Dedicated to Professor R B Bapat on the occasion of his 60th birthday

#### **Foreword**



It gives me immense pleasure that the Department of Statistics of Manipal University is organizing an International Conference on Linear Algebra & its Applications on 18–20 December 2014. Linear Algebra is an important concept of mathematics and is applied in various disciplines like engineering, physics, natural sciences, social sciences and so on. With the participation of national and international professionals, I am confident that the conference will show great results. Dr Ravindra B Bapat, Professor, Indian Statistical Institute is a doyen in the field of statistics and is known for his contributions in this area of Mathematics. He has published several original research papers in numerous international journals. As a mark of appreciation of his contributions, a compilation of his selected publications is being published to be released during the conference on the occasion of his 60th birthday. This volume will provide new

direction and technique to address future research problems in the area of Linear Algebra and its Applications.

Manipal University is happy to be associated with this event and wishes the best to Dr Bapat and the Organizers of this conference. My sincere thanks to all the publishers who have given permission to reproduce the selected articles in this volume, and helped us in this academic cause. I would also like to thank the editorial team lead by Dr Steve Kirkland for their effort in bringing out this volume.

December 2014 Dr H Vinod Bhat
Pro Vice-Chancellor, Manipal University Manipal

### **Preface**

Linear Algebra is a mathematical subject having applications in many branches of science including Biology, Economics, Engineering, Physics, and Statistics. The contribution of Professor Ravindra B Bapat in this area is enormous. The main research interests of Prof Bapat are Nonnegative Matrices, Matrix Inequalities, Matrices in Graph Theory, Matrix Methods in Statistics, and Generalized Inverses. He has published about 130 research papers in the above mentioned areas in reputed national and international journals, and guided three PhD students, all of whom have gone on to make their own contributions to Linear Algebra. He has written books titled "Nonnegative Matrices and Applications" with Prof TES Raghavan, in the Cambridge University Mathematical Series Encyclopedia of Mathematics and its Applications, "Linear Algebra and Linear Models" and "Graphs and Matrices" in the TRIM Series of the Hindustan Book Agency, New Delhi, India, which is co-published by Springer as its UTX Series.

Prof Bapat has more than fifty research collaborators from around the globe. He has served on the editorial boards of several national and international journals, including Linear and Multilinear Algebra, Electronic Journal of Linear Algebra, Indian Journal of Pure and Applied Mathematics, Journal of Mathematical Science of Delhi University, and Kerala Mathematical Association Bulletin. He is an elected Fellow of the Indian Academy of Sciences, Bangalore and the Indian National Science Academy, Delhi. He was awarded the JC Bose Fellowship of DST in 2009. Prof Bapat served as the President of the Indian Mathematical Society during its centennial year 2007–2008. He is also the current President of Academy of Discrete Mathematics and Applications. For the past several years, he has been involved as the National Coordinator of the International Mathematics Olympiad Training Program in India.

The organizers of ICLAA 2014 are proud to honour Prof Bapat on the occasion of his 60th birth-day. This volume 'Linear Algebra with Applications: A Volume in Honour of Professor Ravindra B Bapat' consists of many interesting articles published by Prof Bapat, and will certainly become an inspiring handbook for young researchers, looking for useful mathematical tools to solve diverse problems in the area of 'Linear Algebra and its Applications'.

The editors offer their sincere thanks to the publishers to Elsevier, Taylor & Francis Ltd, Indian Statistical Institute, MATCH Communications in Mathematical and in Computer Chemistry, AKCE International Journal of Graphs and Combinatorics, and other publishers for granting permission to reproduce the selected articles for this academic cause. We are thankful to Peter Semrl, B L S Prakasa Rao, Ivan Gutmann, and S Arumugam for helping and guiding us in the process of obtaining necessary permission.

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In attempting to reproduce the selected articles, a number of friends and colleagues have joined their hands in typing and proofreading the articles. K S Mohana (Mangalore University), P Divya Shenoy (Manipal Institute of Technology), K Arathi Bhat (Manipal Institute of Technology), Vinay Madhusudanan (Manipal Institute of Technology), and Swarup Panda (Indian Institute of Technology Guwahati) have alle contributed their valuable time in retyping these articles. Charlotte Prenzer (Department of Statistics, IAESTE intern), Y Santhi Sheela (Department of Statistics), and Sudhakara G (Manipal Institute of Technology) have proofread the reproduced material meticulously, thus helping us to achieve the present quality of reproduction of the articles. The Editorial Board applauds their dedicated efforts. Finally, the Editors would like to thank MUP and its team for the timely printing of the volume and the quality of print.

19 December 2014

Steve Kirkland K Manjunatha Prasad Sukanta Pati Simo Puntanen

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# **About Ravindra B Bapat**

### Ravindra B Bapat: A Brief Note on His Life and Work

K Manjunatha Prasad

Writing about Prof Bapat gives me an immense pleasure, as our relation has been that of an advisor and a student. Even better, as I may say, he has been my mentor, my friend, and has guided me throughout. My advisor during my PhD at ISI Bangalore was Prof K P S Bhaskara Rao, and Prof S K Mitra during my post doctoral research at ISI Delhi. But, my association with Prof Bapat was for much longer duration and continuous. He visited ISI Bangalore for a duration of five months, when I was in second year term of my research fellowship. That was the occasion when I was just about to finish my course work and get introduced to the research area of generalized inverse by Prof Rao. We (myself, Rao and Bapat) used to meet at the common hall on a regular basis and on most of the oc-



Photograph 1 Prof Ravindra Bapat

casions, we used to start with the question 'What is new today?' and continue our discussion with many stretches of long silence in between. With some conjecture – as an intuitive solution to a problem in my mind, he used to help me in discussion with several mathematical tools to prove the conjecture or to obtain counterexamples. Being among the top in the area of his research interest, he had been continuously promoting mathematics around him and in the nation. The list of collaborators within India and abroad speaks for itself the fact. He was very helpful and readily available to lead the team from the front. He was generous enough to spend his valuable time with research students at Manipal University, and led us in organizing CMTGIM 2012 – an international workshop and conference held in 2012 at Manipal.

In the following, I am making an attempt to compile some information on Prof Bapat, extracted from his bio-data and personal discussion.

K Manjunatha Prasad

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