Nearest Keyword Search in XML Documents

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ABSTRACT

This paper studies the *nearest keyword* (*NK*) problem on XML documents. In general, the dataset is a tree where each node is associated with one or more keywords. Given a node q and a keyword w, an NK query returns the node that is nearest to q among all the nodes associated with w. NK search is not only useful as a standalone operator but also as a building brick for important tasks such as XPath query evaluation and keyword search. We present an indexing scheme that answers NK queries efficiently, in terms of both practical and worst-case performance. The query cost is provably *logarithmic* to the number of nodes carrying the query keyword. The proposed scheme occupies space *linear* to the dataset size, and can be constructed by a fast algorithm. Extensive experimentation confirms our theoretical findings, and demonstrates the effectiveness of NK retrieval as a primitive operator in XML databases.

Categories and Subject Descriptors

H3.1 [**Content analysis and indexing**]: Indexing methods

General Terms

Theory

Keywords

Nearest keyword, XPath, keyword search, group steiner tree

1. INTRODUCTION

We consider the problem of *nearest keyword* (*NK*) search on XML documents. The dataset is a tree $\mathcal{T}$ with undirected edges. Each node is associated with one or more keywords. Define the *distance* between two nodes as the number of edges in the (unique) path linking them. Given a node q in $\mathcal{T}$ and a keyword w, an *NK query* finds the *nearest* w*-neighbor* of q, namely, the node having the minimum distance to q among all the nodes associated with w.

To illustrate, Figure 1 shows part of an XML document, where all nodes have been encoded with the Dewey code [34] for easy reference. An *element node* has its *type* displayed in brackets, while the other nodes are *value nodes*. Given a node $q =$ `012000` and a keyword $w =$ `guard`, an NK query returns node `012010`, whose distance to q is 4 (edges). It is the nearest `guard`-neighbor of q.

1.1 Motivation

NK queries can serve as the building brick to tackle some important problems in XML databases, as elaborated below. For convenience, we assign each value node to a type, whose name concatenates its parent's type and the string `Val` (e.g., node `0100` has type `tnameVal`, and so does node `0200`). Every node carries its type as a keyword. In addition, each value node has its value as another keyword. For example, node `0100` has two keywords: its type `tnameVal`, and its value `Lakers`.

XPath query evaluation. NK search gives a new methodology for efficiently solving a class of XPath queries. An example is:

> Q*: Find the names of all players that originated from Maryland, but are in a team of the west division.*

The XPath statement of Q can be expressed as a *twig pattern* [4, 15] shown in Figure 2a, where a single-lined (double-lined) edge represents parent-child (ancestor-descendent) relationship. The goal is to find *occurrences* of the pattern in the data tree, and for each occurrence, output the value at the position of `pnameVal` (signified as underlined). Figure 2b demonstrates such an occurrence in Figure 1, from which the output is the value `Blake` of node `012000`.

There are two interesting facts about the pattern Q in Figure 2a, with respect to the data of Figure 1:

- Let q be the node in an occurrence corresponding to the node `Maryland` of Q. The type of q is `fromVal`. The nearest `west`-neighbor of q must have distance exactly 6 to q. For example, q is node `012020` in Figure 2b, and its nearest `west`-neighbor is node `0110`.
- Let q be any `fromVal` node that carries the word `Maryland`, but is *not* in any occurrence, i.e., the team of q is in the east division. The nearest `west`-neighbor of q must have distance *greater than* 6 to q, noticing that the neighbor must come from a team different from that of q. For example, let q be node `022020` in Figure 1. Its nearest `west`-neighbor is node `0110`, which has distance 8 to q.

The above facts enable us to process Q via NK search as follows. We enumerate all the `fromVal` nodes that contain `Maryland`. For each such node q, find its nearest `west`-neighbor. If the neighbor retrieved has a distance greater than 6 to q, it is ignored. Otherwise, we have found an occurrence, from which the `pnameVal` should be output. The `pnameVal` node can be found with another NK query, which obtains the nearest `pnameVal`-neighbor of q.

Group steiner tree retrieval. *Keyword search* has emerged as a new paradigm of inquiring XML databases. It enables a user to

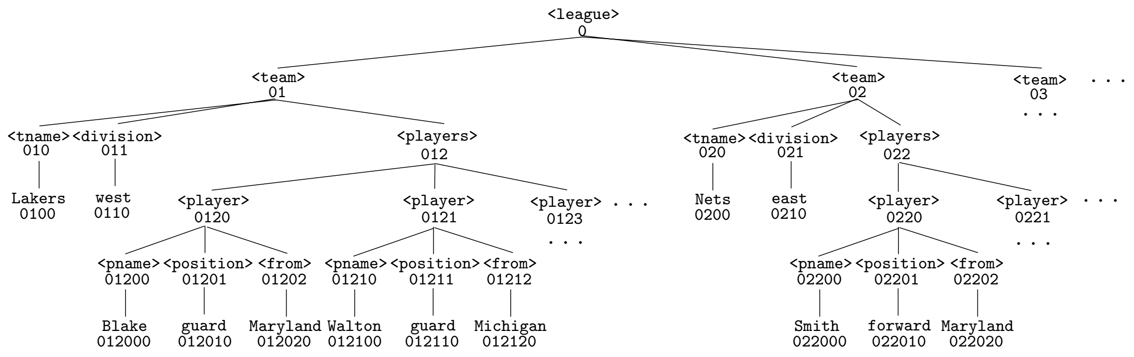
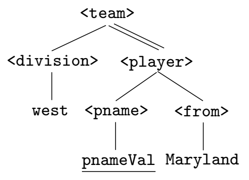
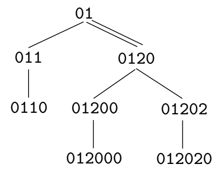


Figure 1: A sample of the NBA dataset





(a) Twig pattern for Q (b) An occurrence in Figure 1

Figure 2: An XPath query example

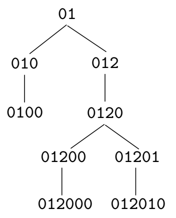


Figure 3: The group steiner tree of {`Lakers`, `Blake`, `guard`} in Figure 1

specify only a few keywords as the query, instead of complying with a rigorous syntax. Its advantage is that the user does not need to learn any query language like XPath, and neither does s/he need to be aware of the data schema. The disadvantage, however, is that what should be the query result becomes heavily dependent on the application backdrop. This has triggered the propositions of a variety of result semantics, among which an intuitive one is to return the *group steiner tree* (*GST*) [12, 20, 23, 25].

More specifically, given a set of query keywords $\{w_1, ..., w_l\}$, a GST is a tree that (i) contains all the query keywords in the texts of its nodes, and (ii) has the fewest edges among such trees. For example, Figure 3 presents the GST for $\{$`Lakers`, `Blake`, `guard`$\}$ on the data of Figure 1. GST computation is known to be NP-hard (even if the dataset is a tree) [17]. Fortunately, as discussed later, NK queries provide an elegant way to extract a good approximate solution, namely, a tree that satisfies requirement (i), and has more edges than the GST by only a small factor.

1.2 Contributions

This paper presents the first study on the NK problem. We propose an indexing scheme that can answer any NK query in $O(\log N_w)$ time, where N_w is the number of nodes associated with the query keyword w. The scheme consumes space *linear* to the size of the dataset. Somewhat surprising is the fact that, despite the complication of the underlying theory, our access method can be implemented as merely a number of binary trees. All the results also hold in disk-oriented environments, where each binary tree is simply replaced with a B-tree. Accordingly, the query cost is $O(\log_B N_w)$ I/Os, where B is the size of a disk block.

