Dedicated to<br>Professor R B Bapat<br>on the occasion of his 60th birthday

## Foreword



It gives me immense pleasure that the Department of Statistics of Manipal University is organizing an International Conference on Linear Algebra \& its Applications on 18-20 December 2014. Linear Algebra is an important concept of mathematics and is applied in various disciplines like engineering, physics, natural sciences, social sciences and so on. With the participation of national and international professionals, I am confident that the conference will show great results. Dr Ravindra B Bapat, Professor, Indian Statistical Institute is a doyen in the field of statistics and is known for his contributions in this area of Mathematics. He has published several original research papers in numerous international journals. As a mark of appreciation of his contributions, a compilation of his selected publications is being published to be released during the conference on the occasion of his 60th birthday. This volume will provide new direction and technique to address future research problems in the area of Linear Algebra and its Applications.

Manipal University is happy to be associated with this event and wishes the best to Dr Bapat and the Organizers of this conference. My sincere thanks to all the publishers who have given permission to reproduce the selected articles in this volume, and helped us in this academic cause. I would also like to thank the editorial team lead by Dr Steve Kirkland for their effort in bringing out this volume.

## Preface

Linear Algebra is a mathematical subject having applications in many branches of science including Biology, Economics, Engineering, Physics, and Statistics. The contribution of Professor Ravindra B Bapat in this area is enormous. The main research interests of Prof Bapat are Nonnegative Matrices, Matrix Inequalities, Matrices in Graph Theory, Matrix Methods in Statistics, and Generalized Inverses. He has published about 130 research papers in the above mentioned areas in reputed national and international journals, and guided three PhD students, all of whom have gone on to make their own contributions to Linear Algebra. He has written books titled "Nonnegative Matrices and Applications" with Prof TE S Raghavan, in the Cambridge University Mathematical Series Encyclopedia of Mathematics and its Applications, "Linear Algebra and Linear Models" and "Graphs and Matrices" in the TRIM Series of the Hindustan Book Agency, New Delhi, India, which is co-published by Springer as its UTX Series.

Prof Bapat has more than fifty research collaborators from around the globe. He has served on the editorial boards of several national and international journals, including Linear and Multilinear Algebra, Electronic Journal of Linear Algebra, Indian Journal of Pure and Applied Mathematics, Journal of Mathematical Science of Delhi University, and Kerala Mathematical Association Bulletin. He is an elected Fellow of the Indian Academy of Sciences, Bangalore and the Indian National Science Academy, Delhi. He was awarded the J C Bose Fellowship of DST in 2009. Prof Bapat served as the President of the Indian Mathematical Society during its centennial year 2007-2008. He is also the current President of Academy of Discrete Mathematics and Applications. For the past several years, he has been involved as the National Coordinator of the International Mathematics Olympiad Training Program in India.

The organizers of ICLAA 2014 are proud to honour Prof Bapat on the occasion of his 60th birthday. This volume 'Linear Algebra with Applications: A Volume in Honour of Professor Ravindra B Bapat' consists of many interesting articles published by Prof Bapat, and will certainly become an inspiring handbook for young researchers, looking for useful mathematical tools to solve diverse problems in the area of 'Linear Algebra and its Applications'.

The editors offer their sincere thanks to the publishers to Elsevier, Taylor \& Francis Ltd, Indian Statistical Institute, MATCH Communications in Mathematical and in Computer Chemistry, AKCE International Journal of Graphs and Combinatorics, and other publishers for granting permission to reproduce the selected articles for this academic cause. We are thankful to Peter Semrl, B LS Prakasa Rao, Ivan Gutmann, and S Arumugam for helping and guiding us in the process of obtaining necessary permission.

In attempting to reproduce the selected articles, a number of friends and colleagues have joined their hands in typing and proofreading the articles. K S Mohana (Mangalore University), P Divya Shenoy (Manipal Institute of Technology), K Arathi Bhat (Manipal Institute of Technology), Vinay Madhusudanan (Manipal Institute of Technology), and Swarup Panda (Indian Institute of Technology Guwahati) have alle contributed their valuable time in retyping these articles. Charlotte Prenzer (Department of Statistics, IAESTE intern), Y Santhi Sheela (Department of Statistics), and Sudhakara G (Manipal Institute of Technology) have proofread the reproduced material meticulously, thus helping us to achieve the present quality of reproduction of the articles. The Editorial Board applauds their dedicated efforts. Finally, the Editors would like to thank MUP and its team for the timely printing of the volume and the quality of print.

