaching and research has not served us well. Our best researchers are concentrated in research institutes and do not teach undergraduate courses in mathematics. This is one of the reasons for a steady decline in the quality of undergraduate mathematics education in the country.

Examinations and tests are a major part of the teaching process and let me say something about them. I find that in physics, students are routinely asked questions which are not in any of their prescribed books and require some extra application of thought. In mathematics however, we are supposed to stick to routine questions, except in olympiad type exams. This is true at the high school board examinations level as well as the college level. Submitting to this requirement makes the subject dull for gifted students. Examination boards need to be persuaded to consider providing for ten percent marks for nonroutine questions, without changing the syllabus. We will then continue to have students securing 100 percent marks but that will mean something much more than in the present system.

Mathematics has been projected as a dry and difficult subject. Being poor in mathematics is considered natural and in fashion, whereas being good in mathematics is taken as being eccentric and queer. This perception has been created by all, including press and popular media. Television interviews of celebrities invariably include the I was awful in maths, just hated it ... bit. All this has its effect on students. The forces that are at work pulling talented students away from mathematics and towards other sciences are too powerful and the only thing that works in favour of mathematics is the enjoyment it provides to some of the students who cannot think of doing anything else. Why cant mathematics be both enjoyable and fashionable, the in-thing, to do? We are repeatedly asked to provide real life examples, motivation, while teaching or while lecturing to a general audience. This is justified to some extent, but mathematics manifests its beauty only when it is stripped off of its worldly connections. It flourishes in the abstract and then again turns messy when brought back to apply. The messy part is the one that engineers and scientists (those who use the mathematics) are to deal with. Why should a mathematician be required to constantly make this back and forth transition? To protect our interests let us make this feeling known, after debating it among ourselves.

Now I turn to the second component of our job, research. Writing our work and then managing to get it published is an important part of our profession. In a subject that is nearly two thousand years old, and in areas that are more than two hundred years old, getting a drop of something original is not easy. The situation is perhaps different in experimental sciences. But a comparison of publication record with other sciences is always made for all policy decisions. The recent debate about the inappropriate use of citation index in mathematics is a case in point. This leads to many ills of our profession. There is too much pressure to publish, quality suffers in the process. Refereeing is a challenging task with no apparent reward except a feeling of satisfaction towards contributing to the health of your area. It is nearly impossible to track all that is published and that results in further narrowing of ones interests.

In this context I wish to propose a scheme, which addresses the question of decreasing the number of publications to some extent. There can be free, possibly electronic, journals, run by well-established societies or academies, in which all papers are by invitation only. Papers must be refereed but the role of the referee must be limited mainly to checking for correctness, style etc. To fix ideas, suppose one such journal is called The Free Journal of Combinatorics. The job of the editorial board of this journal would be to identify promising mathematicians in Combinatorics, at all levels, and invite them to contribute one paper to the journal per year for the next five years. The author in turn should agree to (i) submit the best of his work to the journal and (ii) to restrict his publications in other journals to either zero or a very small number. If the journal acquires enough prestige, then being an invited author of the journal will carry lot of value and then there is no need to publish more. Also in the course of a few years the Free Journal of Combinatorics will be a reflection of the best work in the area of Combinatorics and give a fair picture of the development of the subject.

I am proposing this scheme after giving it some thought from my personal perspective. I would welcome a situation where a reputed publisher will publish one paper of mine per year for a certain number of years. That will then contain the best I have to offer. Apart from that I can stick mainly to expository writing or survey papers. In any event it is true that barring exceptions, the average professional mathematician does not have enough research to report and generate more than one or two papers per year after the initial burst of activity has subsided. In practice however, the number increases since more publications means better salary and more grants, among other things.

The very idea of limiting the role of a referee may appear drastic. Mathematics is an art as much as it is a science. Can one imagine the painting of an accomplished painter being subjected to a refereeing process, before it is exhibited? And with all the stringent refereeing regimen in place in the present system, has it really eliminated erroneous papers or duplication of results? Let us recognize the best talent amongst us and invite them to write for us. There are invited papers at present but an invited paper often means a paper which the author would never care to publish otherwise.

Mathematicians, like other scientists, need support for their research and hence must write proposals for research grants. The mechanics of seeking research grants is designed by and suited to experimental scientists. A natural scientist wishes to propose a theory about an enzyme or a drug and must conduct experiments to test the claims. This requires some equipment, graduate students, site visits and these constitute the bulk of the proposal requirements. This process does not quite suit us mathematicians. We like to think about a problem, and at the same time let our thoughts wander in a random fashion. If something along the way catches our attention then we may follow that route. Thus, in reality, we cannot indeed write an honest research proposal which gives too many specifics about what we are going to achieve. If someone claims in a proposal that he or she is going to investigate bounds for the eigenvalues of a certain class of 0-1 matrices, very likely the proposer has already obtained some such bounds which will only be written up when the proposal is approved. There cannot be any other way. If I do not have any eigenvalue bounds already obtained with me, there is a chance I will not get any, even if a very big grant is awarded.