The proposed index also leads to rigorous results on the usefulness of the NK operator. Specifically:

- We theoretically establish the fact that a large class of XPath queries can be reduced to NK search (in a way similar to how the query of Figure 2a was answered earlier). Our algorithm for processing this query class enjoys a worst-case time complexity that is *irrelevant* to the number of elements whose types appear as an internal node of Q. No previous solution is known to have this feature (as surveyed in Section 5).

- We give a fast solution to finding an approximate GST with an attractive quality guarantee (which is actually optimal if the query has only two keywords). We achieve the running time of $O(N_{min} l \log N_{max})$, plus the cost of outputting the resulting tree, where l is the number of keywords in the query, N_{min} is the number of nodes carrying the *rarest* query keyword (i.e., the one appearing the least times in the XML document), and N_{max} conversely is the number of nodes carrying the *most frequent* query keyword.

Besides confirming our theoretical findings, our experimentation also demonstrates the effectiveness of the NK operator on real XML documents. In particular, we show that XPath queries like the one in Figure 2a can be processed via NK search with performance comparable to or better than that of the existing approaches. Furthermore, for XML keyword search, our NK-based algorithm discovers high-quality approximate GSTs in real time.

Roadmap. The rest of the paper is organized as follows. Section 2 clarifies the problem definition and several technical preliminaries. Section 3 elaborates on the proposed solutions for NK search. Section 4 discusses the applications of NK queries in XPath evaluation and keyword search. Section 5 reviews the previous work related to ours. Section 6 contains extensive experimentation to evaluate the effectiveness and efficiency of our techniques. Finally, Section 7 concludes the paper with a summary of our findings.

2. PRELIMINARIES

For each node u in the data tree $\mathcal{T}$, we use $W(u)$ to represent the set of keywords associated with u. For simplicity, assume that $W(u)$ has at least one keyword. Define the *length* of a path in $\mathcal{T}$ as the number of edges it contains. Denote by $\|u, v\|$ the distance between two nodes u, v, namely, the length of the path connecting u and v. Let $U(w)$ be the set of nodes in $\mathcal{T}$ that include word w (hence, $N_w = |U(w)|$). Given a node q and a keyword w, the result of an NK query is a node $u \in U(w)$ such that

$$\|u, q\| \leq \|v, q\| \qquad \forall v \in U(w).$$

We denote u, the nearest w-neighbor of q, as $NN(q, w)$.

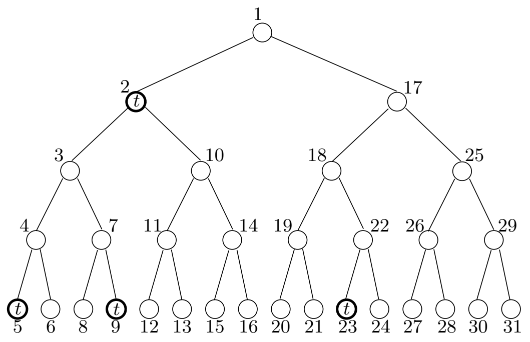


Figure 4: A running example

Figure 4 shows an example $\mathcal{T}$, where each node is labeled an integer. Sometimes we may refer to a node by its label directly, when the meaning is clear. For instance, as node 2 carries a single keyword t, we write $W(2) = \{t\}$. Similarly, as t also appears in nodes 5, 9 and 23, $U(t) = \{2, 5, 9, 23\}$. The keywords of the other nodes are omitted for clarity. Given $q =$ node 17 and $w = t$, an NK query returns node 2, namely, $NN(17, t) = 2$.

Let N be the number of nodes in $\mathcal{T}$, and K the total number of keywords in all the nodes (counting a word twice if it appears in two nodes), i.e., $K = \sum_u |W(u)|$. Note that $\mathcal{T}$ requires $\Omega(K)$ space to store. In other words, *linear* cost should be interpreted as $O(K)$, instead of $O(N)$. We label the levels of $\mathcal{T}$ in a top-down manner, setting the root at level 0. Denote by $level(u)$ the level of a node u, which is also the number of edges on the path from the root to u. In Figure 4, all the leaf nodes are at level 4. Also, we use $sub(u)$ to represent the subtree of u.

In the sequel, we review several basic results useful in our technical discussion.

Interval encoding. For each node u of $\mathcal{T}$, define its *rank*, denoted as $rank(u)$, to be the sequence number of u in the *pre-order* traversal of $\mathcal{T}$. We associate u with an interval $R(u) = [x, y]$, where x is the rank of u, and y is the largest rank of the nodes in $sub(u)$. In Figure 4, the label of each node indicates its rank directly. As an example, the interval $R(10)$ associated with node 10 is $[10, 16]$.

The intervals defined this way have several properties commonly utilized in managing XML data:

- For any two nodes u and v, $R(u)$ contains $R(v)$ *if and only if* u is an ancestor of v. In other words, the ancestor-descendent relationship of u and v can be verified in constant time.
- The intervals of the nodes at the *same* level of $\mathcal{T}$ must be disjoint. In Figure 4, for instance, the nodes at level 2 have disjoint intervals $R(3) = [3, 9]$, $R(10) = [10, 16]$, $R(18) = [18, 24]$, and $R(25) = [25, 31]$.

The above properties allow us to solve the so-called *level-on-path* queries efficiently. Let u be an ancestor of v; given a level $\ell \in [level(u), level(v)]$, a level-on-path query finds the level-ℓ node on the path from u to v. For example, if u (v) is node 1 (15), a level-on-path query with $\ell = 2$ retrieves node 10.

LEMMA 1. *$\mathcal{T}$ can be pre-processed into a structure that occupies $O(N)$ space, such that any level-on-path query can be answered in $O(\log N)$ time. The structure can be built in $O(N \log N)$ time.*

PROOF. We manage the nodes of $\mathcal{T}$ of each level separately. Specifically, create a binary tree to index the ranks of the nodes at the same level (i.e., there are as many trees as the number of levels in $\mathcal{T}$). As each node appears in only one tree, the overall space is $O(N)$. Assume that we want to perform a level-on-path operation to find the level-ℓ node from u to v (which is a descendent of u). We find in $O(\log N)$ time the predecessor of $rank(v)$, among all the ranks indexed in the level-ℓ binary tree. The node whose rank equals that predecessor is exactly what we are looking for. □

Subtree NK search. NK search is easy if attention is restricted to the subtree of the query node. Formally, given a node q and a keyword w, a *subtree-NK query* finds the node u with the smallest distance to q, among all nodes in $sub(q)$ that are associated with w. We refer to u as the *subtree nearest w-neighbor* of q. For instance, let q be node 17 in Figure 4; the subtree-NK query with $w = t$ returns node 23. Note that the (global) nearest t-neighbor of node 17 is in fact node 2.

LEMMA 2. *$\mathcal{T}$ can be pre-processed into a structure that occupies $O(K)$ space, such that any subtree-NK query can be answered in $O(\log N_w)$ time. The structure can be built in $O(K \log K)$ time.*

PROOF. Let us first review a related result. Let S be a set of numbers in the real domain $\mathbb{R}$. Each number $x \in S$ is associated with a *weight* in $\mathbb{R}$. Given an interval I, a *range-min* query finds the number that has the minimum weight among all the numbers in $S \cap I$. We can index S with an SB-tree [36] that uses $O(|S|)$ space, and solve any range-min query in $O(\log |S|)$ time. The tree can be built in $O(|S| \log |S|)$ time.

We can convert subtree-NK search to the range-min problem. Let w be the keyword of concern. Construct S to include the ranks of the nodes in $U(w)$. Each rank is associated with a weight that equals the level of the corresponding node. Subtree-NK search with node q is equivalent to a range-min query on S with interval $R(q)$. We settle the problem with an SB-tree in $O(\log N_w)$ query time. The tree occupies $O(N_w)$ space and can be built in $O(N_w \log N_w)$ time. The SB-trees of all keywords require $O(\sum_w N_w) = O(K)$ space in total. The overall construction time is $O(\sum_w (N_w \log N_w)) = O(K \log K)$. □

Both Lemmas 1 and 2 will be needed to analyze the construction cost of the proposed structure.

