Department of Statistics
VI Floor, Library Building
Manipal University
Manipal, Karnataka, INDIA 576104.

International Workshop and Conference on Combinatorial Matrix Theory and Generalized Inverses of Matrices

02 - 07 January 2012 & 10 - 11 January 2012

Venue: Innovation Center Building MIT Campus Manipal.

Message

Dr. Ramdas M Pai

PRESIDENT & CHANCELLOR

I am glad the department of Statistics of our University has organized an International Workshop and Conference to deliberate at length on the applications of Combinatorial Matrix Theory in di erent branches of modern science. I am sure, the exchange of thoughts and opinions among the participants will enliven the proceedings. I send my best wishes for the success of both the programmes.

Dr. H.S. Ballal

PRO CHANCELLOR

Generalized Inverses of matrices and Matrix Theory have embraced a wide range of scientific advances resulting new innovations. It is good that experts in this field are able to come together at Workshop and Conference organized by our University Department of Statistics. The discussion and interactions, I have no doubt, will lead to new procedures and formulae to benefit the advancement of the Matrix Theory. My best wishes to all the participants for a successful get together.

Dr. K. Ramnarayan

VICE CHANCELLOR

It is good to see Department of Statistics is conducting an International Workshop and Conference on Combinatorial Matrix Theory and Generalized Inverses of Matrices in January 2012. The Department of Statistics in a very short span of time has bloomed into a full-fledged department o ering courses ranging from certificate courses to doctoral programs in statistics.

It is important that researchers, academicians are abreast of the latest techniques and developments in this area. This conference will give a rare opportunity to meet and listen to the statisticians and mathematicians of international repute. I am certain that researchers, faculty and students of not only Manipal University, but also from across the country will take advantage of this programme.

I wish the organizers of this conference all the very best for the success of the program and wish the participants a fruitful and comfortable stay in Manipal.

Dr. H. Vinod Bhat

PRO VICE-CHANCELLOR

Department of Statistics, Manipal University is hosting an International Workshop & Conference on Combinatorial Matrix Theory and Generalized Inverses of Matrices. I laud the department taking the bold first step in setting up a division of Mathematics in the University.

The division of Physics in Manipal center for Natural Sciences is already operational. Mathematics will soon follow. The center is actively considering oceanography, natural history and archeology as other streams that will be added on.

I welcome the delegate of the workshop and conference to this knowledge town. I wish everyone the very best.

Dr. G.K. Prabhu

REGISTRAR

International workshop and conferences play a great role in bringing scholars from all over the world leading to chains of collaborations. It gives me a great pleasure to note that Department of Statistics is hosting events - "International workshop and conference on combinatorial matrix theory and generalized inverses of matrices" at Manipal in January 2012.

I wish the department and organizers every success in hosting these events. I take pleasure in welcoming all the delegates participating in these events. We hope these workshop and conference will fetch some newer perceptions in the field of Mathematics and Statistics leading enormous progress in research.

Dr. N. Sreekumarn Nair

Professor & Head, Department of Statistics

Dear Participant, we are delighted to have you with us during the international workshop and conference on "Combinatorial Matrix Theory and Generalized Inverses of Matrices. During the last five years of existence, the department of statistics has initiated a Masters programme in Biostatistics and another certificate programme in Epidemiology and Biostatistics. The department faculty regularly conducts workshops and conferences at Regional, National and International level. Department has collaboration with National and International agencies and Industries. You are welcome to the department and please feel free to visit us. We the faculty, sta s and students of department of Statistics, Manipal University wish you a very warm welcome, happy stay and fruitful deliberations.

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Our sincere thanks to

International Linear Algebra Society(http://www.ilasic.math.uregina.ca/iic/) for endorsing this events.

Society for Industrial and Applied Mathematics(http://www.siam.org/) for being cooperating society. Professor S Kirkland is the SIAM representative for the events.

The Abdus Salam International Center for Theoretical Physics(http://www.ictp.it/) for supporting us by awarding travel grant for selected speakers.

The speakers nominated for partial support of ICTP travel grant are

1. Prof. Yimin Wie, Fudan Univesity, Shanghai, China.
2. Prof. Md. Maruf Ahmed, BRAC University, Dhaka, Bangladesh.
3. Prof. Oskar Maria Baksalary, Adam Mickiewicz University, Poland.
4. Prof. Om Prakash Niraula, Tribhuvan University, Kathmandu, Nepal.

We register our sincere thanks to

Central Statistical Office, Ministry of Statistics and Programme Implementation (http://www.mospi.nic.in) for supporting us by awarding grant-in-aid.

Council of Scientific and Industrial Research (http://www.csir.res.in) for supporting us by awarding separate grant-in-aid for workshop and conference.

National Board for Higher Mathematics (http://www.nbhm.dae.gov.in) for supporting us by awarding grant-in-aid.

Preface

Professor Ravindra B Bapat

Indian Statistical Institute, Delhi.
CHAIRMAN - Scientific Advisory Committee, CMTGIM

Generalized inverses of matrices and combinatorial matrix theory are among areas of matrix theory with a strong theoretical component as well as with applications in diverse areas that have seen rapid advances in recent years. Significant contribution to generalized inverses has come from the Indian school including luminaries such as C.R. Rao, S.K. Mitra and C.G. Khatri. Incidentally, the eightieth birthday of Professor S.K. Mitra falls in the current month on January 23, 2012. The interaction between graph theory and matrix theory is known to be very fruitful and example abound where generalized inverses of matrices arising from graphs play an important role. Just to give one example, an expression for the resistance distance between two vertices in a graph in terms of a generalized inverse of the Laplacian matrix is a key tool. It is hoped that the six-day Workshop and the two-day Conference will provide an ideal opportunity for participants from India and abroad to interact and exchange ideas in pleasant surroundings. I extend a warm welcome to all the participants, invited speakers and students to the program.

Professor K. Manjunatha Prasad

Department of Statistics, Manipal.
Organizing Secretary, CMTGIM

It is believed in India that the divine power, to be used for the welfare of the society, is obtained from "Agni" (fire) and "Surya"(Sun) through "Yaga". In the modern days, the knowledge is the divine power and this could only be attained through meeting the scholarly people and learn from them. The workshop and conferences are the platforms (yaga of present days) of di erent level, where we can bring scholars from di erent places together and enhance the knowl-

edge power of each individual participated. I thank University for providing an opportunity for taking an initiation of this kind in the area of "Combinatorial Matrix Theory and Generalized Inverse of Matrices" at Department of Statistics, Manipal. When Professors such as Bapat, Styan and Puntanen are available as advisors for the conduct of events and with administrative support from University, the success was partially assured. I am delighted to the response received from Mathematicians and Statisticians, to our invitation. Though, few among participants have to drop their plan of participation due to their academic responsibility at their organizations, we remain with about 30 invited speakers, about 15 speakers presenting contributed papers and about 120 participants altogether. Many speakers and other participants from abroad and national institutes have utilized their research fund to come here and participate, I thank the organizations supported them.