This special nature of mathematical research needs to be kept in mind by one and all, particularly the funding agencies. Past achievements can be given more weight and a sketchy proposal should not necessarily mean automatic rejection. Travel and short visiting appointments are a big attraction to mathematicians. If a mathematician can spend a few months in a place with all facilities, an intellectually stimulating atmosphere, and no teaching or administrative duties, then he can achieve wonderful results. Needs of a mathematician are meagre compared to that of an experimental scientist but they need to be addressed sensitively.

Even though outsiders may recognize one of us simply as a mathematician, within our community there are many subdivisions. One is not just a mathematician, he is a differential geometer or a quantum probabilist or a commutative ring theorist. Mathematics is neatly divided into areas: algebra, geometry, analysis, topology are some of the respected ones and there are many others which do not enjoy a similar standing. A research mathematician must make his or her area known and then should stick to it, if he/she doesnt want the professional career to suffer. Contacts must be developed in that particular area, journals should be identified, one must become known to the editorial board members and then life may be easier. Except that if you get bored with the same type of problems and want to be adventurous and venture into new territory, you better be first rate and adapt quickly, otherwise getting a foothold in the new area is not easy. So if your Ph.D. thesis has been in uniform bounded cohomology of sections of holomorphic vector bundles, twenty years later you would at best be venturing into the locally nonuniform case. More specialization has created further divisions among the dwindling number of mathematicians.

In ground reality, however mathematicians are indeed divided, but these divisions are of a different kind. There are those who enjoy teaching and mentoring students, researchers who excel in what they do but are hardly comfortable or efficient in a classroom, good expositors who can make a difficult subject look simple in their writing, people who enjoy organizing conferences; they dont mind if their own area is far removed from the area of the conference, good Ph.D. guides; their number of students is in double digits in a short time, those who are interested in foundations, those who like to write proposals and have several projects to their credit simultaneously, those who like to chair departments and so on.

Can we recognize this classification and take it into account in our decision making, rather than the narrow area-wise breakup? Some of our sister areas, notably Economics, are devoid of the sort of tight division that we mathematicians have imposed upon ourselves.

Now, I come to a topic where mathematicians are not really at fault, but it is made to appear that they are doing something wrong, or rather, not doing something right. And here I am referring to the task of communicating our work to others, especially to a general audience. Explaining mathematics to non-mathematicians, even educated ones, is an ordeal. Our subject does not lend itself to explanation to nonspecialist, period. However most people, including mathematicians, believe that it is a shortcoming of mathematicians that they are not able to explain there subject to others. Explaining to a layman (an educated one) what a group means, appears interesting - talk about geometrical figures, rotations, reflections and so on. But that is just the beginning. Can we get to normal subgroups and still retain the same clarity? And then what about trivial torsion units in G-adapted group rings?

A recent book review by Daniel Bliss in the Notices of the AMS (June/July 2007) brings forth this dilemma in a very interesting fashion. I recommend reading it in original, but the highlight is that the author (John Stillwell) of the book under review (Yearning for the Impossible, A.K.Peters Limited, 2006) tries very sincerely and very hard to explain what an ideal is, develops the concept very patiently and finally has an explanation. But at the end of it one wonders whether it is worth the trouble. Why cant we be frank and say that, look, these concepts are really very abstract and cannot be explained but then it is also required that we justify why we are doing what we do. In any event whatever good nontechnical exposition of mathematics has been achieved has remained confined to a few areas, notably discreet mathematics, where more ground can be covered. Even the great Martin Gardner has had to restrict himself in terms of range of topics. But then we are not conveying a true picture of the vastness and depth of mathematics.

This difficulty is also faced by other areas of arts and science. But it is interesting to see how they get around the problem. We are made to believe that physics, chemistry or biology are easier to explain to the general audience. Nothing can be farther from the truth but this belief is implanted successfully in our minds by the way scientists in these areas present their work. Their abstract concepts are presented in a way as if they are actual realities. Atoms, quarks, dark energy, strings, black holes, are all concepts but people believe in them. In comparison mathematicians are awfully shy of presenting anything for which they do not have a refereed proof. We need to be bold. Our ongoing investigations should be presented with a passion and sense of self-belief. Our deceptively simple terminology is also a culprit. How can anyone believe that simple sounding concepts like group, ring and field can have anything deep to connote?

I have tried to present some views about our profession and as remarked in the beginning, it will be helpful if it generates more exchange of ideas. Our subject, known as the Queen of Sciences and one with a long history, is losing its image in the eyes of the young student, policy makers and general public. Riding on the wave of computer science does not solve the problem, since the nature of mathematics is unique and computer science is at best a glimpse into a small portion of it. We must strive to effectively communicate the unique nature of our subject, its beauty as perceived by us, and its applications to the betterment of life, to the layman, as well as scientists and policy makers in order to create a positive feeling towards our profession.

I conclude by wishing all the members and delegates a very fruitful and memorable conference.