Lowest common ancestor (LCA). We use $lca(u, v)$ to denote the LCA of nodes u, v in $\mathcal{T}$ (e.g., $lca(20, 26)$ is node 17 in Figure 4). In general, the distance of two nodes can be calculated in constant time, once their LCA has been identified, as can be seen from the following equation:

$$\|u, v\| = (level(u) - level(z)) + (level(v) - level(z))$$

where $z = lca(u, v)$.

LCA computation has been thoroughly studied. Harel and Tarjan [18] were the first to observe that the problem can be settled optimally in constant time using linear space. Their structure, however, is rather theoretical and difficult to implement. To remedy the drawback, several (much) simpler structures [1, 2, 13] have been developed, keeping the same space and query performance. As a corollary, we can obtain $\|u, v\|$ of any u, v in constant time.

3. NEAREST KEYWORD SEARCH

We pre-process the data tree $\mathcal{T}$ by building a separate structure for each distinct keyword w that appears in $\mathcal{T}$. This is reminiscent of the *inverted index*, which also has an *inverted list* dedicated to each w. Instead of a simple list, however, our structure for w is a binary tree constructed in a more sophisticated manner.

3.1 Overview

We concentrate on NK queries with a specific keyword w, as the structure is identical for all keywords. The term "nearest w-neighbor" will be abbreviated as *nearest neighbor* (*NN*), when no ambiguity arises. Accordingly, we simplify notation $NN(u, w)$ to $NN(u)$.

A straightforward solution to answering an NK query is to perform a *breath first traversal* (*BFT*) starting from q. Namely, the BFT explores the nodes of $\mathcal{T}$ in ascending order of their distances to q, and stops as soon as it encounters a node associated with w. This approach is efficient only if the NN of q is close, and may end up visiting a large number of nodes otherwise.

Alternatively, we can pre-compute the NN of every node in $\mathcal{T}$. Each query can be answered in constant time, because we can simply return the (pre-computed) NN of the query node q. This approach, however, has the severe drawback that, the pre-computation incurs $\Omega(N)$ space for *every* keyword appearing in T. The number of distinct keywords can be easily $\Omega(N)$. In this case, the space complexity of the above approach is $\Omega(N^2)$, which is prohibitively large in practice.

The chief observation towards reducing the space is that, many nodes of $\mathcal{T}$ have the same NN, thus raising the hope that we could capture them collectively with much less information. Recall that each node can be uniquely identified by its rank, while the ranks of all nodes come from the *rank domain* $\mathbb{D} = [1, N]$ (e.g., $\mathbb{D} = [1, 31]$ in Figure 4). We can always partition $\mathbb{D}$ into a set $\mathcal{I}$ of disjoint intervals such that, for each interval $I \in \mathcal{I}$, the nodes with ranks in I have the same NN, which can be associated with I. Given an NK query with node q, we can solve it by identifying the (only) interval I that covers $rank(q)$, and returning the NN associated with I. This can be easily achieved by indexing $\mathcal{I}$ with a binary tree, which consumes $O(|\mathcal{I}|)$ space and has query cost $O(\log |\mathcal{I}|)$. Figure 5 illustrates the contents of a possible $\mathcal{I}$ for the data of Figure 4 when the keyword w of concern is t.

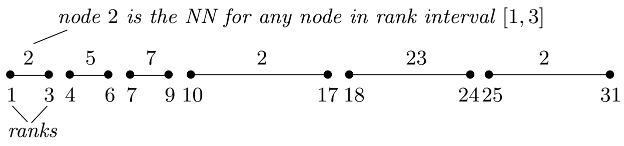


Figure 5: A tree Voronoi partition

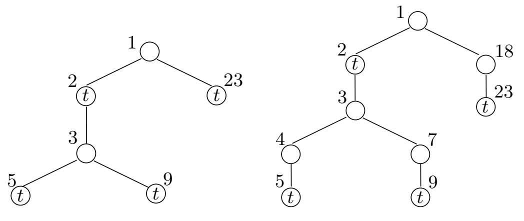
We refer to $\mathcal{I}$ as a *tree Voronoi partition* (*TVP*) *of* w. An immediate issue is whether a small $\mathcal{I}$ *always* exists. Fortunately, we will show in Section 3.2 that there is definitely an $\mathcal{I}$ with size $O(N_w)$, where N_w is the size of $U(w)$ (i.e., the number of nodes in $\mathcal{T}$ carrying w). Furthermore, the size of $O(N_w)$ is asymptotically tight because $|\mathcal{I}|$ needs to be at least N_w – every node in $U(w)$ apparently finds itself as the NN.

Another important issue is how to compute $\mathcal{I}$ efficiently. Naively, one could first compute the NN of every node in $\mathcal{T}$, and then go over the nodes in ascending order of their ranks, merging consecutive nodes into an interval if their NNs are the same. This approach, however, entails $\Omega(N)$ time, which would render the total pre-computation cost (for all keywords) prohibitively expensive in practice. In Section 3.3, we will give a significantly faster algorithm to produce $\mathcal{I}$ in $O(N_w \log N_w)$ time.

3.2 TVP characteristics

This subsection will establish our first main result:

THEOREM 1 (TVP THEOREM). *For any keyword w appearing in $\mathcal{T}$, there is a tree Voronoi partition $\mathcal{I}$ with size less than $8N_w$, where N_w is the number of nodes in $\mathcal{T}$ associated with w.*



(a) $CT(t)$ (b) $ECT(t)$

Figure 6: Compact and extended compact trees

Let us start the proof by introducing the *compact tree* of w, denoted as $CT(w)$. First, all the nodes of $U(w)$ are in $CT(w)$, and termed the *data nodes*. Second, a non-data node u belongs to $\mathcal{T}$, *if and only if* there are *at least* two child nodes of u whose subtrees contain a data node. We call u a *branching node*. Consider $w = t$ in Figure 4. There are four data nodes 2, 5, 9, 23, and two branching nodes 1, 3. Node 17, for example, is not a branching node because only *one* of its child nodes (i.e., node 18) has a data node in its subtree. Let S be the set of all data and branching nodes. We form $CT(w)$ by drawing an edge from each node $u \in S$ to its lowest ancestor in S. Figure 6a shows the $CT(t)$ for the data of Figure 4. Node 5, for instance, is connected to node 3, because among all the data and branching nodes, node 3 is the lowest ancestor of node 5.

LEMMA 3. *$CT(w)$ has at most $2N_w - 1$ nodes.*

PROOF. Each branching node must have at least two child nodes in $CT(w)$. As $CT(w)$ has at most N_w leaf nodes, the total number of branching nodes cannot be more than $N_w - 1$. $\square$

Consider any edge (u, v) in $CT(w)$. Let us walk, in the data tree $\mathcal{T}$, along the path from u to v. As we go, monitor the $NN(z)$ of the node z being visited, and count how many *changes* in $NN(z)$ there are in total. Call each of those changes an *NN-change* on (u, v). As an example, consider edge $(1, 23)$ in the $CT(t)$ of Figure 6a. Now we walk from node 1 to node 23 in the $\mathcal{T}$ of Figure 4. Along the path there is only a single NN-change, which happens as we move from node 17 to node 18 (i.e., $NN(17) = 2$, but $NN(18) = 23$). The next lemma gives an important fact:

LEMMA 4. *There can be at most one NN-change on each edge of $CT(w)$.*

PROOF. Let (u, v) be an edge of $CT(w)$. The removal of (u, v) cuts $CT(w)$ into two connected components. Let C_u (C_v) be the component including u (v). Denote by P the path from u to v in $\mathcal{T}$. Suppose that $u^\star$ ($v^\star$) is the data node in C_u (C_v) closest to u (v). We claim that, for any node z on P, $NN(z)$ must be either $u^\star$ or $v^\star$. In fact, for any node $u' \in C_u$, it holds that

$$\|z, u^\star\| = \|z, u\| + \|u, u^\star\| \leq \|z, u\| + \|u, u'\| = \|z, u'\|.$$

Similarly, for any node $v' \in C_v$, we have $\|z, v^\star\| \leq \|z, v'\|$. Therefore, except $u^\star$ and $v^\star$, no other data node in $C_u \cup C_v$ can be the NN of z.