Financial support is very essential for the success of any event. International Centre for Theoretical Physics (ICTP), Council of Scientific and Industrial Research (CSIR), National Board for Higher Mathematics (NBHM) and Ministry of Statistics and Programme Implementation (MOSPI) have come forward with generous grant. Discussions with Dr. K. Jayakumar of CSIR, Dr. N. Ekhambaram of CSO, Shri V.V. Bhat, IAS of Govt. of India and Dr. S. Ganesan of BARC were morale booster. When we had an apprehension, once regarding the amount we may receive from the national agencies, many of my **colleagues and friends** *have approached social organizations for the possible support. I thank all individuals and organizations for evaluating the programme and come forward in support.*

We have planned for bringing out "Proceeding of the Conference" through 'Springer' and the "Lecture Notes" of the workshop through 'Manipal University Press'. I thank all the speakers providing a quality material for these issues.

The 'Knowledge Town' - Manipal is with pleasant and beautiful surrounding and six-day Workshop and the two-day Conference will provide an ideal opportunity for participants from India and abroad to interact and exchange ideas. I extend a warm welcome to all the participants, invited speakers and students to the program and we assure of our e orts in making these events very successful with cooperation of all participants, colleagues and university.

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Committees for CMTGIM, January 2012

Patrons:

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Dr. H.S. Ballal (Pro Chancellor, Manipal University

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Souvenir & Lecture Notes:

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Mr. Vinay Madhusudanan (Department of Mathematics, MIT)
Mr. K.S. Mohana (Department of Mathematics, MIT)
Ms. H.S. Sujatha (Department of Mathematics, MIT)

International Workshop on Combinatorial Matrix Theory and Generalized Inverses of Matrices
02 - 07 January 2012

List of speakers in the workshop:

1. Oskar Maria Baksalary, *Adam Mickiewicz University, Poland.*
2. R. Balakrishnan, *Bharathidasan University, Tiruchirapalli.*
3. Rajendra Bhatia, *Indian Statistical Institute, New Delhi.*
4. Jeffrey J Hunter, *Auckland University of Technology, New Zealand.*
5. Steve S Kirkland, *Hamilton Institute, Ireland.*
6. Bhaskara Rao Kopparty, *Indiana State University, USA.*
7. Sukanta Pati, *Indian Institute of Technology, Guwahati.*
8. Simo Puntanen, *University of Tampere, Finland.*
9. T.E.S. Raghavan, *University of Illinois at Chicago, USA.*
10. Sivaramakrishnan Sivasubramanian, *Indian Institute of Technology, Bombay.*
11. George PH Styan, *McGill University, Canada.*
12. Götz Trenkler, *Technische Universitat Dortmund, Germany.*
13. Yimin Wei, *Fudan University, China.*
14. Hans Joachim Werner, *University of Bonn, Germany.*

Hyderabad 2000 - Manipal 2012 - Over 11 years with Projectors

Oskar Maria Baksalary

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Abstract

In December 2000 I participated in The Ninth International Workshop on Matrices and Statistics which was held in Hyderabad, India, and was devoted to the celebration of the 80th birthday of C.R. Rao. Within the student session organized at the meeting I gave a talk entitled "Idempotency of linear combinations of idempotent and tripotent matrices", which was inspired by a joint research with my father, Jerzy K. Baksalary. It was also my father who introduced me to another participant of the workshop, namely Götz Trenkler. During over 11 years following the meeting in Hyderabad an essential part of my research activities was focused on idempotent matrices (i.e., projectors), and I enjoyed working on them jointly with two excellent collaborators, Jerzy K. Baksalary and Götz Trenkler. The talk will review some of the results obtained during these years. Particular attention will be paid to linear combinations, eigenvalues, and commutativity of projectors as well as properties of generalized and hypergeneralized projectors.

The talk is based on joint research with Jerzy K. Baksalary and Götz Trenkler.

Keywords: g-inverse, Projector.

Spectral Properties of Graphs

R. Balakrishnan

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Abstract

The spectrum of a simple graph is the set of eigenvalues of its adjacency matrix. In these lectures, we discuss the spectral properties of graphs. As an application, we deal with the energy of a graph, a graph parameter that has close links to chemistry.

Keywords: Adjacency Matrix, Eigenvalue, Energy of Graph.

Matrix Inequalities

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Abstract

Inequalities, like the arithmetic-geometric mean inequality and the Cauchy-Schwartz inequality, have long been used in almost all areas of mathematics. Their matrix versions have been discovered in the last few years, and are equally important in several subjects. In this talk we will illustrate some general principles that have been found to be very useful in deriving such inequalities.
We will focus on two themes. The first is the arithmetic-geometric mean inequality

$$\|A^{\overline{2}} B^{\overline{2}}\| \leq \frac{1}{2}(A+B),$$

where A and B are positive semidefinite matrices. The second is the problem of estimating $\|f(A) \quad f(B)\|$ in terms of $\|A \quad B\|$ for functions of matrices. Both problems have common ingredients, and both have generalizations that make them more attractive as well as useful.

Keywords: Matrix inequalities, Positive definite matrix, Arithmetic-geometric mean, Perturbation bound.

Generalized Matrix Inverses and their use in Applied Probability

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Abstract

In many stochastic models, in particular Markov chains in discrete or continuous time and Markov renewal processes, a Markov chain is present either directly or indirectly through some form of embedding. The analysis of many problems of interest associated with these models, e.g. stationary distributions, moments of first passage time distributions and moments of occupation time random variables, often concerns the solution of a system of linear equations involving $I \quad P$, where P is the transition matrix of a finite, irreducible, discrete time Markov chain. Generalized inverses play an

important role in the solution of such singular sets of equations. In this talk we survey the application of generalized matrix inverses to the aforementioned problems.

Keywords: generalized inverse, Markov renewal process, Markov Chain, Transition matrix

Graph Structure Revealed by Spectral Graph Theoretic Methods

Steve S Kirkland - SIAM Representative

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Abstract

Suppose that we are given a simple undirected graph G on vertices labelled $1, 2, 3, ..., n$, and for each $i = 1, 2, 3, ..., n$, let d_i be the degree of vertex i. From G we may form the following two matrices:

1) the *Laplacian matrix* L whose entries are given by

$$L_{ij} = \begin{cases} \quad 1 & \text{if } i \sim j, \\ 0 & \text{if } i \neq j \text{ and } i \quad\ j, \\ d_i & \text{if } i = j; \end{cases}$$

2) the *signless Laplacian matrix* Q whose entries are given by

$$Q_{ij} = \begin{cases} 1 & \text{if } i \sim j, \\ 0 & \text{if } i \neq j \text{ and } i \quad\ j \\ d_i & \text{if } i = j. \end{cases}$$

In this talk we will give an introductory overview of how certain eigenvalues and eigenvectors of the Laplacian matrix and the signless Laplacian matrix can help to reveal structural properties of a graph. Specifically, we will focus on the second smallest eigenvalue of the Laplacian matrix, and present a number of results that reveal the connection between that eigenvalue and the connectivity properties of the graph. We will also consider the smallest eigenvalue of the signless Laplacian matrix, and discuss its role as a measure of bipartiteness of the graph.

Keywords: Laplacian Matrix, Connectivity of graph, Bipartite graph

An Overview of Generalized Inverses of Matrices Over Rings

Bhaskara Rao Kopparty

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Abstract

To be received.