19 December 2014
Steve Kirkland
K Manjunatha Prasad
Sukanta Pati
Simo Puntanen

## Contents

Ravindra B Bapat: A Brief Note on His Life and Work ..... 3
K Manjunatha Prasad
Ravindra B Bapat ..... 13
T E S Raghavan
Welcoming Ravi Bapat to the Senior Citizens’ Club ..... 17
V S Sunder
A Generalization of a Theorem of Ky Fan on Simplicial Maps ..... 21
R B Bapat
On Diagonal Products of Doubly Stochastic Matrices ..... 27
R B Bapat and T E S Raghavan
$D_{1} A D_{2}$ Theorems for Multidimensional Matrices ..... 31
R B Bapat
Doubly Stochastic Matrices with Equal Subpermanents ..... 37
R B Bapat
A Matrix of Permanents ..... 43
R B Bapat
On Majorization and Schur Products ..... 49
R B Bapat and V S Sunder
An Extremal Property of the Permanent and the Determinant ..... 59
R B Bapat and V S Sunder
Application of an Inequality in Information Theory to Matrices ..... 69
R B Bapat
Multinomial Probabilities, Permanents and a Conjuncture of Karlin and Rinott ..... 77
R B Bapat
Order Statistics for Nonidentically Distributed Variables and Permanents ..... 85
R B Bapat and M I Beg
Mixed Discriminants of Positive Semidefinite Matrices ..... 99
R B Bapat
Permanents in Probability and Statistics ..... 113
R B Bapat
Generalized Inverses over Integral Domains ..... 129
R B Bapat, K P S Bhaskara Rao, and K Manjunatha Prasad
The Generalized Moore-Penrose Inverse ..... 141K Manjunatha Prasad and R B Bapat
The Moore-Penrose Inverse Over a Commutative Ring ..... 149
R B Bapat and Donald W Robinson
On Likelihood-Ratio Ordering of Order Statistics ..... 161
R B Bapat and Subhash C Kochar
Generalized Inverses with Proportional Minors ..... 169
R B Bapat
König's Theorem and Bimatroids ..... 175
R B Bapat
Singular Values and Maximum Rank Minors of Generalized Inverses ..... 185
R B Bapat and Adi Ben-Israel
A Max Version of the Perron-Frobenius Theorem ..... 195
R B Bapat
Algebraic Connectivity and the Characteristic Set of a Graph ..... 209
R B Bapat and S Pati
Generalized Power Symmetric Stochastic Matrices ..... 219R B Bapat, S K Jain, and K Manjunatha Prasad
Generalized Matrix Tree Theorem for Mixed Graphs ..... 229
R B Bapat, Jerrold W Grossman, and Devadatta M Kulkarni
Linear Estimation in Models Based on a Graph ..... 239
R B Bapat
Moore-Penrose Inverse of Set Inclusion Matrices ..... 247
R B Bapat
The Perturbed Laplacian Matrix of a Graph ..... 257R B Bapat, S J Kirkland, and S Pati
Outer Inverses: Jacobi Type Identities and Nullities of Submatrices ..... 275
R B Bapat
On Distance Matrices and Laplacians ..... 289
R B Bapat, S J Kirkland, and M Neumann
A $q$-Analogue of the Distance Matrix of a Tree ..... 305
R B Bapat, Arbind Kumar Lal, and Sukanta Pati
On Euclidean Distance Matrices ..... 321
R Balaji and R B Bapat
Resistance Matrices of Blocks in a Graph ..... 331
R B Bapat and Somit Gupta
Identities for Minors of the Laplacian, Resistance and Distance Matrices ..... 341
R B Bapat and Sivaramakrishnan Sivasubramanian
On the First Passage Time of a Simple Random Walk on a Tree ..... 353
R B Bapat

## About Ravindra B Bapat

# Ravindra B Bapat: A Brief Note on His Life and Work 

K Manjunatha Prasad

Writing about Prof Bapat gives me an immense pleasure, as our relation has been that of an advisor and a student. Even better, as I may say, he has been my mentor, my friend, and has guided me throughout. My advisor during my PhD at ISI Bangalore was Prof K P S Bhaskara Rao, and Prof S K Mitra during my post doctoral research at ISI Delhi. But, my association with Prof Bapat was for much longer duration and continuous. He visited ISI Bangalore for a duration of five months, when I was in second year term of my research fellowship. That was the occasion when I was just about to finish my course work and get introduced to the research area of generalized inverse by Prof Rao. We (myself, Rao and Bapat) used to meet at


Photograph 1 Prof Ravindra Bapat the common hall on a regular basis and on most of the occasions, we used to start with the question 'What is new today?' and continue our discussion with many stretches of long silence in between. With some conjecture - as an intuitive solution to a problem in my mind, he used to help me in discussion with several mathematical tools to prove the conjecture or to obtain counterexamples. Being among the top in the area of his research interest, he had been continuously promoting mathematics around him and in the nation. The list of collaborators within India and abroad speaks for itself the fact. He was very helpful and readily available to lead the team from the front. He was generous enough to spend his valuable time with research students at Manipal University, and led us in organizing CMTGIM 2012 - an international workshop and conference held in 2012 at Manipal.

In the following, I am making an attempt to compile some information on Prof Bapat, extracted from his bio-data and personal discussion.

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