Hence, if there were at least two NN-changes on (u, v), there would have to be three nodes z_1, z_2, z_3 on P such that z_2 was on the path from z_1 to z_3, but $NN(z_1) = NN(z_3) \neq NN(z_2)$. It is trivial to show that such a scenario cannot happen. $\square$

Next, $CT(w)$ is augmented with the nodes where NN-changes occur. Consider any edge (u, v) of $CT(w)$ on which there is an

NN-change. Let z be the first node on the u-to-v path in $\mathcal{T}$ such that $NN(z) \neq NN(u)$. If z is different from v, we add it to $CT(w)$, breaking (u, v) into two edges (u, z) and (z, v). Denote by $ECT(w)$ the resulting tree, after applying such transformation on all edges of $CT(w)$ with NN-changes. In case $root(\mathcal{T})$ is not already in $ECT(w)$, we add it as the parent of the current root of $ECT(w)$. The final $ECT(w)$ is called the *extended compact tree* of w. Let us illustrate the transformation with edge $(1, 23)$ in the $CT(t)$ of Figure 6a (i.e., $u = 1, v = 23$). As mentioned earlier, on the path from node 1 to node 23 in Figure 4, there is an NN-change as we cross from node 17 to node 18. Hence, $z = 18$, and accordingly, $(1, 23)$ is broken into two edges $(1, 18)$ and $(18, 23)$ in the extended compact tree $ECT(t)$, as shown in Figure 6b.

We are ready to generate a tree Voronoi partition $\mathcal{I}$ of w. It suffices to invoke the following algorithm for every node u of $ECT(w)$ in turn (ordering does not matter):

```
algorithm voronoiIntv($u$)
/* $u$ is a node in $ECT(w)$ */
1.  $S = \{R(u)\}$
2.  for each child node $v$ of $u$ in $ECT(w)$ do
3.      $I \leftarrow$ the (only) interval in $S$ covering $R(v)$
4.      break $I$ into intervals $I_1, R(v), I_2$
        /* $I_1$ ($I_2$) is the part of $I$ to the left (right) of $R(v)$ */
5.      remove $I$ from $S$, and add $I_1, I_2$
6.  add to $\mathcal{I}$ the intervals in $S$, after associating them with $NN(u)$
```

For example, let u be node 1 in the $ECT(t)$ of Figure 6b. At Line 1, `voronoiIntv` sets $S = \{R(1)\} = \{[1, 31]\}$. Since node 1 has two child nodes in $ECT(t)$, the for-loop in Lines 2-5 is executed twice. The first time trims $R(2) = [2, 16]$ away from $[1, 31]$, after which $S = \{[1, 1], [17, 31]\}$. The second execution cuts $R(18) = [18, 24]$ out of $[17, 31]$, leaving $S = \{[1, 1], [17, 17], [25, 31]\}$. Line 6 adds all three intervals of S to $\mathcal{I}$, after associating them with $NN(1) = 2$, indicating that node 2 is the NN of any node (whose rank falls) in those intervals.

LEMMA 5. *Applying* `voronoiIntv` *to all nodes of $ECT(w)$ creates a tree Voronoi partition $\mathcal{I}$ with less than $8N_w$ intervals.*

PROOF. We start by proving that the intervals of $\mathcal{I}$ are disjoint, and their union covers the rank domain $\mathbb{D}$. Observe that the intervals inserted in $\mathcal{I}$ at Line 6 are disjoint with the $R(v)$ of any child node v of u, where u is the input to the current execution of `voronoiIntv`. None of those intervals can overlap with the intervals added to $\mathcal{I}$ by the execution of `voronoiIntv` invoked with v (which only adds intervals within $R(v)$). On the other hand, every value $x \in \mathbb{D}$ is covered by an interval in the final $\mathcal{I}$. Such an interval is inserted in $\mathcal{I}$ by running `voronoiIntv` with the lowest node u in $ECT(w)$ whose $R(u)$ covers x.

We proceed to show that, for each interval $I \in \mathcal{I}$, the NN associated with I is indeed the NN of all nodes in I. Consider any node u in $ECT(w)$. Denote its child nodes in $ECT(w)$ as $v_1, ..., v_f$ for some $f \geq 0$. For each $1 \leq j \leq f$, cutting $R(v_j)$ out of $R(u)$ at Line 4 effectively removes $sub(v_j)$ from $sub(u)$ (recall that $sub(.)$ represents the subtree of a node). Let $sub^{\triangle}(u)$ be the set of nodes in $sub(u)$, but not in the $sub(v_j)$ of any j. It suffices to prove that all nodes in $sub^{\triangle}(u)$ have the same NN as u. For each edge (u, v_j) of $ECT(w)$, define $P(u, v_j)$ as the path in $\mathcal{T}$ from node u to v_j, but *excluding* node v_j. Each node $z \in sub^{\triangle}(u)$ is either (i) on $P(u, v_j)$ for some j, or (ii) has an ancestor z' in $\mathcal{T}$ that is on $P(u, v_j)$ for some j. In the former case, $NN(z)$ must be $NN(u)$ by the way $ECT(w)$ is constructed. In the latter case, $NN(z) = NN(z')$, while z' is a node of case (i), implying $NN(z) = NN(u)$ as well.

It remains to bound the size of $\mathcal{I}$. By Lemma 3, $CT(w)$ has at most $2N_w - 2$ edges. As each of them may generate two edges in $ECT(w)$, the number of edges in $ECT(w)$ is at most $4N_w - 4 + 1 = 4N_w - 3$, including the (potential) extra edge due to $root(\mathcal{T})$. In `voronoiIntv`, each edge breaks $\mathbb{D}$ into at most 2 more intervals. Therefore, the final $|\mathcal{I}|$ is bounded above by $1 + 2(4N_w - 3) = 8N_w - 5$. $\square$

The proof of Theorem 1 is thus completed.

3.3 Finding the minimum TVP

This subsection completes our discussion of NK search by elaborating an algorithm for computing a TVP $\mathcal{I}$ of a keyword w with the *minimum* size. In fact, the main idea of the algorithm has been mentioned in Section 3.2, as summarized below:

```
algorithm computeTVP($w$)
1.  build $CT(w)$
2.  build $ECT(w)$ from $CT(w)$
3.  $\mathcal{I} = \emptyset$
4.  for each node $u$ in $ECT(w)$ do
5.      voronoiIntv($u$)
    /* at the end of the for-loop, $|\mathcal{I}| < 8N_w$ */
6.  merge consecutive intervals in $\mathcal{I}$ that are associated with
    the same NN
7.  return $\mathcal{I}$
```

Next, we clarify the details of Lines 1 and 2, because `voronoiIntv` has been presented in the previous subsection.

Construction of $CT(w)$. We construct $CT(w)$ in two steps: first collect the set S of nodes in $CT(w)$, and then connect them properly to meet the definition of $CT(w)$. The first step applies the algorithm below to compute S:

```
algorithm collectCTnodes($U(w)$)
1.  $S = U(w)$
2.  sort the nodes of $U(w)$ in ascending order of ranks
3.  for each pair of consecutive nodes $u, v$ do
4.      $S = S \cup lca(u, v)$
```

We claim that the final S includes all the branching nodes. Let z be any branching node. By definition, it has at least two child nodes that have a data node in their subtrees, respectively. Let u_1, u_2 be the two left-most ones of those child nodes, with u_1 being the left-most one. Denote by v_1 (v_2) the data node with the largest (smallest) rank in the subtree of u_1 (u_2). By the property of pre-order ranking, v_1 and v_2 constitute a pair of consecutive data nodes in the rank domain. Hence, z will be discovered as $lca(v_1, v_2)$.