Algebraic Connectivity of Graphs

Sukanta Pati

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Abstract

The lectures are providing an introduction to the Laplacian matrix of a graph. It aims to explain some of the basic results and techniques related to 'algebraic connectivity', a measure of connectivity of a graph based on the Laplacian matrix.

Matrix Trix for Statistical Model

Simo Puntanen

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Abstract

In teaching linear statistical models to first-year graduate students or to final-year undergraduate students there is no way to proceed smoothly without matrices and related concepts of linear algebra; their use is really essential. Our experience is that making some particular matrix tricks very familiar to students can substantially increase their insight into linear statistical models (and also multivariate statistical analysis). In matrix algebra, there are handy, sometimes even very simple "tricks" which simplify and clarify the treatment of a problem both for the student and for the professor. Of course,

the concept of a *trick* is not uniquely defined by a trick we simply mean here a useful important handy result.

Reference:

1. Puntanen, Simo; Styan, George P. H. & Isotalo, Jarkko (2011). *Matrix Tricks for Linear Statistical Models: Our Personal Top Twenty.* Springer. http://www.springer.com/statistics/statistical+theory+and+methods/book/978-3-642-10472-5

Introduction to Graph Theoretic Applications to Computing the Nucleolus of an Assignment Game

T.E.S. Raghavan

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Abstract

Computing the nucleolus for assignment games is a major algorithm that involves, facets of matrix theory from the point of view of permanents, doubly stochastic matrices, Frobenius Konig Theorem and graphs with many vertices and directed arcs that slowly shrink to a single vertex by a suitable equivalence relation all lead to the important solution to the problem of assignment. The game theoretic solution resolves the split between the sellers of houses and buyers of houses where sellers try to get the maximum out of their houses and buyres get the best deal from the best among the sellers and the real estate agent cashes on the total gains. The secrecy of the seller and buyer before the final sale takes place is crucial for the nucleolus split.

Keywords: Assignment game, Nucleolus, Permanents, Frobenius Konig Theorem.

Reference:

1. L. S. Shapley and M. Shubik, *Assignment gamesInternational* J. Game Theory (1971)

2. Tamas Solymosi and TES Raghavan, *An algorithm for locating the nucleolus for assignment games*, International J. Game Theory (1994)

3. R.B. Bapat and TES Raghavan, *Nonnegative Matrices and Applications*, Paperback edition, Cambridge University Press (2009).

Applications of the Laplacian Matrix of Graphs

Sivaramakrishnan Sivasubramanian

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Abstract

Let G be a connected graph with n vertices and κ spanning trees. Let L be its Laplacian matrix. Let $L(v)$ be the $(n \quad 1) \times (n \quad 1)$ sub matrix of L obtained by deleting the row and the column corresponding to vertex v.
The famous Matrix Tree Theorem says that $\det(L(v)) = \kappa$. It is known that the matrix L is singular. We generalize the matrix L by changing its entries to have polynomials in one variable q. This matrix $\mathcal{L}_q$ with polynomials is such that when $q = 1$, we get $\mathcal{L}_q = L$. Further, $\mathcal{L}_q$ is non-singular.
Thus, we have a polynomial generalization of the Laplacian matrix of connected graphs. This matrix $\mathcal{L}_q$ has very nice properties both when restricted to trees and otherwise. Some applications of $\mathcal{L}_q$ include

1. Connections to spanning trees
2. The Ihara zeta function where one enumerates prime and reduced cycles in a connected graph.
3. Relations between minors of $\mathcal{L}_q$ and the exponential distance matrix $\mathsf{E}_{\quad T}$ of a tree T. For connected graphs, this becomes a relation between minors of $\mathcal{L}_q$ and its resistance matrix.
4. Counting the number of integer points in a zonotope related to distances arising from trees.

The first one-hour will be an introduction where we start from the basics and recall known facts about L, the Laplacian of a connected graph and then give some basic facts about $\mathcal{L}_q$. The second one-hour will focus only on $\mathcal{L}_q$ where we outline some of its interesting properties.

Yantra Magic Squares

George P.H. Styan

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Abstract

We study various properties of Yantra magic squares, i.e., magic squares associated with Yantras: a "Yantra" is a "geometrical diagram used like an icon usually in meditation". We begin with 3×3

magic squares depicted on “Planet Yantras”: Yantras associated with one of the 9 “planets”: (1) Sun/Surya, (2) Moon/Chandra, (3) Mars/Mangala, (4) Mercury/Budha, (5) Jupiter/BrihaspatiGuru, (6) Venus/Shukra (7) Saturn/Shani, (8) Rahu, and (9) Ketu. The lunar nodes Rahu and Ketu are the points where the moons path in the sky crosses the suns path. We also consider “Navagraha Yantras”: 3×3 arrangements of the nine Planet Yantras, usually with a (1) Sun/Surya Planet Yantra in the centre. Placing (5) Jupiter/BrihaspatiGuru Planet Yantra in the centre, we construct 8 Navagraha Yantras each in the form of a 9×9 composite fully-magic square.

We examine, in some detail, the magic matrices defined in the magic-square planetary talismans by

- Heinrich Cornelius Agrippa von Nettesheim (14861535),
- Gerolamo Cardano (15011576),
- Paracelsus [born Philippus Aureolus Theophrastus Bombastus von Hohenheim] (14931541),
- Athanasius Kircher (16021680).

We present a matrix algorithm, which we believe to be new, for generating “Agrippa magic matrices” A_n with odd n; and find that A_9 is the 9×9 Yantra magic square given at the Gemstone website. We construct a related 2-parameter family of 9×9 “ChandraGemstone Yantra magic squares”. We also present procedures which, using “magic-basis matrices”, generate classic rank-3 $n \times n$ “AgrippaMatlab magic matrices” with n doubly-even, and classic nonsingular “Agrippa Fermat magic matrices” with n singly-even.

A three-parameter family of 4×4 magic squares is constructed from the “personal Yantra magic square” presented by Richard Webster in his Numerology Magic. In addition we consider magic knights tours, with special reference to the tour created in 1852 by Krishnaraj Wadiar, Rajah of Mysore (17901868), and to recent results by Awani Kumar.

We continue our report by studying a new four-parameter general form for a 4×4 most-perfect pandiagonal fully-magic (MPPM) matrix [151, Th. 2] which allows us to find simple conditions for a keyed 4×4 MPPM matrix to be EP or bi-EP. We introduce the concepts of EP-, bi-EP-, and bP-multipliers to measure how far a magic matrix is from being, respectively, EP, bi-EP or bP (bi-EP but not EP). We also look at Schur complements in magic matrices and obtain several results which we believe are new.

We have tried to illustrate our findings as much as possible, and whenever feasible with images of postage stamps or other philatelic items. We end our report with an extensive bibliography of over 100 references, many with hyperlinks.

Keywords: Magic Square, Yantra, Schur Compliment.

Projectors in the Linear Regression Model

Götz Trenkler

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Abstract

First we give an introduction to the most important properties of oblique and orthogonal projectors. The second and main part is dedicated to the application of projectors in the linear regression model. The following topics will be presented:

- BLUE vs OLSE,
- Residual analysis (outliers, correlation structure),
- Prediction,
- Aggregation,
- Instrumental variables,
- Estimation of disturbance variance,
- F-test,
- Estimation under restrictions,
- Coefficients of determination,
- Mean square error comparisons of estimators,
- Analysis of multicollinearity,
- Pretesting,
- Proxy variables,
- Analysis of misspecification (over- and underfitting, wrong model).

