Anisotropic and Strain-Dependent Model of Magnetostriction in Electrical Steel sheets

A. Belahcen[1,2], D. Singh[1], P. Rasilo[1,3], F. Martin[1], S. G. Ghalamestani[3], and L. Vandevelde[3]

[1]Aalto University, Dept. of Electrical Engineering and Automation, POBox 13000, FIN-00076, Aalto, Finland
[2]Tallinn University of Technology, Dept. of Electrical Engineering, Ehitajate tee 5, Tallinn, Estonia
[3]Ghent University, Dept. of Electrical Energy, Systems and Automation, Sint-Pietersnieuwstraat 41, BE-9000, Ghent, Belgium

This paper presents an anisotropic and mechanical strain-dependent model of magnetostriction in electrical steel sheets and its application in finite element computations. The presented model is bidirectional and the data needed for its derivation is extracted solely from unidirectional measurements under mechanical loading. The model has six parameters that describe the magnetic and strain behavior and two parameters that describe the anisotropy. The validation of the model is carried out through measurements and computations on a single-phase transformer-like device. The comparison between computation and measurement results seems to be reasonable regardless of the fact that the magnetic behavior is modeled as single valued, isotropic, and anhysteretic. Original magnetostriction measurements are also presented and the importance of magnetostriction anisotropy in a priori isotropic electrical steel sheets is demonstrated. The model is easy to implement in existing codes and the anisotropic behavior is straightforward to modify according to a specific material.

***Index Terms*—Anisotropy, magnetomechanical effects, magnetostriction, soft magnetic materials, transformers, vibrations.**

I. Introduction

MAGNETOSTRICTION of electrical steel sheets has been researched on several fronts and from different points of view. On one hand, extensive measurements have been conducted to characterize the magnetostrictive behavior of different materials [1]-[3] and on the other hand models of varying complexity have been developed to describe the observed magnetostrictive behavior [2]-[5]. Furthermore, incorporation of the developed models of magnetostriction into the simulation tools, mainly finite element (FE) programs, of electrical machines and devices has been presented [5], [6]. The magnetostriction model's complexity emanates from the complexity of the phenomenon itself, which depends on mechanical quantities such as stress or strain as well as on the microstructure of the magnetic material. These dependencies result in intrinsic and mechanically induced anisotropy of the magnetostriction deformation even if the magnetic behavior of the material can be to a large extent isotropic.

In this paper we present a bidirectional, anisotropic, and mechanical strain-dependent model of magnetostriction. The bidirectionality is derived using the principle of isochoric deformation and the fact that the magnetostriction strain-tensor is diagonal in the coordinate system defined by the magnetization direction. The model is said to be strain-dependent since it is the strain, rather than the stress, which is responsible for the behavioral change of magnetostriction. Although there is a natural relationship between the stress and the strain, several measurements presented in the literature, e.g. [1], bear out the strong influence of the strain on the magnetic and magnetostrictive properties of the material. Further, the developed model of magnetostriction is implemented and incorporated into in-house FE software and applied to the computation of a single phase transformer-like device for the purpose of validation.

The strain dependency of the presented model is verified through unidirectional measurements of a non-oriented (NO) electrical steel. The measurements have been carried out through a modified Epstein frame that allows the samples to be mechanically loaded [7]. The magnetostrictive anisotropy of the material is based on measurements made on NO samples with a single sheet tester equipped with a dual heterodyne laser interferometer [8].

The computation results show that both the strain-dependency and anisotropy of magnetostriction can be solved in a coupled scheme, which incorporates the solution of the magnetic field and circuit equations of the windings too. However, some convergence issues due to the implicit nonlinear relation between the stress and strain might arise.

The paper is organized so that in Section II the magnetostriction model is derived and presented, in Section III the methodology for incorporating the model into the finite element code is presented and in Section IV the validation of the model with appropriate measurement setup is shown. At the end, a conclusion section suggests how the present work can be used to derive a new version of the so called equivalent stress and the needed further developments of the model.

