

# Equality of BLUEs or BLUPs under two linear models using stochastic restrictions

Stephen J. Haslett · Simo Puntanen

Received: 17 July 2008 / Revised: 23 February 2009 / Published online: 13 March 2009  
© Springer-Verlag 2009

**Abstract** In this paper, we consider mixed linear models, possibly with singular covariance matrices, by supplementing a particular fixed effects model with appropriate stochastic restrictions. We show that all representations of the best linear unbiased estimator (BLUE) and best linear unbiased predictor (BLUP) can be obtained through the augmented model including stochastic restrictions. Using this approach, we consider two mixed linear models,  $\mathcal{M}_1$  and  $\mathcal{M}_2$ , say, which have different covariance matrices. We give necessary and sufficient conditions that the BLUP and/or BLUE under the the model  $\mathcal{M}_1$  continue to be BLUP and/or BLUE also under the model  $\mathcal{M}_2$ .

**Keywords** BLUE · BLUP · Generalized inverse · Linear fixed effects model · Linear mixed effects model · Stochastic restrictions

**Mathematics Subject Classification (2000)** 62J05 · 62F10

## 1 Introduction

A mixed linear model can be presented as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\gamma} + \boldsymbol{\varepsilon}, \quad (1.1)$$

---

S. J. Haslett  
Institute of Fundamental Sciences, Massey University,  
Palmerston North, New Zealand  
e-mail: s.j.haslett@massey.ac.nz

S. Puntanen (✉)  
Department of Mathematics and Statistics,  
University of Tampere, 33014 Tampere, Finland  
e-mail: simo.puntanen@uta.fi