Keywords: g-inverse, Linear Regression, Regression Model

Solving Singular Linear Equations and Generalized Inverses

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Abstract

In this lecture, we will present some recent results on solving singular linear equations and generalized inverses.

Keywords: Linear equation, Singularity, g-inverse.

G-Inverses, Projectors and the General Gauß-Markov Model

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Abstract

In the framework of the general (possibly singular) Gauß-Markov model, we will review some prominent estimation and prediction procedures. We will investigate their basic properties, explain the motivations behind, and discuss the connections between them. Most observations may be obtained by employing rather elementary, yet powerful, concepts from matrix algebra, such as g-inverses and projectors.

Keywords: BLUE, BLIMBE, BLUP, BLIMBIP, Gauß-Markov model, g-inverse, η-inverse, GLS, projector.

International Conference
on
Combinatorial Matrix Theory and Generalized Inverses of Matrices
10 - 11 January 2012

List of speakers presenting invited talk in the conference:

1. Rafikul Alam, *Indian Institute of Technology Guwahati.*
2. Oskar Maria Baksalary, *Adam Mickiewicz University, Poland.*
3. R. Balakrishnan, *Bharathidasan University, Tiruchirapalli.*
4. Rajendra Bhatia, *Indian Statistical Institute, New Delhi.*
5. P. Bhimasankaram, *Indian School of Business, Hyderabad.*
6. Francisco Carvalho, *Polytechnic Institute of Tomar, Portugal*
7. N. Ekambaram, *Ministry of Statistics and Program implementation New Delhi.*
8. Jeffrey Joseph Hunter, *Auckland University of Technology, New Zealand.*
9. Surender Kumar Jain, *Ohio University, USA*
10. Steve S Kirkland, *Hamilton Institute, Ireland.*
11. Tõnu Kollo, *University of Tartu, Estonia.*
12. Bhaskara Rao Kopparty, *Indiana State University, USA.*
13. S.H. Kulkarni, *Indian Institute of Technology, Madras.*
14. Augustyn Markiewicz, *Poznan University of Life Sciences, Poland.*
15. A. R. Meenakshi, *Annamalai University, Chennai.*
16. Simo Puntanen, *University of Tampere, Finland.*
17. TES Raghavan, *University of Illinois at Chicago, USA.*
18. Sharad S Sane, *Indian Institute of Technology, Bombay.*
19. K.C. Sivakumar, *Indian Institute of Technology, Madras.*
20. Sivaramakrishnan Sivasubramanian, *Indian Institute of Technology, Bombay.*
21. Murali Srinivasan, *Indian Institute of Technology, Bombay.*

22. George PH Styan, *McGill University, Canada.*
23. Götz Trenkler, *Technische Universitat Dortmund, Germany.*
24. Yimin Wei, *Fudan University, China.*
25. Hans Joachim Werner, *University of Bonn, Germany.*

List of speakers presenting contributed paper in the conference:

1. Mohammad Maruf Ahmed, *BRAC University, Dhaka.*
2. A. Anuradha, *Bharathidasan University, Tiruchirapalli.*
3. Ravi Shankar Bhat, *Manipal Institute of Technology, Manipal.*
4. Sachindranath Jayaraman, *Indian Institute of Science Education and Research, Kolkata*
5. Debajit Kalita, *Indian Institute of Technology, Guwahati.*
6. K. Kamaraj, *University College of Engineering, Arni.*
7. Kuncham Syam Prasad, *Manipal Institute of Technology, Manipal.*
8. K. Shreedhar, *K. V. G College of Engineering, Sullia.*
9. Kedukodi Babushri Srinivas, *Manipal Institute of Technology, Manipal.*
10. G. Sudhakara, *Manipal Institute of Technology, Manipal.*
11. H. S. Sujatha, *Manipal Institute of Technology, Manipal.*
12. Surekha, *Manipal Institute of Technology, Manipal.*

Abstracts of invited papers.

On Structured Mapping Problems for Matrices

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Abstract

Given an appropriate class of structured matrices $\mathbb{S}$, we characterize matrices X and B for which there exists matrix $A \in \mathbb{S}$ such that $AX = B$. We determine all matrices in $\mathbb{S}$ mapping X to B. We also determine all matrices in $\mathbb{S}$ mapping X to B and having the smallest norm. For matrices X and B when there does not exists a matrix in $\mathbb{S}$ mapping X to B. We determine all matrices A in $\mathbb{S}$ for which $\|AX \quad B\|_F$ is minimized. As a consequence, we determine $\triangle B$ having the smallest norm and a matrix A in $\mathbb{S}$ also having the smallest norm such that $AX = B + \triangle B$. We use these results to investigate structured backward errors and perturbations of Hamiltonian eigen problem.

Keywords: Structured matrices, Hamiltonian eigen problem

On Subspaces attributed to Functions of Oblique Projectors

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Abstract

In the literature one can find a number of characteristics involving column and null spaces of functions of two projectors, but most of them concern orthogonal projectors (i.e., Hermitian idempotent matrices) only. In the present talk, several original characteristics of column and null spaces of various functions of a pair of oblique projectors (i.e., idempotent matrices) will be presented. These results often generalize known characteristics of a pair of orthogonal projectors. The approach exploited is based on partitioned representation of the pair, which proves to be very useful for investigations devoted to subspaces attributed to functions of projectors, both orthogonal and oblique.

The talk is based on joint research with Götz Trenkler.

Keywords: g-inverse, projector.

Polynomial Time Computation of the Hosoya Index of Some Families of Graphs

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Abstract

We show that the Hosoya index (a molecular-graph based structure descriptor) of graphs in which the cycles, if they exist, are edge-disjoint and of odd lengths can be computed in polynomial time. This is done by orienting the edges of these graphs and computing their skew energy. We also extend a classical formula for the Hosoya index of trees given by Gutman and Shalabi to these graphs. This incidentally brings out a connection between a skew energy of graphs and chemistry.

Keywords: Adjacency Matrix, Eigenvalue, Energy of graph.

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Means of Matrices

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Abstract

Averaging operations on positive definite matrices have been of interest in operator theory, engineering, physics and statistics. Because of noncommutativity of matrix multiplication and because of subtleties of the partial order $A \geq B$ on positive definite matrices, the geometric mean presents special difficulties.
The problem for a pair of matrices was solved in the 1970s, and the theory continues to find new uses in matrix analysis and applications. When more than two matrices are involved, a satisfactory definition has been found only recently. In 2005 the Riemannian barycentre of m matrices was proposed as a candidate for their geometric mean. One of its important properties monotonicity in the m variableshas been established in 2010. Meanwhile the concept has been used in diverse applications such as elasticity, imaging, radar, machine learning etc. We will discuss the problem from the perspective of matrix analysis and operator theory.

Keywords: Geometric mean of positive definite matrices, Riemannian manifold, Barycentre, Matrix monotonicity.