II. Model of Magnetostriction

There are many publications dealing with either the magnetic or magnetostrictive anisotropy on one hand or the stress-dependency of magnetic and magnetostrictive properties on the other hand. However, very little has been done on the coupling of anisotropy and stress-dependency. In this section we present an attempt to deal with this complex problem in a rather simple way.

A. The strain dependency

Previous unidirectional measurements on samples that are mechanically loaded (stressed) in the same direction as they are magnetized showed that there is a strong coupling between the measured magnetostrictive strain in the magnetization direction and the applied mechanical stress. Most researchers agree on this fact and they partially explain the physics behind this dependency. Recent bidirectional measurements [1], where the stress could be applied in all the plane directions combined with a magnetization of arbitrary direction revealed that the magnetostriction is actually rather strain- than stress-dependent. Such a conclusion was not presented in [1] but it could be drawn from the results. This suggested that the magnetostriction under bidirectional mechanical loading could be described through a dependency on the strain component parallel to the direction of magnetization as:

$$\varepsilon_{ms}^{\boldsymbol{M}//} = p(\boldsymbol{M}^2)h(\varepsilon^{\boldsymbol{M}//}), \tag{1}$$

where $p(x) = a_1x + a_2x^2 + a_3x^3$ is a third order polynomial, $\varepsilon^{\boldsymbol{M}//}$ is the total strain in the direction of magnetization $\boldsymbol{M}$ and h a strain-dependency function given by:

$$h(x) = a_4 + \tanh\left(\frac{a_5 - x}{a_6}\right). \tag{2}$$

$a_1 ... a_6$ are model parameters, which can be identified from unidirectional measurements as shown in Fig. 1. The bidirectional model will build on (1) and the assumption of isochoric magnetostrictive deformation.

B. The anisotropy

Since the magnetostrictive strain tensor is assumed to be solely defined by the strain component parallel to the magnetization direction, the anisotropy of magnetostriction can be described by the dependency of the amplitude of the magnetostrictive strain in the magnetization direction on the angle of the magnetization $\theta_{\boldsymbol{M}}$ with respect to the *xy*-reference frame. In this case (1) becomes:

$$\varepsilon_{ms}^{\boldsymbol{M}//} = f_{ani}(\theta_{\boldsymbol{M}})p(\boldsymbol{M}^2)h(\varepsilon^{\boldsymbol{M}//}), \tag{3}$$

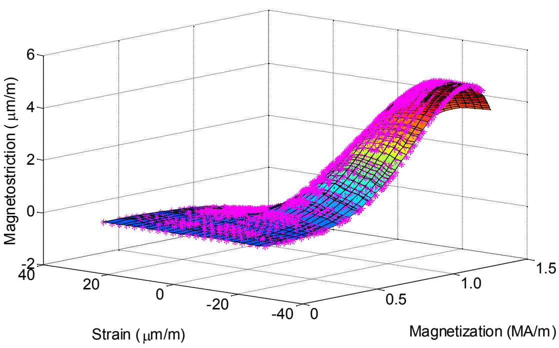

Fig. 1. Modeled (surface) and measured (stars) magnetostriction at different mechanical strains and magnetization levels. The results are from unidirectional rolling direction measurement and they are used in the bidirectional model after appropriate scaling of the transverse direction.

where f_{ani} is an anisotropy function. In this work f_{ani} is represented by the radius of an ellipse with major and minor axes as parameters as well as the direction of the major axis with respect to the *xy*-reference frame.

