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Comparing the BLUEs Under Two Linear Models

JAN HAUKE[1], AUGUSTYN MARKIEWICZ[2],
AND SIMO PUNTANEN[3]

[1]Institute of Socio-Economic Geography and Spatial Planning,
Adam Mickiewicz University, Poznań, Poland
[2]Department of Mathematical and Statistical Methods,
Poznań University of Life Sciences, Poznań, Poland
[3]School of Information Sciences, University of Tampere,
Tampere, Finland

In this article, we consider two linear models, $\mathscr{M}_1 = \{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \mathbf{V}_1\}$ *and* $\mathscr{M}_2 = \{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \mathbf{V}_2\}$*, which differ only in their covariance matrices. Our main focus lies on the difference of the best linear unbiased estimators,* BLUE*s, of* $\mathbf{X}\boldsymbol{\beta}$ *under these models. The corresponding problems between the models* $\{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \mathbf{I}_n\}$ *and* $\{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \mathbf{V}\}$*, i.e., between the* OLSE *(ordinary least squares estimator) and* BLUE*, are pretty well studied. Our purpose is to review the corresponding considerations between the* BLUE*s of* $\mathbf{X}\boldsymbol{\beta}$ *under* $\mathscr{M}_1$ *and* $\mathscr{M}_2$*. This article is an expository one presenting also new results.*

Keywords BLUE; Gauss–Markov model; Linear sufficiency; Löwner ordering; OLSE; Orthogonal projector.

Mathematics Subject Classification 15A42; 62J05; 62H12; 62H20.

1. Introduction

In this article, we consider the general linear model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \quad \text{or in short } \mathscr{M} = \{\mathbf{y}, \mathbf{X}\boldsymbol{\beta}, \mathbf{V}\},$$

where $\mathbf{X}$ is a known $n \times p$ model matrix, the vector $\mathbf{y}$ is an observable n-dimensional random vector, $\boldsymbol{\beta}$ is a $p \times 1$ vector of unknown parameters, and $\boldsymbol{\varepsilon}$ is an unobservable vector of random errors with expectation $\mathrm{E}(\boldsymbol{\varepsilon}) = \mathbf{0}$, and covariance matrix $\mathrm{cov}(\boldsymbol{\varepsilon}) = \mathbf{V}$. The non negative definite (possibly singular) matrix $\mathbf{V}$ is known.

As regards the notation, we will use the symbols $\mathbf{A}'$, $\mathbf{A}^-$, $\mathbf{A}^+$, $\mathscr{C}(\mathbf{A})$, $\mathscr{C}(\mathbf{A})^\perp$, and $\mathscr{N}(\mathbf{A})$ to denote, respectively, the transpose, a generalized inverse, the Moore–Penrose inverse, the column space, the orthogonal complement of the column space, and the

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Address correspondence to Simo Puntanen, School of Information Sciences, FI-33014 University of Tampere, Tampere, Finland; E-mail: sjp@uta.fi