Moore-Penrose Inverse and Star Order under General Inner Products

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Abstract

We consider Moore-Penrose inverse, singular value decomposition, simultaneous singular value decomposition and star order under general inner products and enumerate some interesting properties. Several unique inverses and associated matrix orders have been developed and their properties studied piecewise. Notable among them are Moore-Penrose inverse, sharp inverse, core inverse and $A^{+}AA$. One notices striking similarities and some dissimilarity in the properties of these different inverses and orders. The object of this talk is to relate these developments to the Moore-Penrose inverse under general inner products.

Keywords: Moore-Penrose inverse, Core inverse, Partial order.

Models with Commutative Orthogonal Block Structure: Inference and Structured Families

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Abstract

Models with orthogonal block structure having variance-covariance matrices

$$= \left\{ \sum_{j=1}^{m} \gamma_j \mathbf{Q}_j \right\},$$

where the $\mathbf{Q}_j$ are known pairwise orthogonal orthogonal projection matrices and $\gamma_1, \ldots, \gamma_m$ are unknown non negative constants, continue to play an important part in the theory of randomized block designs. We now study an important class of these models, these with commutative orthogonal block structure, in which $\mathbf{T}$, the orthogonal projection matrix on the range space spanned by the mean vector commutes with $\mathbf{Q}_1, \ldots, \mathbf{Q}_m$.

We use commutative Jordan algebras of symmetric matrices in order to study the algebraic structure of these models and to obtain unbiased estimators, namely since **T** and commute, the least squares estimators for estimable vectors will be BLUE and under quite general conditions, unbiased estimators for variance components having sub-optimal properties, may be obtained.
Once normality is assumed, inference using pivot variables is quite straightforward.
To show the interest of this class of models we will present unbalanced examples before considering families of models.
When the models in a family correspond to the treatments of a base design, the family is structured. It will be shown how, under quite general conditions, the action of the factors in the base design on estimable vectors, can be studied.

This talk is based on joint research with João T. Mexia.

Keywords: COBS, Inference, Commutative Jordan algebras.

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Abstract

Yet to recieve

The Role of Kemeny's Constant in Properties of Markov Chains

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Abstract

In a finite m-state irreducible Markov chain with stationary probabilities $\{p_i\}$ and mean first passage times m_{ij} (mean recurrence time when $i = j$) it was first shown, by Kemeny and Snell, that $\sum_{j=1}^{m} \pi_j m_{ij}$ is a constant, K, not depending on i. This constant has since become known as Kemeny's constant. We consider a variety of techniques for finding expressions for K, derive some bounds for K, and explore various applications and interpretations of these results. Interpretations include the expected number of links that a surfer on the World Wide Web located on a random page needs to follow before reaching a desired location, as well as the expected time to mixing in a Markov chain. Various applications have been considered including some perturbation results, mixing on directed graphs and its relation to the Kirchhoff index of regular graphs.

Keywords: Markov Chain, Kemeny's constant, Kirchhoff index, Regular graphs.

Some Results on Semiring (Semigroup) of Nonnegative Matrices

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Abstract

Part I of the talk is to give a characterization of the multiplicative semigroup of a semiring of nonnegative matrices A with diag$A \geq I$,if the semigroup is cyclic. It follows from the characterization that the multiplicative semigroup of a semiring need not be finite as is the case for a field whose multiplicative group is cyclic. Part II of the talk is to give a characterization of nonnegative matrix A under a condition $AA^{(1)}$ or $AA^{(1)}$ is nonnegative, where $\mathrm{A}^{(1)}$ denotes a 1-inverse of A . This provides a sufficient condition on the matrix A that guarantees the linear system $Ax = b$, where b is nonnegative, to have a nonnegative best approximate solution. We produce a nonnegative full rank factorization of A under the above hypothesis. These results generalize earlier results (Jain-Tynan, LAMA, 2003) and provide new characterizations of group monotone matrices obtained in (Jain-Kwak-Goel, Trans AMS, 1980).
The results stated in this talk come from the joint work with Adel Alahmedi of King Abdulaziz University and and Yousef Alkhamees of King Saud University).

Keywords: Nonnegative matrices, Group monotone matrices, Rank factorization.

The Group Inverse and Conditioning of Stationary Vectors for Stochastic Matrices

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Abstract

A Square Matrix T is called *stochastic* if its entries are nonnegative and its rows sums are equal to one. Stochastic Matrices are the centrepiece of the theory of discrete-time, time homogenous Markov chains on a finite state space. If some power of the stochastic matrix of T has all positive entries, then

there is a unique left eigenvector for T, known as the *stationary distribution vector*, to which the iterates of the Markov chain converge, regardless of what the initial distribution for the chain is. Thus, in this setting, the stationary distribution vector can be thought of as giving the probability that the chain is in a particular state over the long run. Suppose that such a stochastic matrix T is perturbed slightly to yield another stochastic matrix, say $\widetilde{T}$; how large is the change in the corresponding stationary distribution vector? It turns out that the group inverse of $I \quad T$ is key to answering that question.
In this talk we will outline the basics on how, the group inverse of $I \quad T$ can be used to develop a measure of the conditioning of the stationary distribution vector when T is perturbed. We will then move on to give some results that connect the eigenvalue of T with the conditioning of the corresponding stationary distribution vector.

Matrix Problems of Skewed Multivariate Distributions

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Abstract

Multivariate elliptical distributions have usually two multivariate parameters: a location vector and a positive definite dispersion matrix. Skewed multivariate elliptical distributions are obtained from elliptical distributions by transformations where a multivariate elliptical density is multiplied to a distribution function or a function with similar properties. For example, the skew normal distribution $SN_p(\boldsymbol{\mu}, \mathbf{R}, \quad)$ has the density

$$f_{p.SN}(\mathbf{x}, \boldsymbol{\mu}, \mathbf{R}, \quad) = 2 f_{N_p(\ ,\ )}(\mathbf{x}) \Phi(\quad (\mathbf{x} \quad \boldsymbol{\mu})) \tag{1}$$

Where $\boldsymbol{\mu}$ is the mean vector of the multivariate normal distribution $\mathbf{R}$ is the correlation matrix and $\Phi(.)$ the distribution function of the univariate standard normal distribution. In the argument of $\Phi(.)$ a parameter-vector is called the shape parameter. In some generalizations the number of multivariate parameters is bigger. It is not possible to get maximum likelihood estimates in explicit form for such distributions, the same time he moment generating function has relative simple form and in many cases we can find the first moments by differentiating the moment generating function $M(\mathbf{t})$. Moments $m_k(\mathbf{x})$ can be found as matrix derivatives from $M(\mathbf{t})$:

$$m_k(\mathbf{x}) = \frac{d^k M(\mathbf{t})}{d\mathbf{t}^k}_{\mathbf{t}=0}, \quad k = 1, 2, ...$$

It follows from definitions of moments and matrix derivative that $m_1(\mathbf{x})$ is a transposed p-vector, $m_2(\mathbf{x})$ is a $p \times p$ matrix and $m_3(\mathbf{x})$ is given by a $p^2 \times p$ matrix of the third order mixed moments. To apply the method of moments we have to transform the $p^2 \times p$ matrix of third order moments into a p-vector in a meaningful way. The idea can be realized by star-product of matrices. This operation is also useful when defining multivariate skewness and kurtosis characteristics. We shall apply the method of moments to some skew elliptical distributions and find estimates of parameters by compressing the matrices of the third and fourth order moments via the star-product.