C. The total magnetostriction tensor

From the isochoric assumption, the magnetostrictive strain tensor in the reference frame of magnetization is written as:

$$\varepsilon_{ms}^{\boldsymbol{M}} = \begin{bmatrix} \varepsilon_{ms}^{\boldsymbol{M}//} & 0 \\ 0 & -\frac{1}{2}\varepsilon_{ms}^{\boldsymbol{M}//} \end{bmatrix}. \tag{4}$$

Using the transformation matrix $\boldsymbol{R}$ from the $\boldsymbol{M}$-reference frame to the *xy*-reference frame, we can write the magnetostrictive strain tensor in its general form as:

$$\varepsilon_{ms}^{xy} = \boldsymbol{R}^{\mathrm{T}}\varepsilon_{ms}^{\boldsymbol{M}}\boldsymbol{R}. \tag{5}$$

III. Incorporation into the FE Software

The starting point of the implementation is an existing in-house 2D software, where the magnetic field is formulated in terms of the z-component of the magnetic vector potential and an electric scalar potential that serves to couple the field equations with the circuit equations of any possible windings. The magnetic problem is formulated in a conventional way and does not need any further increments as the anisotropy in this work concerns only the magnetostriction behavior.

The elastic mechanical problem is formulated as a displacement field problem, which is derived from the constitutive equations following a similar procedure as in [9]. For the simplicity of the implementation, the magnetization dependency is written as a flux density dependency since the effect of the applied magnetic field strength is very low at the levels of magnetization we are working with. The needed equations for the implementation of the magnetostriction model are explained hereafter.

First the total mechanical stress $\boldsymbol{\tau}$ is written as:

$$\boldsymbol{\tau}(\boldsymbol{\varepsilon}, \boldsymbol{B}) = \boldsymbol{\tau}_{mech}(\boldsymbol{\varepsilon}) + \boldsymbol{\tau}_{ms}(\boldsymbol{\varepsilon}, \boldsymbol{B}), \tag{6}$$

where $\boldsymbol{\varepsilon}$ is the total mechanical strain and the two terms on the right of (6) stand for a mechanical stress corresponding to the total strain, and the magnetostrictive stress respectively. They are calculated following the generalized Hooke's law as:

$$\begin{aligned} \boldsymbol{\tau}_{mech} &= \boldsymbol{c}\boldsymbol{\varepsilon} \\ \boldsymbol{\tau}_{ms} &= -\boldsymbol{c}\boldsymbol{\varepsilon}_{ms}(\boldsymbol{\varepsilon}, \boldsymbol{B}) \end{aligned} \tag{7}$$

where $\boldsymbol{c}$ is the material stiffness tensor for either plane stress or plane strain. In this work the plane stress is used for better consistency with the isochoric assumption.

Eq. (6) was first linearized to the first order and then the principle of virtual work applied, which resulted in:

$$\int_{\Omega}\left(\hat{\boldsymbol{\varepsilon}}^{\mathrm{T}}\frac{\partial \boldsymbol{\tau}_{mech}}{\partial \boldsymbol{\varepsilon}}\Delta\boldsymbol{\varepsilon}\right)d\Omega+\int_{\Omega}\left(\hat{\boldsymbol{\varepsilon}}^{\mathrm{T}}\frac{\partial \boldsymbol{\tau}_{ms}}{\partial \boldsymbol{B}}\Delta\boldsymbol{B}\right)d\Omega+$$
$$\int_{\Omega}\left(\hat{\boldsymbol{\varepsilon}}^{\mathrm{T}}\frac{\partial \boldsymbol{\tau}_{ms}}{\partial \boldsymbol{\varepsilon}}\Delta\boldsymbol{\varepsilon}\right)d\Omega+\int_{\Omega}\left(\hat{\boldsymbol{\varepsilon}}^{\mathrm{T}}\boldsymbol{\tau}_{0}\right)d\Omega- \quad , \quad (8)$$
$$\int_{\Omega}\hat{\boldsymbol{u}}^{\mathrm{T}}(\boldsymbol{f}_{mech}+\boldsymbol{f}_{inert})d\Omega-\int_{\partial\Omega}\hat{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{f}_{surf}ds=0$$

where $\boldsymbol{u}$ is the mechanical displacement vector, $\boldsymbol{f}_{mech}$, $\boldsymbol{f}_{inert}$, $\boldsymbol{f}_{surf}$ are respectively mechanical, inertia body forces and surface forces. $\boldsymbol{\tau}_0$ stands for any additional stress e.g. thermal stresses. The ^ over the variables means that they are virtual ones. The magnetic forces resulting from the local application of the principle of virtual work (Generalized Nodal Forces [5]) are also included in the term $\boldsymbol{f}_{mech}$ and thus treated as mechanical load.

