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Effect of adding regressors on the equality of the BLUEs under two linear models

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ABSTRACT

In this paper we consider the estimation of regression coefficients in two partitioned linear models, shortly denoted as $\mathscr{M}_{12} = \{\mathbf{y}, \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2, \mathbf{V}\}$, and $\underline{\mathscr{M}}_{12} = \{\mathbf{y}, \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2, \underline{\mathbf{V}}\}$, which differ only in their covariance matrices. We call $\mathscr{M}_{12}$ and $\underline{\mathscr{M}}_{12}$ full models, and correspondingly, $\mathscr{M}_i = \{\mathbf{y}, \mathbf{X}_i\boldsymbol{\beta}_i, \mathbf{V}\}$ and $\underline{\mathscr{M}}_i = \{\mathbf{y}, \mathbf{X}_i\boldsymbol{\beta}_i, \underline{\mathbf{V}}\}$ small models. We give a necessary and sufficient condition for the equality between the best linear unbiased estimators (BLUEs) of $\mathbf{X}_1\boldsymbol{\beta}_1$ under $\mathscr{M}_{12}$ and $\underline{\mathscr{M}}_{12}$. In particular, we consider the equality of the BLUEs under the full models assuming that they are equal under the small models.

1. Introduction

In this paper we consider the partitioned linear model

$$\mathbf{y} = \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2 + \boldsymbol{\varepsilon}, \tag{1.1}$$

or more succinctly,

$$\mathscr{M}_{12} = \{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \mathbf{V}\} = \{\mathbf{y}, \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2, \mathbf{V}\}, \tag{1.2}$$

where $\mathrm{E}(\mathbf{y}) = \mathbf{X}\boldsymbol{\beta}$, $\mathrm{E}(\boldsymbol{\varepsilon}) = \mathbf{0}$, $\mathrm{cov}(\mathbf{y}) = \mathrm{cov}(\boldsymbol{\varepsilon}) = \mathbf{V}$. We denote the expectation vector and covariance matrix, respectively, by $\mathrm{E}(\cdot)$ and $\mathrm{cov}(\cdot)$.

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