Keywords: Method of moments, Multivariate kurtosis, Mulivariate skewness, Skew elliptical distributions.

An Overview of Generalized Inverses of Matrices Over Rings

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Abstract Abstract to be received.

Generalized Inverses and Approximation Numbers

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Abstract

Let $\mathbb{C}^{m\times n}$ denote the set of all matrices of order $m \times n$ with complex entries. For a natural number k and $A \in \mathbb{C}^{m\times n}$, the k-th approximation number of A, denoted by $s_k(A)$ is defined by

$$s_k(A) := inf\{\| A \quad B \|: B \in \mathbb{C}^{m\times n}, \quad rank(B) \quad k \quad 1\}$$

Here the norm is the operator norm induced on $\mathbb{C}^{m\times n}$ by vector norms on $\mathbb{C}^n$ and $\mathbb{C}^m$. If these vector norms are l^2 norms, then $s_k(A)$ coincides with the k-th singular value $\sigma_k(A)$.
In this talk we attempt to relate approximation numbers of a matrix with norms of its generalized inverses. In particular, the following result is obtained:
Let $A \in \mathbb{C}^{m\times n}$ be of rank r and $X \in \mathbb{C}^{n\times m}$ satisfy $AXA = A$. Then

$$\frac{1}{\| X \|} \quad s_r(A) \quad \frac{\| XA \|}{\| X \|}$$

In particular, if $X = A^\dagger$, the Moore-Penrose inverse of A, and the matrix norms are induced by l^2 norms, then $s_r(A) = \sigma_r(A) = \dfrac{1}{\| A^\dagger \|}$.

An Optimality of Neighbour Designs Under Interference Models

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Abstract

The concept of neighbour designs was introduced and defined by Rees (1967) who gave also some methods of their construction. Henceforth many methods of construction of neighbour designs as well as of their generations are available in the literature. However there are only few results on their optimality. Therefore the aim of this talk is to give an overview of study on this problem. It will include some recent results on optimality of specified neighbour designs under various linear models.

Reference:

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Regular Matrices over an Incline

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Abstract Inclines are a generalization of Boolean and Fuzzy algebras and can represent intensity of relationships in a more dynamic way. The notion of inclines and their applications were described by Cao, Kim and Roush in the monograph "Incline Algebra and Applications". Inclines are additively idempotent semirings in which products are less than (or) equal to either factor. We discuss the invertibility of matrices over DL, the set of idempotent elements in an incline and for matrices over an integral incline. We discuss the regularity of matrices over an incline with elements of DL are linearly ordered. We apply our results for matrices over special types of inclines such as regular inclines whose elements are linearly ordered, for distributive lattices and for integral inclines. We obtain equivalent conditions for the existence of various generalized inverses of an incline matrix. We provide an algorithm for the regularity of incline matrices and illustrate with suitable examples.

Keywords: Regular matrix, Incline, Semiring.

Equalities of the BLUEs and/or BLUPs in two Linear Models

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Abstract

In this talk we consider two linear models, M_1 and M_2, which differ in their covariance matrices. We review conditions under which the best linear unbiased estimator (BLUE) of an estimable parametric function under M_1 continues to be BLUE also under M_2. Similarly, we consider linear models, L_1 and ML_2, with new unobserved future observations, and give conditions that the best linear unbiased predictor (BLUP) of the new observation under the model L_1 continues to be BLUP also under the model L_2. These results can be applied for studying the BLUPs of random effects in two mixed models

Introduction to Graph Theoretic Applications to Computing the Nucleolus of an Assignment Game

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Abstract

Computing the nucleolus for assignment games is a major algorithm that involves, facets of matrix theory from the point of view of permanents, doubly stochastic matrices, Frobenius Konig Theorem and graphs with many vertices and directed arcs that slowly shrink to a single vertex by a suitable equivalence relation all lead to the important solution to the problem of assignment. The game theoretic solution resolves the split between the sellers of houses and buyers of houses where sellers try to get the maximum out of their houses and buyres get the best deal from the best among the sellers and the real estate agent cashes on the total gains. The secrecy of the seller and buyer before the final sale takes place is crucial for the nucleolus split.

Reference:

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3. R.B. Bapat and TES Raghavan, *Nonnegative Matrices and Applications*, Paperback edition, Cambridge University Press (2009).

Linear Algebra of Strongly Regular Graphs and related objects

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Abstract

This exposition will deal with combinatorial configurations that includes strongly regular graphs and their linear algebra. The talk will introduce various classes of strongly regular graphs. It will also discuss the Krein conditions that are necessary for the existence of a strongly regular graph. The talk will also give various interlinks between these objects and other combinatorial structures.

Keywords: strongly regular graph, Krein condition.

Weak Monotonicity of Interval Matrices*

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Abstract

This talk is concerned with weak monotonicity of interval matrices, with specific emphasis on its relationship with a certain class of proper splittings. Some new results will be presented.

This talk is based on joint research with Agarwal N. Sushama and K. Premkumari

Keywords: Interval matrix, M-matrix, Weak monotonicity, range kernal regularity, weak row regular splitting, Moore-Penrose inverse.

The second immanant of two combinatorial matrices

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Abstract

Let $A = (a_{i,j})_{1 \leq i,j \leq n}$ be an $n \times n$ matrix where $n \geq 2$. Let $\mathbf{det2}(A)$, its second immanant be the immanant corresponding to the partition $\lambda_2 = 2, 1^{n\ \ 2}$. Let G be a connected graph with blocks $B_1, B_2, \ldots B_p$ and q-exponential distance matrix being $\mathbf{ED}_G$. We show that $\mathbf{det2}(\mathbf{ED})_G$ is independent of the manner in which the blocks are connected. Our result is similar to the result of Bapat, Lal and Pati who show that $\det(\mathbf{ED}_T)$ where T is a tree is independent of the structure of T and only depends on n, the number of vertices of T. Similar results are shown for the q-analogue of T's laplacian, and in general to a matrix associated to connected graphs G. aph.

The Complexity of the q-analog of the n-cube

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Abstract

We represent a positive, combinatorial formula for the complexity (= number of spanning trees) of the q - analog of the n cube.

Some comments on old Magic Squares from India illustrated with Postage Stamps

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Abstract

The talk covers demonstrations of magic squares observed in ancient Indian mathematics and archaeology. The same is illustrated with postage stamps.

Functions of Orthogonal Projectors Involving the Moore-Penrose Inverse

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Abstract

Several results scattered in the literature express an oblique projector having given onto and along spaces in terms of a pair of orthogonal projectors. The results were established in various settings, including finite and infinite dimensional vector spaces over either real or complex numbers, but their common feature is that they are valid merely under the assumption of the non singularity of certain functions of the involved projectors. In the present paper, these results are unified and re-established in a generalized form in a complex Euclidean vector space, with the generalization obtained by relaxing the non singularity assumption and use of the Moore-Penrose inverse instead of the ordinary inverse. Additionally, several new formulae of the type are proved.