The electromagnetic problem and the elastic equation (8) are then discretised with first order triangular elements in a standard finite element fashion. The nonlinearities of the magneto-elastic problem are handled with the Newton-Raphson iteration scheme. The elastic problem is further dealt with using a relaxation procedure, which is needed to overcome the fact that the magnetostriction is not a monotone function of the magnetization and/or strain and the fact that an explicit full jacobian matrix is not possible to derive because in (7) the elastic stress depends on the total elastic strain, which is itself an implicit function of the magnetization, or in our implementation of the magnetic flux density.

IV. Results and Discussion

The validation of the presented model is carried out as a two-fold process. At a first stage, the magnetization and strain-dependency of the magnetostriction are verified through the single directional measurements under a given range of elastic strain. This first stage guarantees the validity of (1) and (2). The results from this validation are seen in Fig. 1 as a very good match between the measurements and the model-predicted magnetostriction. The measurements are carried out by applying a mechanical load in the same direction as the magnetization. In future work we will present results where the mechanical load is orthogonal to the magnetization.

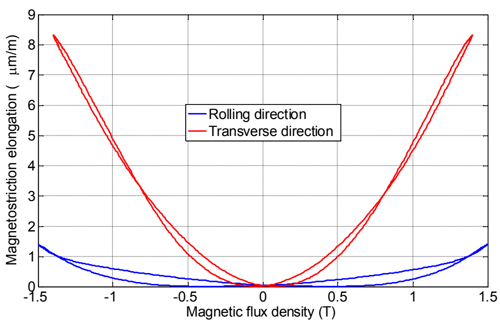

Fig. 2. Measured magnetostriction of the sample material in rolling and transverse directions at 50 Hz. The anisotropy is well seen as the magnetostriction in transverse direction is more than six times higher than in the rolling direction. The hysteretic behavior is very low.

In a second stage, the validity of the anisotropy formulation is carried out on a single-phase transformer-like device. The lamination of this device has been cut and the device constructed so that two legs are in the rolling direction of the material and the two others in the transverse direction. Three magnetizing coils connected in series have been wound around three legs of the device, whereas the fourth leg has been kept free for measuring its deformation. The lamination used to construct the device has been characterized using a single-sheet tester measurement setup equipped with a dual heterodyne laser interferometer as described in [8]. The magnetostriction curves of these electrical sheets in both rolling and transverse directions are shown in Fig. 2, where a strong magnetostrictive anisotropy can be noticed. Both curves correspond to parallel magnetostriction.

The structure of the verification device is shown in Fig. 3 altogether with the computed magnetic flux density distribution and the scaled deformation at the time instant when it is at its maximal value. The simulation of the device from which these results are obtained, has been carried out following the procedure presented in the previous section. An air region containing the other coil-sides (not shown in Fig. 3) has been used to ensure good accuracy of the magnetic computations with an outside Dirichlet boundary condition.

The boundary conditions for the mechanical problem have been set so that the lower left corner in Fig. 3 was fixed in both x- and y-directions and the lower right and upper left corners fixed in the y-direction and x-direction respectively. For clarity and ease of comparison purposes, the deformed structure has been shifted back to its center, using the displacement of the upper right corner of the core (Fig. 3). The device has been fed with a sinusoidal voltage, which produced

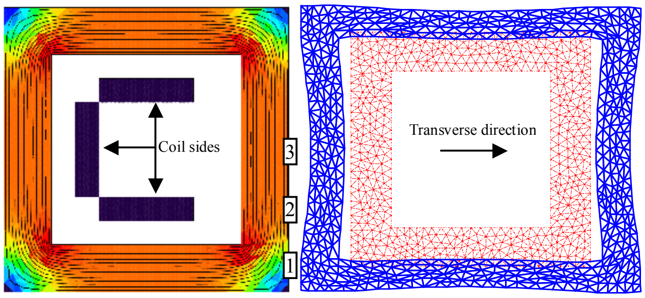

Fig. 3. Computed magnetic flux density distribution (left) and deformation (right) of the single-phase transformer-like device. The flux density in the legs is 1.22 T and the deformation in blue is scaled by a factor of 2.5x10[5]. The anisotropy effect on the deformation is seen as a larger horizontal deformation compared to the vertical one. 1, 2, and 3 are control points.