Recall that each node in $CT(w)$ should be connected to its lowest ancestor (if any) among all the nodes in $CT(w)$. We achieve the purpose using a stack J. Specifically, we process the nodes of S in ascending order of their ranks, and maintain $CT(w)$ for the nodes already seen. At each moment, J keeps the right-most root-to-leaf path of the current $CT(w)$. Nodes of the path are pushed in J in the same order they are scanned, i.e., with the leaf (root) at the top (bottom).

```
algorithm computeCTedges($w$)
1.  sort the nodes of $S$ in ascending order of ranks
2.  $J = \emptyset$
3.  while $S \neq \emptyset$ do
4.      $u \leftarrow$ the first node of $S$; remove $u$ from $S$
```

```
5.      keep popping the top node $v$ of $J$ as long as $R(v)$ is
        disjoint with $R(u)$
6.      if $J$ is not empty then
7.          add an edge between $u$ and the top node of $J$
8.      push $u$ in $J$
```

Let us illustrate the algorithm using the set of nodes in the $CT(t)$ of Figure 6a. Here, $S = \{1, 2, 3, 5, 9, 23\}$. Node 1 is the first scanned, and directly pushed to J. For node 2, Line 5 has no effect, while Lines 6-7 add an edge between nodes 1 and 2. $J = \{2, 1\}$ at this time (with node 2 at the top). Similarly, the scanning of nodes 3 and 5 create edges $(2, 3)$ and $(3, 5)$ respectively, after which $J = \{5, 3, 2, 1\}$. Next, the algorithm comes to node 9. Line 5 pops node 5 out of J because $R(5)$ is found to be disjoint with $R(7)$. Line 6 then spawns another edge $(3, 9)$ in $CT(t)$. Finally, the handling of node 23 pops nodes 9, 3, 2, and adds one more edge $(1, 23)$. The $CT(t)$ now becomes final as S has been exhausted.

Construction of $ECT(w)$. Let (u, v) be an edge in $CT(w)$, with u being an ancestor of v. Lemma 4 states that there can be at most one NN-change on (u, v). In case no NN-change exists, (u, v) is kept directly in $ECT(w)$. Otherwise, we should create two edges (u, z), (z, v) in $ECT(w)$, where z is the first node (on the u-to-v path in $\mathcal{T}$) such that $NN(u) \neq NN(z)$.

Our algorithm for building $ECT(w)$, named `computeECT`, processes the edges (u, v) of $CT(w)$ in ascending order of $level(u)$. It keeps the invariant that, at the time (u, v) is to be processed, we have already determined $NN(u)$. It is easy to see that $NN(v)$ can only be either $NN(u)$ or $NN_{sub}(v)$, where $NN_{sub}(v)$ is the subtree nearest w-neighbor of u (see Section 2). Another key to the algorithm is that, if $NN(v)$ turns out to be different from $NN(v)$, the node z splitting (u, v) can be identified by a level-on-path query. This is because the level ℓ of z can be calculated as:

$$\ell = \left\lceil \frac{\delta + level(u) + level(v)}{2} \right\rceil \quad (1)$$

where $\delta = \|v, NN(v)\| - \|u, NN(u)\|$.

```
algorithm computeECT($CT(w)$)
1.   sort the edges $(u, v)$ of $CT(w)$ in ascending order of
     $level(e)$; $E \leftarrow$ the sorted list
2.   find the NN of the root of $CT(w)$ with a subtree NK query
3.   while $E \neq \emptyset$ do
4.       $(u, v) \leftarrow$ the first edge of $E$; remove it from $E$
5.       $NN_{sub}(v) \leftarrow$ the subtree nearest $w$-neighbor of $v$
6.       $NN(v) \leftarrow$ whichever of $NN(u)$ and $NN_{sub}(v)$ that
         is closer to $v$
7.       if $NN(u) = NN(v)$ then
8.           create edge $(u, v)$ in $ECT(w)$
9.       else
10.          $z \leftarrow$ the result of a level-on-path query using
             $(u, v)$ and $\ell$ (Equation 1)
11.          create edges $(u, z)$ and $(z, v)$ in $ECT(w)$
```

We demonstrate the algorithm by explaining how to derive the $ECT(t)$ in Figure 6b from the $CT(t)$ in Figure 6a. First, Line 1 arranges the edges of $CT(t)$ in the order of $E = \{(1, 2), (1, 23), (2, 3), (3, 5), (3, 9)\}$. Line 2 obtains $NN(1) = 2$. The subsequent execution examines each edge of E in turn. The first one, $(1, 2)$, is easy to handle because $NN(2) = NN(1) = 2$. Hence, $(1, 2)$ is included in $ECT(t)$ directly. Consider the second edge $(1, 23)$ of E. As $NN(23) = 23$ is different from $NN(1)$, Line 10 calculates $\ell = \lceil(-1 + 0 + 4)/2\rceil = 2$, and issues a level-on-path query to find the level-2 node z on the path from node 1 to node 23 in the data tree of Figure 4. The z retrieved is node 18. Line 11 then adds edges $(1, 18)$ and $(18, 23)$ into $ECT(t)$. The rest of the algorithm proceeds in the same manner.

Analysis. Line 1 of `computeTVP` invokes `collectCTnodes` and `computeCTedges`, both of which terminate in $O(N_w \log N_w)$ time. The same complexity applies to at Line 2, which executes `computeECT`. Lines 4-5 essentially spend constant time on each edge of $ECT(w)$, and hence, incur only $O(N_w)$ cost. Line 6 apparently requires $O(N_w)$ time. Therefore, the overall complexity of `computeTVP` is $O(N_w \log N_w)$.

THEOREM 2. *$\mathcal{T}$ can be pre-processed into a structure that occupies $O(K)$ space, such that any NK query can be answered in $O(\log N_w)$ time, where N_w is the number of nodes in T carrying the query keyword. The structure can be built in $O(K \log K)$ time.*

PROOF. The result follows from the discussion in Section 3.1, Theorem 1, the analysis of this subsection, and the fact that (i) the total construction cost of the structures of all keywords is $O(\sum_w (N_w \log N_w)) = O(K \log K)$, and (ii) the space of all these structures is $O(\sum_w N_w) = O(\sum_u |W(u)|) = O(K)$. $\square$

When B-trees, instead of binary trees, are deployed, the space and query complexities of our structure are $O(K/B)$ disk blocks and $O(\log_B N_w)$ I/Os, respectively, where B is the size of a block.

4. NEAREST KEYWORD SEARCH AS AN OPERATOR

In the introduction, we outlined why NK search can be deployed as a primitive operator to support other tasks efficiently. Sections 4.1 and 4.2 elaborate this for XPath query answering and group steiner tree computation, respectively.

4.1 XPath evaluation

This subsection aims at a theoretical justification that many XPath queries can be reduced to NK search. We consider queries that can be represented as a twig pattern Q as follows. Each internal node of Q gives an element type. A leaf node is designated as the *output*, indicating the information solicited (extension to multiple output nodes is trivial). Every other, non-output, leaf node carries a keyword, which imposes a predicate that must hold on each occurrence of Q in the data tree $\mathcal{T}$. See Figure 2a for an example. Without loss of generality, we assume that Q is compatible with the data schema; otherwise, it can be rejected by standard syntax checking with the DTD.