Condition Numbers for Moore-Penrose Inverse, Linear Least Squares, Total Least Squares, Matrix Equations and Tikhonov Regularization

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Abstract

Classical condition numbers are normwise: they measure the size of both input perturbations and output errors using some norms. To take into account the relative of each data component, and, a possible data sparseness, componentwise condition numbers have been increasingly considered. These are mostly of two kinds: mixed and componentwise. In this talk, we give explicit expressions, computable from the data, for the mixed and componentwise condition numbers for the computation of the Moore-Penrose inverse as well as for the computation of solutions and residues of linear least squares problems. In both cases the data matrices have full column (row) rank. We also discuss the total least squares, matrix equations and Tikhonov regularization.

Keywords: Condition number, least square, Tikhonov regularization.

Weak Complementarity, Non-Testability and Restricted Moore-Penrose Inverses

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Abstract

In the literature, COMPLEMENTARY matrices have been studied because of their importance when analyzing OVER-PARAMETERIZED linear statistical models. In this talk, the somewhat more general concept of WEAK COMPLEMENTARITY is considered. Observing the fact that the usual F-TEST in ANOVA is applicable only for "TESTABLE hypotheses, that in practice however - e.g. in nonorthogonal settings or incomplete layouts - NON-TESTABLE hypotheses can be of importance, a variant of the F-TEST, which is based on RESTRICTED MOORE-PENROSE INVERSES, is discussed that allows to decide for significant deviations also in NON-TESTABLE situations and to detect NON-TESTABILITY, too.

Keywords: F-test, Gauß-Markov model, missing observations, non-testability, restricted Moore-Penrose inverse, weak complementarity.

Abstracts of contributed papers.

Assorted Representations of Generalized Inverse with Numerical Solutions

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Abstract

This paper deals with different representations of Generalized Inverse. Here Pseudo Inverse of a matrix A has been shown as $A^+ = G^*(F^*AG^*)\ {}^1F^*$ where $A = FG$ is a full rank factorization. Other results show different full-rank representations of Generalized Inverse of a rectangular matrix over the complex field. Here A^+ has been shown as a Lagrange-Silvester interpolation polynomial in powers of A, A^* with examples. Here I have discussed two theorems that represent the Generalized Inverse using contour integral formula. These two theorems verify all the four conditions of a Pseudo Inverse. Numerical examples will support my proof. Another representation of Pseudo Inverse by Caley-Hamilton Theorem has been proved numerically. I have used special Mathematica code **PseudoInverse** in this paper.

Keywords: full-rank factorization, interpolation polynomial , contour, contour integral, mathematica code.

Nonisomorphic Cospectral Oriented Hypercubes

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Abstract

Let $G\ \ = (V,\ \ )$ be any oriented graph obtained by assigning an orientation σ to the edge set E of a simple undirected graph $G = (V, E)$ where $V = \{v_1, v_2, ..., v_n\}$. The energy $E(G)$ of a graph G is the

sum of the absolute values of its eigenvalues. This concept was recently generalized to oriented graphs in [1]: if G is an oriented graph and its skew spectrum $\{\mathbf{i}\lambda_1, \mathbf{i}\lambda_2..., \mathbf{i}\lambda_n\}$, is the spectrum of its skew adjacency matrix $S(G)$, then the skew energy $\epsilon_S(G)$ of G is given by $\epsilon_S(G) = \sum_{i=1}^{n} |\lambda_i|$.
In [1], Adiga et al., have proved that $\epsilon_S(G) \leq n\sqrt{\triangle}$, where $\triangle$ is the maximum degree of the underlying undirected graph G of G. In [2], Bryan Shader and Wasin So, using matrix techniques, have shown that a graph G is bipartite if, and only if, there is an orientation σ of G such that $S_p(G) = \mathbf{i}S_p(G)$. In [3], G X Tian considered the above two properties in hypercubes by constructing two nonisomorphic orientations of the hypercube Q_d such that the first orientation yields the maximum possible skew energy among all d-regular graphs of order 2^d, namely, $2^d\sqrt{d}$ while in the second orientation, the skew energy equals the energy of the underlying undirected hypercube. In this presentation, we construct two other nonisomorphic orientations of the hypercube Q_d with respect to which the skew energy is equal to the energy of the underlying Q_d.

The talk is based on joint research with Prof. R. Balakrishnan.

Keywords: Adjacency Matrix, Eigenvalue, Energy of graph.

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Strong/Weak Edge Vertex Mixed Domination Number of a Graph

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Abstract

An edge x, m-dominates a vertex v if $v \in N[x]$. An edge x strongly(weakly) m-dominates a vertex v if $v \in N[x]$ and $deg(x) \geq deg(y)(deg(x) \leq deg(y))$ for every y which m-dominates the vertex v. A set $L \subseteq E$ is an Edge Vertex Dominating set (EVD-set) if every vertex in G is m-dominated by an edge in

L. The edge vertex domination number $\gamma_{ev}(G)$ is the minimum cardinality of an EVD-set.
A set $L \subseteq E$ is a Strong Edge Vertex Dominating Set (SEVD -set) [Weak Edge Vertex Dominating Set (WEVD -set)] if every vertex in G is strongly(weakly) m-dominated by an edge in L. The strong(weak) edge vertex domination number $\gamma_{sev}(G)$ ($\gamma_{wev}(G)$) is the minimum cardinality of a SEVD -set(WEVD -set). Besides finding the relationship between the existing graph parameters, we prove a Gallais type result for edges. A new parameter called Edge Vertex degree of an edge is defined and a bound in terms of the maximum and minimum EV degree is established.

The talk is based on joint research with Prof. S. S. Kamath and Mrs.Surekha

Keywords: Edge vertex Dominating sets, (EVD sets) Strong Edge Vertex Dominating sets (SEVD sets), Weak Edge Vertex dominating sets (WEVD sets), EV degree.

Matrix Partial Orders in Indefinite Inner Product Spaces

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Abstract

An indefinite inner product in $\mathbb{C}^n$ is a conjugate symmetric sesquilinear form $[x, y]$ together with the regularity condition that $[x, y] = 0, \forall y \in \mathbb{C}^n$ only when $x = 0$. Associated with any indefinite inner product is a unique invertible Hermitian matrix J (called a weight) with complex entries such that $[x, y] = \langle x, Jy \rangle$ where $\langle ., . \rangle$ denotes the Euclidean inner product on $\mathbb{C}^n$ and vice versa. An additional, non-restrictive assumption on J, namely, $J^2 = I$, is also made that allows us to present our results more elegantly and also compare with the existing results in the Euclidean case. The indefinite matrix product of two matrices A and B of sizes $m \times n$ and $n \times l$ complex matrices, respectively, is defined to be the matrix $A \circ B = AJ_nB$. The adjoint is defined to be the matrix $A^{[*]} = J_nA^*J_m$ (Refer [1] for more details about the indefinite matrix product).
For $A \in \mathbb{C}^{m\times n}, X \in \mathbb{C}^{n\times m}$ is called the Moore-Penrose inverse of A (with respect to the indefinite matrix product) if it satisfies the following equations : $A \circ X \circ A = A$, $X \circ A \circ X = X$, $(A \circ X)^{[*]} = A \circ X$;$(X \circ A)^{[*]} = X \circ A$. Such an X will be denoted by $A^{[\dagger]}$ and has the representation $A^{[\dagger]} = J_nA^{[\dagger]}J_m$. Similarly, for $A \in \mathbb{C}^{n\times n}, X \in \mathbb{C}^{n\times n}$ is called the group inverse of A, denoted by A , if it satisfies the equations: $A \circ X \circ A = A$, $X \circ A \circ X = X$, $A \circ X = X \circ A$. An analogous formula for the group inverse similar to that of the Moore-Penrose inverse does not exist in indefinite inner product spaces. Our aim is to extend the notions of the sharp and the star orders to indefinite inner product spaces (with respect to the indefinite matrix product) and investigate reverse order laws and other related results for the group inverse (when it exists) and the Moore-Penrose inverse, respectively.