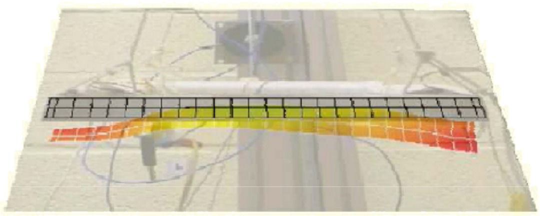

Fig. 4. Measured deformation of the single-phase transformer-like device presented on top of the photography of the set up. The maximum displacements are at 0.40 µm/m, corresponding to the dark-red regions. A slight skew of the deformation is present due to the support of the device.

a maximum flux density in the legs of 1.22 T.

We simulated 600 time steps, which corresponds to two periods of the line voltage (300 steps per period). Each time step took between 5 and 15 iterations depending on the level of magnetization and strain. At high values of the magnetostrictive strain, a relaxation factor of 0.5 was used and the number of iterations increased to 30-40.

Measurements have been conducted on the same device under the same conditions as in the simulations. The device was suspended and the displacements of the free leg measured with a scanning laser vibrometer PSV-400 Polytec. The measurement set up altogether with the measured deformation of the outer surface of the free leg is shown in Fig. 4. A comprehensive and detailed description of the measurement setup and the results from different measurements has been reported in [10].

The comparison between measurements and simulations is made based on the maximum displacement of three control points on the surface of the free leg, marked as 1, 2, and 3 in Fig. 3, as well as the visual comparison of the shape of the deformation of the surface. Figs. 3 and 4 show that the simulated and measured deformations match well. Further, the measured and simulated amplitudes of deformation, at 100 Hz, of three control points are respectively 0.381µm vs. 0.252 µm for point 1, 0.207 µm vs. 0.238 µm for point 2, and 0.130 µm vs. 0.225 µm for point 3. Displacements at the 200 Hz harmonic were also measured. They are 0.0459 µm, 0.0533 µm, and 0.0630 µm for points 1, 2, and 3 respectively, which ensures that the vibrations of the coil due to Lorentz forces are not interfering in the measurements.

The above results show that the proposed bidirectional and strain-dependent model of magnetostriction is able to reproduce the behavior of electrical steel, and that its incorporation into a FE code can be used to simulate complex geometries.

A close look at Fig. 4 shows that the measured deformation at the surface of the free leg is not uniform. It is affected by the quality of the ad hoc suspension system and the bending of the electrical sheets. Considering these inaccuracies and the tolerably low relative error of the modeling, we can say that the model is accurate. The validation of the model was made separately for the strain dependency and the anisotropy. This is rather a simplistic validation approach, which needs more development in order to verify the full coupled model. In future works we are planning to make measurements and simulations on a rotational single sheet tester with possibility of mechanical loading to tackle this shortcoming.

On the other hand, the presented model of strain-dependent magnetostriction can be used to define an equivalent uniaxial stress as it was attempted in [11]. This unidirectional equivalent stress will produce the same strain in the magnetization direction as the full bidirectional stress would produce. In this respect, the equivalent unidirectional stress will be dependent on the direction of magnetization contrary to the one presented in [11]. Further developments of this issue are under work.