We will prescribe two conditions whose satisfaction *guarantees* the success of reducing Q to a set of NK queries. These conditions are conservative, in the sense that one may still carry out reduction even if the conditions do not hold (as shown in the experiments). We will see that the class of reducible queries can be processed by a new algorithm with an attractive worst-case performance bound.

Type-sequence condition. Let P be any path of $\mathcal{T}$ starting from $root(\mathcal{T})$. Define the *type sequence* of P as the ordering of the node types encountered as we walk along P. For instance, if P is the path from node `0` to node `010` in Figure 1, its type sequence is (`league`, `team`, `tname`).

CONDITION 1. *For each node u of the same type, the root-to-u path in $\mathcal{T}$ has the same type sequence.*

In Figure 1, for example, the path from the root to every `player` node has type sequence (`league`, `team`, `players`, `player`).

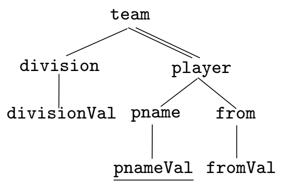


Figure 7: Type pattern of the query in Figure 2a

Anchor condition. As the next condition is more complex, we first explain the idea using an example. Consider Q to be the pattern in Figure 2a. Let us examine its *type pattern*, which is also a tree, and results from replacing each node of Q with its type, as shown in Figure 7. Denote the type pattern of Q as $type(Q)$.

Let us fix a leaf node, `fromVal`, in $type(Q)$ as the *anchor*, denoted as anc. Then, find the LCA, called a *critical LCA*, of anc and every other leaf, namely, (i) $clca_1$ = `team`, which is the LCA of anc and $leaf_1$ = `divisionVal`, and (ii) $clca_2$ = `player`, which is the LCA of anc and $leaf_2$ = `pnameVal`. Each pair $(clca_i, leaf_i)$ decides a *critical set* represented as $cset_i$, which includes all the types on the path from $clca_i$ to $leaf_i$. That is, $cset_1$ = {`team`, `division`, `divisionVal`}, and $cset_2$ = {`player`, `pname`, `pnameVal`}.

In the $\mathcal{T}$ of Figure 1, for each type-$clca_i$ node u, $sub(u)$ (i.e., the subtree of u) has *at most one* node of each type in $cset_i$. To be specific, let us inspect $clca_1$ = `team`. The subtree of a `team` node can have at most one node of type `team`, `division`, and `divisionVal`, respectively. Similarly, this is also true for $clca_2$, i.e., every `player` node can have in its subtree at most one `player`, `pname`, and `pnameVal` node, respectively. In this case, we say that Q is *unambiguous*.

Not all the choices of anc would make Q unambiguous. For example, it is not hard to see that Q is not unambiguous if `divisionVal` is selected as the anchor. The second condition requires:

CONDITION 2. *At least one choice of anchor makes Q unambiguous.*

To grasp the intuition behind the condition, consider again the strategy explained in Section 1.1 for processing the query Q of Figure 2a. Recall that we used the `fromVal` nodes carrying `Maryland` as the query nodes q for NK search. Instead, let us choose q as the `divisionVal` nodes associated with `west`. That is, for each such q, say node `0100`, the algorithm finds its nearest `Maryland`-neighbor (i.e., node `012020`) in the set of `fromVal` nodes, and then checks whether the neighbor has distance 6 to q. The answer is yes, so the presence of an occurrence has been detected. The problem, however, is that we can no longer find the output value `Blake` by identifying the nearest `pnameVal`-neighbor of q. This is because multiple `pnameVal` nodes have an *identical* (smallest) distance 6 to q and, hence, the NK search with `pnameVal` as the query word may happen to return a node (e.g., `012100`) that is not describing the same player as node `012020`.

We point out that, although the definitions and notations leading to Condition 2 were introduced with an example, they can be generalized in a straightforward manner to arbitrary Q.

Reducibility. The theorem below formally establishes the reduction from XPath evaluation to NK search.

THEOREM 3. *An XPath query can be reduced to NK search under Conditions 1 and 2.*

PROOF. As in Section 1.1, we set the type of each node u in $\mathcal{T}$ as a keyword in u. Each value node has its value as an extra keyword. An XPath query Q is processed with the following algorithm:

```
algorithm xpath($Q$)
1.  for each node $q$ in $\mathcal{T}$ having type $anc$ do
2.      for each non-output leaf node $u$ of $Q$ do
3.          $w \leftarrow$ the keyword of $u$
4.          $v \leftarrow$ the result of $NN(q, w)$ among all nodes in $\mathcal{T}$
            of the same type as $u$
            /* This can be done with minor extension to our technique
            in Section 3. For example, one simple solution is to prefix
            each word with the type of the node containing it. Such a
            prefix is added to $w$, which automatically restricts the
            search to the nodes of the designated type. */
5.          if $\|q, v\| \neq$ the correct value as in an occurrence then
6.              mark $q$ as pruned
7.              break for
8.      if $q$ has not been pruned then
9.          $w \leftarrow$ the type of the output node of $Q$
10.         $v \leftarrow NN(q, w)$
11.         if $\|q, v\| =$ the correct value as in an occurrence then
12.             report the value of $v$
```

Let us refer to the type-anc node in an occurrence of Q as an *anchor node*. Denote by f be the number of leaf nodes in $type(Q)$ other than anc. Lines 2-12 of `xpath` are collectively called an *iteration*. We will prove that (a) the output value in every occurrence is reported by `xpath`, and (b) every value reported is indeed the output value of an occurrence.

Proof of statement (a). Let occ be any occurrence with q as its anchor node. Denote by u_i the type-$leaf_i$ node in occ ($1 \leq i \leq f$). Each u_i must be retrieved by the NK-query at Line 4 (or 10) in the iteration for q, and pass the if-condition at Line 5 (or 11). The iteration reports the output value of occ at Line 12.

Proof of statement (b). Under Condition 1, it suffices to consider Q with single-lined edges only, because any double-lined edge can be expanded into a set of single-lined edges without affecting the query result. Condition 1 also allows us to focus on Q (i) that has only a single node, or (ii) whose root has at least two child nodes. If $root(Q)$ has a single child, we can remove the root while still obtaining the same result.

Assume that `xpath` reports a value in an iteration with q as the anchor node. Line 4 or 10 must have fetched a type-$leaf_i$ node u_i ($1 \leq i \leq f$). Let occ be the tree that (i) is rooted at the LCA of $u_1, ..., u_f, q$, and (ii) includes (only) the edges on the path from the LCA to each of $u_1, ..., u_f, q$. The rest of the proof argues that occ is an occurrence. Towards this, we consider tree $type(occ)$, which is obtained by replacing each node of occ with its type. Our goal is to show that $type(occ)$ and $type(Q)$ are exactly the same.

Condition 1 implies that all nodes of a type must be at the same level. Given a node type z, we use $seq(z)$ to represent the type sequence of the path in the data tree $\mathcal{T}$ that goes from the root (of $\mathcal{T}$) to an arbitrary type-z node u (recall that $seq(z)$ is unique regardless of u). Also, suppose that $clca_1$, ..., $clca_f$ are in the top-down order.

As $clca_i \in seq(anc)$ for each $i \in [1, f]$, q has a (unique) type-$clca_i$ ancestor in $\mathcal{T}$, which we represent as v_i. A key observation is that u_i must be in the subtree of v_i. Otherwise, $\|q, u_i\|$ would have been at least 2 more than the correct value in an occurrence, noticing that the path from q to u_i would need to go outside $sub(v_i)$, and then eventually descend to the level where type-$leaf_i$ nodes are. From $type(Q)$, we know that $clca_i$ is the last common type in $seq(leaf_i)$ and $seq(anc)$, implying that v_i must be the LCA of u_i and q, and belong to occ. Hence, the root of occ has type $clca_1$.

