

# **INEQUALITIES AND APPLICATIONS**

**THEMISTOCLES M. RASSIAS  
DORIN ANDRICA  
(eds.)**

**Cluj University Press**

### About the Book

In this volume the contributing authors deal with an extensive study of several important inequalities useful in several problems in mathematical analysis, geometry and their applications. Subjects dealt with include: Inequalities of Jensen type and approximation processes, inequalities and equalities for the generalized efficiency function in orthogonality partitioned linear models, reverses of the Schwarz inequality in inner product spaces, new sharp bounds for the Bernoulli numbers and refinement of Becker-Stark inequalities, Ulam stability problem for approximately biquadratic mappings and functional inequalities, orthogonality equation on a bounded domain, generalized Hosszú's functional equation, Weyl multipliers and numerical series, stability of homomorphisms in quasi-Banach algebras, inequalities for d-isometric isomorphisms on linear d-normed  $C^*$ -algebras, rate of growth of polynomials not vanishing inside a disk, new Redheffer type inequalities, inequalities connected with the Hyers' stability theorem, Jensen inequality and Popoviciu's and related functional equations, nonlinear variational inclusion systems, iterative algorithm and convergence analysis, Hilbert-type inequalities with best constant factors, reverse of Hilbert-Hong's inequality, Jordan's inequality and applications.

In addition to these inequalities, applications to certain problems in pure and applied mathematics are considered.



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## PREFACE

We wish to mention here three of the most influential books on inequalities:

1. G.H. Hardy, J.E. Littlewood and G. Pólya, *Inequalities*, Cambridge University Press, Cambridge, 1934.
2. E.F. Beckenbach and R. Bellman, *An Introduction to Inequalities*, Random House, New York, 1961.
3. D.S. Mitrinović (with P.M. Vasić), *Analytic Inequalities*, Springer-Verlag, Berlin, 1970.

It is generally acknowledged that the classic book *Inequalities* by G.H. Hardy, J.E. Littlewood and G. Pólya transformed the field of inequalities from a collection of isolated formulas into a systematic discipline. The modern theory of inequalities, as well as the continuing and growing interest in this field, undoubtedly stems from that work.

Richard Bellman said during the Second International Conference on General Inequalities (Oberwolfach, 30th July-5th August 1978), "There are three reasons for the study of inequalities: practical, theoretical, and aesthetic". On the aesthetic aspects he said: "As has been pointed out, beauty is in the eyes of the beholder. However, it is generally agreed that certain pieces of music, art, or mathematics are beautiful. There is an elegance to inequalities that makes them very attractive."

After the classic book by Hardy, Littlewood and Pólya, the book of D.S. Mitrinović is the next most cited book in the field of inequalities. Mitrinović often used to say: "There are no equalities, even in the human life the inequalities are always present".

The book of G.V. Milovanović, D.S. Mitrinović, and Th.M. Rassias, *Topics in Polynomials: Extremal Problems, Inequalities, Zeros*, World Scientific Publishing Co., Singapore, New Jersey, London, 1994, presents a vast subject of polynomial inequalities and approximation theory in an integrated and self-contained fashion. Some 1200 references have been cited in it.

In this volume the contributing authors deal with an extensive study of several important inequalities useful in several problems in mathematical analysis, geometry and their applications. Subjects dealt with include: Inequalities of Jensen type and approximation processes, inequalities and equalities for the generalized efficiency function in orthogonality partitioned linear models, reverses of the Schwarz inequality in inner product spaces, new sharp bounds for the Bernoulli numbers and refinement of Becker-Stark inequalities, Ulam stability problem for approximately biquadratic mappings and functional inequalities, orthogonality equation on a bounded domain, generalized Hosszú's functional equation, Weyl multipliers

and numerical series, stability of homomorphisms in quasi-Banach algebras, inequalities for d-isometric isomorphisms on linear d-normed  $C^*$ -algebras, rate of growth of polynomials not vanishing inside a disk, new Redheffer-type inequalities, inequalities connected with the Hyers' stability theorem, Jensen inequality and Popoviciu's and related functional equations, nonlinear variational inclusion systems, iterative algorithm and convergence analysis, Hilbert-type inequalities with best constant factors, reverse of Hilbert-Hong's inequality, Jordan's inequality and applications.

In addition to these inequalities, applications to certain problems in pure and applied mathematics are considered.

It is a pleasure to express our deepest appreciation to all the mathematicians, who contributed to this volume. Finally, we wish to acknowledge the superb assistance provided by the staff of Cluj-University Press.

July 2008

Themistocles M. Rassias

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## Inequalities and Approximation Theory

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**Abstract.** The purpose of this paper is twofold. Firstly, we present an equivalence property involving isotonic linear functionals. Secondly, by using the contraction principle, we give a method for obtaining the limit of iterates of some classes of linear positive operators.

### 1. Introduction

In Approximation Theory a tool with rich mathematical content and great potential for applications is given by linear methods of approximation generated by sequences of linear operators, the essential ingredient being that of positivity.

The main objective of this survey paper is to present results which spring from standard inequalities enriching the mentioned research field.

In this respect, the paper is organized in two main sections. Taking into account that the class of convex functions is characterized by the well-known inequality of Jensen, the following question arises in a natural way: what are the connections between the Jensen's inequality on  $C([a, b])$ , the existence of a sequence of approximating and convexity-preserving positive linear polynomial operators which reproduce the affine functions and Bohman-Korovkin's theorem? The aim of Section 2 is to show that the three above mentioned basic results together with a certain generalization of Jensen's inequality due to B. Jessen are equivalent. This equivalence property emphasizes the role of convexity and convexity-preserving operators in the approximation of functions by positive linear operators. Once again, the powerful criterion due to T. Popoviciu, H. Bohman and P.P. Korovkin is pointed out. It helps us to decide if a sequence of positive

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## Inequalities and Equalities for the Generalized Efficiency Function in Orthogonally Partitioned Linear Models

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**Abstract.** We consider the estimation of regression coefficients in orthogonally partitioned linear models and focus on the Watson efficiency of the ordinary least squares estimator of the full set of the parameters with respect to the best linear unbiased estimator and how this full Watson efficiency relates to the product of the Watson efficiencies of two subsets of the parameters. Building upon our recent paper [19], we introduce a new and apparently very useful generalized efficiency function and show how it is related to the Watson efficiency; several new inequalities and equalities are established.

### 1. Introduction and mise-en-scène

#### 1.1 Introduction

In this paper we consider the general partitioned linear (or Gauß-Markov) model

$$\mathbf{y} = \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2 + \boldsymbol{\varepsilon}, \quad (1.1)$$

or in another notation,

$$\mathcal{M}_{12} := \{\mathbf{y}, \mathbf{X}_{12}\boldsymbol{\beta}_{12}, \mathbf{V}\} := \{\mathbf{y}, \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2, \mathbf{V}\}, \quad (1.2)$$

with

$$E(\mathbf{y}) = \mathbf{X}_{12}\boldsymbol{\beta}_{12}, \quad E(\boldsymbol{\varepsilon}) = \mathbf{0}, \quad \text{cov}(\mathbf{y}) = \text{cov}(\boldsymbol{\varepsilon}) = \mathbf{V}, \quad (1.3)$$

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