It should be mentioned here too, that the anisotropy has been dealt with only from the point of view of magnetostriction as related to the strain-dependency. This means that the model is valid only for materials that are non-oriented. A more general approach is to account for the magnetic anisotropy too. This could be done if the presented model is combined with the methodology presented in [12] for example. However, the convergence of such models and the related computation time could be a drawback in this approach especially if the magnetic and magnetostrictive anisotropy have different directions.

V. Conclusion

Besides the fact that the presented anisotropic and strain-dependent model of magnetostriction can describe to a great level of accuracy the behavior of electrical steel sheets, it could also be used to derive a version of the so called equivalent stress. This equivalent stress is simply the unidirectional stress parallel to the magnetization direction that produces the same strain in that direction as the total applied bidirectional stress. In future works we will develop more on this aspect as well as on the hysteretic behavior and the limitations associated with the isochoric assumption.

References

[1] Y. Kai, Y. Tsuchida, T. Todaka, M. Enokizono, “Influence of stress on vector magnetic property under rotating magnetic flux conditions,” *IEEE Trans. Magn.*, vol. 48, no. 4, pp. 1421-1424, 2012.

[2] S. Somkun, A.J. Moses, P.I. Anderson, P. Klimczyk, “Magnetostriction anisotropy and rotational magnetostriction of a nonoriented electrical steel,” *IEEE Trans. Magn.*, vol. 46, no. 2, pp. 302-305, 2010.

[3] G. Shilyashki, H. Pfüetzner, V. Galabov, F. Hofbauer, E. Mulasalihovic, J. Anger, K. Gramm, “Magnetostriction of transformer core steel considering rotational magnetization,” *IEEE Trans. Magn.*, 2013.

[4] P. Rasilo, D. Singh, A. Belahcen, A. Arkkio, “Iron losses, magnetoelasticity and magnetostriction in ferromagnetic steel laminations,” *IEEE Trans. Magn.*, vol. 49, no. 5, pp. 2041-2044, 2013.

[5] A. Belahcen, “Vibrations of rotating electrical machines due to magnetomechanical coupling and magnetostriction,” *IEEE Trans. Magn.*, vol. 42, no. 4, pp. 971-974, 2006.

[6] H. Ebrahimi, G. Yanhui, A. Kameari, H. Dozono, K. Muramatsu, “Coupled magneto-mechanical analysis considering permeability variation by stress due to both magnetostriction and electromagnetism,” *IEEE Trans. Magn.*, vol. 49, no. 5, pp. 1621-1624, 2013.

[7] A. Belahcen, M. El Amri, “Measurement of stress-dependent magnetisation and magnetostriction of electrical steel sheets,” *Intern. Conf. on Elect. Machines ICEM*, Cracow, Poland, 5-8.9.2004, 4 p.

[8] S.G. Ghalamestani, T. Hilgert, L. Vandevelde, J. Dirckx, J.A.A. Melkebeek, “Magnetostriction measurement by using dual heterodyne laser interferometers,” *IEEE Trans. Magn.*, vol.46, no.2, pp.505-508, Feb. 2010

[9] K. Fonteyn, A. Belahcen, R. Kouhia, P. Rasilo, A. Arkkio, “FEM for directly coupled magneto-mechanical phenomena in electrical machines,” *IEEE Trans. Magn.*, vol. 46, no. 8, pp. 2923-2926, Aug. 2010.

[10] S. Gorji Ghalamestani, L. Vandevelde, J.J.J. Dirckx, P. Guillaume, J.A.A. Melkebeek, “Magnetostrictive deformation of a transformer: A comparison between calculation and measurement,” Inter J of App Electromagn. and Mech., vol. 44, no. 3-4, pp. 295-299, 2014.

[11] L. Daniel, “An analytical model for the effect of multiaxial stress on the magnetic susceptibility of ferromagnetic materials,” *IEEE Trans. Magn.*, vol. 49, no. 5, pp. 2037-2040, May. 2013.

[12] O. Bıro, S. Außerhofer, K. Preis. and Y. Chen. “A modified elliptic model of anisotropy in nonlinear magnetic materials,” COMPEL, vol. 29, pp. 1482-1492, 2010.