Let P_Q (or P_{occ}) be the path in $type(Q)$ (or $type(occ)$) from the root to anc. The earlier analysis indicates that P_Q is identical to P_{occ}. Next we prove that $type(occ)$ and $type(Q)$ are the same in the other parts as well. Let us walk down P_Q and P_{occ} synchronously, and stop as soon as encountering a $clca_i$ (of any $i \in [1, f]$) in both paths. Let T_Q (or T_{occ}) be the subtree of $clca_i$ in $type(Q)$ (or $type(occ)$), removing the subtree rooted at z, where z is the child node of $clca_i$ that is an ancestor of anc.

Define $S = \{j \mid clca_j = clca_i\}$ (it is possible for several critical LCAs to coincide on one node). Let z be any type in the $cset_j$ of any $j \in S$. Under Condition 2, there is a unique type-z node in the subtree of v_i in $\mathcal{T}$. Therefore, each type in T_Q (T_{occ}) can appear only once. For each $leaf_j$ ($j \in S$), define a type-sequence s_j that equals the suffix of $seq(leaf_j)$ starting from $clca_i$. Regarding each type as a symbol, both T_Q and T_{occ} are in fact a trie of the same set of strings $\{s_j \mid j \in S\}$.

The above reasoning of T_Q and T_{occ} is independent of i. We thus have established the equivalence of $type(Q)$ and $type(occ)$. $\square$

Remark 1. We outline the main ideas on how to verify Conditions 1 and 2 efficiently, while leaving the complete details to the full paper. We regard each rule of the DTD as having the form $e \to s$, where e is an element type and s a string specifying the possible child element types of e. Combine multiple rules whose left hand sides are the same type. Denoting by S the set of resulting rules, we can show that Condition 1 is satisfied if and only if every element type appears on the right hand side (RHS) of exactly one rule in S.

To check Condition 2, we resort to a *schema tree* ST, where each node is an element type e, whose child nodes are the element types in the RHS of rule $e \to s$. Recall that every element type c in s may carry a star (e.g., $e \to c^*$ means that there can be multiple instances of c below e directly). In this case, the edge (in ST) from e to c is a *star edge*; otherwise, it is a *non-star edge*. Given a query Q, ST allows us to verify Condition 2 as follows. First pick a leaf node of $type(Q)$ as the anchor anc. Perform the following for every other leaf node u: identify the LCA v of anc and u, and examine if the path from v to u in ST has a star edge. If the answer is no for all u, we assert that Q satisfies Condition 2. Otherwise, pick a different anchor and repeat the above process. If all anchors have been attempted and Q has not been confirmed to satisfy Condition 2, we conclude that it violates the condition. Clearly, the time of Condition-2 checking depends only on the schema, is irrelevant to the size of the XML document, and hence, typically accounts for only a fraction of the total query cost.

Note that Condition 1 (2) is a constraint on the data (queries). It is worth mentioning that the two conditions are satisfied by many real datasets and meaningful queries. In particular, we note that all the datasets experimented in [4, 6, 10, 34, 35] and at least one in [7, 15, 16, 20, 27, 28, 29, 30, 32, 37] satisfy Condition 1 (see also our experiments). We point out that simple *aliasing* can often be carried out to make a dataset satisfy Condition 1. For example, in the well-known *DBLP* dataset, an `author` element may be under an `article` or `inproceedings` element. To meet Condition 1, we can rename the `author` of former type as `a-author`, and that of the latter type as `i-author`.

Remark 2. Denote by $leaf_1, ..., leaf_f$ the leaf nodes of Q that are not the anchor anc. By Theorem 2, algorithm `xpath` terminates in $O(N_{anc} \sum_{i=1}^{f} \log N_{leaf_i})$ time, where N_x is the number of nodes in the data tree $\mathcal{T}$ fulfilling the *predicate* implied by the node x in $type(Q)$. For example, for the Q of Figure 2a, the execution time is $O(N_{\texttt{Maryland}}(\log N_{\texttt{west}} + \log N_{\texttt{pnameVal}}))$. In general, the time complexity of `xpath` is independent on the number of nodes in $\mathcal{T}$ whose types are *internal* nodes of $type(Q)$. This is a unique performance characteristic that is not shared by any previous solution.

4.2 Finding approximate group steiner trees

This section discusses the GST problem as motivated in Section 1. The dataset is a tree $\mathcal{T}$ as described in Section 2. A query specifies a set of keywords $QS = \{w_1, ..., w_l\}$. Recall that $U(w_i)$ is the set of nodes in $\mathcal{T}$ associated with w_i ($1 \le i \le l$). Any l nodes $u_1, ..., u_l$, where $u_i \in U(w_i)$ for each i, uniquely determines a *minimum connecting tree* (*MCT*) [20] M as follows. M has the LCA of all $u_1, ..., u_l$ as its root, and includes (all and only) the edges on the path in $\mathcal{T}$ from the LCA to every u_i. Referring to $(u_1, ..., u_l)$ as a *vector*, the GST problem is equivalent to discovering the vector that minimizes the number of edges in M. This problem is NP-hard [17].

As an example, assume $\mathcal{T}$ to be the data tree in Figure 1, and $w_1 = \texttt{Lakers}$, $w_2 = \texttt{Blake}$, $w_3 = \texttt{guard}$ ($l = 3$). Given u_1 = node `0100`, u_2 = node `012000`, and u_3 = node `012010`, Figure 3 demonstrates the corresponding MCT M. Notice that the root of M is the LCA of u_1, u_2 and u_3.

We assume that $w_1, ..., w_m$ have been arranged in such a way that $|U(w_1)| \le ... \le |U(w_l)|$. The following algorithm extracts an MCT with a small number of edges.

```
algorithm approxGST($w_1, ..., w_l$)
1.  $d_{min} \leftarrow \infty$
2.  for each node $u_1 \in U(w_1)$ do
3.      for $i \leftarrow 2$ to $l$ do
4.          $u_i \leftarrow NN(u_1, w_i)$
5.      $d \leftarrow \sum_{i=2}^{l} \|u_1, u_i\|$
6.      if $d < d_{min}$ then
7.          remember $(u_1, ..., u_l)$ as the best vector
8.          $d_{min} = d$
9.  return the MCT $M$ determined by the best vector
```

The for-loop in Lines 2-8 enumerates each node $u_1 \in U(w_1)$. Lines 3-4 find the nearest w_i-neighbor of u_1 for every other query keyword w_i. The resulting neighbors, together with u_1, determine an MCT. The key of the algorithm is to measure the quality of the tree as the sum of the distance from u_1 to each retrieved neighbor (see Line 5). The algorithm eventually returns the best tree according to this quality metric.

We say that an MCT M is a *c-approximate GST* if it has at most c times more edges than the GST. Formally, if $cost(M)$ represents the number of edges in M, it holds that $cost(M) \le c \cdot cost(M^\star)$, where $M^\star$ is the (optimal) GST.

LEMMA 6. *The output of* `approxGST` *is an* $(l-1)$-*approximate GST.*

PROOF. For each $i \in [1, l]$, let $u_i^\star$ be a node in $M^\star$ associated with keyword w_i (if there are several such nodes, $u_i^\star$ can be any of them). Define $d^\star = \sum_{i=2}^{l} \|u_1^\star, u_i^\star\|$. Since every edge of $M^\star$ is included at most once in the path from $u_1^\star$ to each $u_i^\star$ ($2 \le i \le l$), it holds that:

$$d^\star \le (l-1) \cdot cost(M^\star). \tag{2}$$

Let M_{apx} be the output of `approxGST`. The d_{min} in the sequel refers to the final d_{min} of `approxGST`. Notice that $\{u_1^\star$, $NN(u_1^\star, w_2)$, ..., $NN(u_1^\star, w_l)\}$ is a set $\{u_1, ..., u_l\}$ that must have been inspected at Line 5 of `approxGST`. In other words:

$$d_{min} \le \sum_{i=2}^{l} \|u_1^\star, NN(u_1^\star, w_i)\| \le d^\star. \tag{3}$$

where the second $\le$ is because $\|u_1^\star, NN(u_1^\star, w_i)\| \le \|u_1^\star, u_i^\star\|$.