Keywords: Indefinite inner product spaces, indefinite matrix product, Moore-Penrose inverse, group inverse, star order, sharp order, reverse order laws.

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The Singularity of Weighted Directed Graphs

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Abstract

A mixed graph is a graph with some directed and some undirected edges. In this talk we present a more general structure than that of mixed graphs, namely the weighted directed graphs. Further, we introduce the notion of 3-colored digraphs which generalize the notion of mixed graphs but is much restricted in comparison to the weighted directed graphs. We define the Laplacian matrix of weighted directed graphs and supply several characterizations of the singularity of these matrices. Appropriate generalization of several existing results for mixed graphs related to singularity of the corresponding Laplacian matrix is supplied here. We also prove many new combinatorial results relating the Laplacian matrix and graph structure.
The talk is based on joint research with Prof. R. B Bapat and S. Pati.

Characterization of Normal Matrices in an Indefinite Inner Product Spaces

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Abstract

Grone et al (Lin. Alg. and Appl., Vol. 87, pp. 213-225, 1987) established seventy conditions equivalent to normality of a matrix. Elsner and Ikramov (Lin. Alg. and Appl., Vol.303, pp.285-291, 1998) extended the utility of normal matrices by supplementing the original list with additional equivalent conditions to the normality. In this paper, we extend those conditions (to the possible extend) in the indefinite inner product space. We classify the indefinite normal matrices into two classes :

1. Indefinite normal matrices that have Moore-Penrose inverses
   (K. Kamaraj and K.C.Sivakumar, J. Applied Mathematics and Computing Vol. 40, pp. 1233-1240, 2005).

2. Indefinite normal matrices that have Spectral property
   (K. Kamaraj and K.C.Sivakumar, J. Analysis, Vol. 12, pp. 143-152, 2005).

The talk is based on joint research with Prof. K. Ramanathan.

Insertion of factors property in Matrix Nearrings

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Abstract

Let N stands for a zero symmetric right nearring with identity. For an arbitrary natural number n, we denote N^n as the direct sum of n-copies of $(N,+)$. The nearring of $n \times n$ matrices over N is the subnearring of $M(N^n)$ generated by the set $\{f^r_{ij}; r \in N, 1 \leq i,j \leq n\}$ where $f^r_{ij} : N^n \to N^n$ defined by $f^r_{ij} = i_i f^r \pi_j$ for all $1 \leq i,j \leq n$ (here i_i, π_j are respectively the ith injection and the jth projection maps, and $f(s) = rs$ for all $s \in N$. It is denoted by $M_n(N)$, the matrix nearring). In [1] the authors obtained that certain properties are preserved in matrix nearrings. The concept of insertion of factors property is well known in nearrings. In this note we present simple observations concerning to this property in $M_n(N)$ and related results.

The talk is based on joint research with Prof. Bhavanari Satyanarayana.

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Metric Dimension of Graphs by Using Distance Matrices

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Abstract

The distance matrix of a graph G of order n is the square matrix of order n in which the ijth element is the distance of ith vertex from the jth vertex in the graph. For an ordered set $W = \{w_1, w_2, w_3, ...w_k\}$ of vertices and a vertex v in a connected graph G, the representation of v with respect to W is the ordered k-tuple $r(v|W) = (d(v, w_1), d(v, w_2), d(v, w_3), ..., d(v, w_4))$ where $d(x, y)$ represents the distance between the vertices x and y. The set W is called a resolving set for G if every two vertices of G have distinct representations. A resolving set containing a minimum number of vertices is called a basis for G. The elements of a metric basis are called land marks and the metric dimension of G, denoted by (G), is the number of vertices in a basis of G. In this Paper we discuss the metric dimension concept of graphs in terms of distance matrices and also obtain some results on the metric dimension of graph powers.

The talk is based on joint research with Prof. B. Sooryanarayana.

Matrices associated with $_I\ N)$

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Abstract

$G_I(N)$ represents the graph of a nearring N with respect to an ideal I of N. We first note that not all matrices have the representation $G_I(N)$. We find properties of matrices associated with $G_I(N)$ and identify some matrices that can be represented as $G_I(N)$.

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Realization of Product of Adjacency Matrices of Graphs

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Abstract

In an attempt to understand the structural implication of product of adjacency matrices of two graphs, we try to answer the following questions: "when is the product of adjacency matrices of two graphs G and H realizes a graph?". In this paper we answer the question with usual multiplication and derive some related results. Using the above, we define a new product of graphs on the same set of vertices and also we realize the problem as a problem of edge coloring of graphs.

The talk is based on joint research with Prof. K Manjunatha Prasad.

Keywords: Adjacency matrix, Graph Products, Graph equations, edge coloring.

Realization of Product of Adjacency Matrices of Graphs

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Abstract

In this paper we try to solve some graph equations involving the adjacency matrices of a graph G and its complement $\overline{G}$. Only modulo two products are considered. In particular, we study the modulo two products of Cycle graph and its distance graph.

The talk is based on joint research with Prof. Sudhakara G.

Keywords: Adjacency matrix, Graph Products, Graph equations.

Relationship Between Block Domination Parameters of Graphs

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Abstract

Two vertices u and w, vv-dominate each other if they incident on the same block. A set $S \in V$ is a vv- dominating set (VVD- set) if every vertex in $V \quad S$ is vv-dominated by a vertex in S. The *vv- domination number* $\gamma_{vv} = \gamma_{vv}(G)$ is the minimum cardinality of a VVD- set of G. Analogously, we define a similar parameter for blocks. Two blocks $b_1, b_2 \in B(G)$, bb-dominate each other if there is a common cutpoint incident with b_1 and b_2. A set $L \in B(G)$ is said to be a *bb-dominating set* (BBD set) if every block in G is bb-dominated by some Block in L. The *bb-domination number* $\gamma_{bb} = \gamma_{bb}(G)$ is the minimum cardinality of a BBD- set of G. In [4] Mixed domination parameters are defined. We say that a vertex v and a block b are said to b-dominate each other if v is incident with the block b. Then *vb-domination number* $\gamma_{vb} = \gamma_{vb}(G)$ (*bv-domination number* $\gamma_{bv} = \gamma_{bv}(G)$) is the minimum number of vertices (blocks) needed to b-dominate all the blocks of G. In this paper we study the properties of these block domination parameters and establish a relation between these parameters giving an inequality

chain consisting of seven parameters.

The talk is based on joint research with Prof. P. G Bhat.

Keywords: vv-domination number, bb-domination number, vb-domination number, bv-domination number.