Let $\{\hat{u}_1, ..., \hat{u}_l\}$ be the set $\{u_1, ..., u_l\}$ that determines M_{apx} at Line 7 of approxGST. Then, $d_{min} = \sum_{i=2}^{l} \|\hat{u}_1, \hat{u}_i\|$. Every edge of M_{apx} is used at least once in the union of the paths from $\hat{u}_1$ to $\hat{u}_2$, ..., $\hat{u}_l$, respectively. Hence:

$$cost(M_{apx}) \leq d_{min}. \tag{4}$$

Combining Inequalities 2-4 gives $cost(M_{apx}) \leq (l-1) \cdot cost(M^\star)$. $\square$

By Theorem 2, the execution time of approxGST is bounded by $O(N_{min} l \log N_{max})$, plus the cost of outputting the tree, where $N_{min} = |U(w_1)|$ and $N_{max} = |U(w_l)|$.

Remark 3. Sometimes it is useful to return k MCTs with small cost, where k is a user-specified parameter. In this case, we maintain the k best vectors currently found, as opposed to only the top-1. This can be achieved with minor modification to approxGST.

5. RELATED WORK

NK search has not been studied previously. In the sequel, we review the existing work on other topics related to our discussion.

The first topic is the processing of *holistic twig joins*, where the goal is to enumerate *all* occurrences of a twig pattern. The existing algorithms can be classified as *sequential* or *indexed*. A *sequential algorithm* [4, 5, 8, 7, 15, 21, 30, 32, 37] scans synchronously the nodes, whose types appear in the query pattern, in ascending order of their ranks. The drawback of these algorithms is that they must access every such node *at least once*, even though it does not participate in any occurrence. Motivated by this, an *indexed algorithm* [4, 21] utilizes a data structure that can be used to efficiently retrieve a particular ancestor/descendent of a node. Such an ability allows the algorithm to skip many nodes not involved in any occurrence and, therefore, to terminate much earlier.

In this paper, we do not attempt to attack general holistic twig joins. Instead, our focus is to improve the efficiency of those XPath queries that can be processed with NK search. As far as these queries are concerned, our method significantly outperforms all the solutions in the sequential category because (similar to indexed algorithms) it only needs to access a small number of nodes that may form an occurrence. Regarding the indexed category, the comparison is more subtle, mainly because the algorithms of [4, 21] are heuristic in nature, and are not accompanied by any nontrivial complexity analysis. We will experimentally compare our technique against the state of the art, *TSGeneric*$^+$ [21], of that category. Noteworthily, an advantage of our algorithm xpath (Section 4.1) is that, in practice, its cost can be accurately estimated by a query optimizer. This is not possible for [4, 21], as their behavior is sensitive to the data distributions.

We note that no existing algorithm has the performance characteristic pinpointed in Remark 2. The only approach that comes close to having the characteristic is TJFast [30], which is a sequential algorithm that needs to scan the nodes matching only the leaves of the query pattern. Unfortunately, the *extended dewey codes* that TJFast relies on can be as long as the height of the XML tree. The time of reading a node is proportional to the length of its extended dewey code. The length, unfortunately, is linear to the number of ancestors of the node, which can be asymptotically identical to the total number of nodes in the tree in the worst case. It is also worth mentioning that there exist some other methods [9, 24], just like ours, that are designed for certain special classes of XPath queries.

Another related topic is keyword search in XML databases. A bulk of the existing research [6, 10, 16, 20, 27, 28, 29, 35] explores various semantics of query results that is suitable for different scenarios. In this work, we showed the applicability of NK search to the GST semantics [12, 20, 23, 25]. This choice does not imply our preference of GST; in fact, the potential application of the NK operator to the other semantics is an exciting direction for future work. As mentioned before, the GST problem on trees is NP-hard, and remains so even if the goal is changed to finding an $O(\log^{2-\epsilon} n)$-approximate solution for arbitrarily small $\epsilon > 0$ [17], where n is the number of nodes in the data tree. The best known approximation ratio achievable in polynomial time is $O(\log n \log l)$ [14], where l is the number of query keywords (and can be as large as n). The algorithm of [14], which is based on linear programming, is highly theoretical and not appropriate for practical implementation.

In the database area, research on GST computation (e.g., [3, 12, 19, 20, 22, 25]) is largely motivated by the observation that the value of l can often be regarded as a constant in reality. In this case, the $l-1$ approximation ratio guaranteed by our solution (Lemma 6) can be (much) lower than $O(\log n \log l)$. A small l also considerably shrinks the search space for discovering the GST. This fact is leveraged in [20] to enumerate all the MCTs (defined in Section 4.2), and thereby, eventually come across the GST (which is also an MCT). While the method of [20] works on trees only, the approaches reviewed below apply to general graphs[1]. Algorithms for computing approximate GSTs (faster than extracting the GST with [20]) are presented in [3, 22]. Somewhat surprisingly, there is a dynamic-programming algorithm [12] for finding the *exact* GST, in even less time than the approximate methods of [3, 22].

The above approaches do not rely on pre-computation, whereas the BLINKS system [19] leverages a sophisticated access method constructed in advance to produce l-approximate GSTs efficiently. The most serious drawback of BLINKS, however, is that it occupies $\Omega(n^{4/3})$ space[2], where n is the number of nodes in the graph. For n at the order of millions, $\Omega(n^{4/3})$ amounts to duplicating the database 100 times, which is too expensive in many environments. Furthermore, the query algorithm of BLINKS is ad-hoc and has no interesting worst-case performance bound. Note that our result in Lemma 6 in fact dominates the performance of BLINKS, i.e., we guarantee a slightly better approximation ratio, with a *linear* space access method and worst-case efficient query time. It should be noted, however, that the improvement is made possible by utilizing properties of a tree (recall that BLINKS deals with general graphs). The work of [23, 25] considers the *steiner tree* problem, which is a special case of the GST problem where there can be only a single node in the data tree associated with each query keyword. Note that, while the steiner tree problem is NP-hard on graphs, it is not on trees, as is clear from the way that MCT is built from a vector in Section 4.2.

Finally, in the special case where only one distinct keyword exists, NK retrieval is similar to *nearest neighbor search on spatial networks*. In that problem, the data consist of a graph G and a set S of points, each located at a node of G. Given a node q of G, a query returns the point in S that has the smallest shortest-path distance to q. This problem has been well studied in various settings with different performance goals (see [11, 26, 31, 33] and the references therein). The existing solutions (i) perform Dijkstra-like expansion from the query point (e.g., [11]), (ii) pre-compute the answer for each node of G (e.g., [31]), or (iii) rely on specialized structures that leverage properties of spatial data (e.g., [26, 33]). In our con-

[1]Given a set of keywords, the GST in a graph is the minimum tree (i) whose nodes and edges come from the underlying graph, and (ii) that has the minimum number of edges among all trees satisfying (i). When the graph is a tree, this definition degenerates into the one in Section 4.2.

[2]This can be derived from Theorem 3 of [19] by observing that, $\sum_b N_b^2 + BP \geq n^2/B + B^2$, which is $\Omega(n^{4/3})$.

text, solutions of (i) degenerate into the BFT approach explained in Section 3.1, those of (ii) incur prohibitive space when the number of keywords is large, whereas those of (iii) are simply inapplicable to XML data.

6. EXPERIMENTS

This section empirically evaluates the performance of the proposed techniques. We used two real XML documents:

- *NBA*, which contains all the teams and players of the leagues during 1946-2004.
- *DBLP*, which includes all the conference papers during 1959-2010 collected by the DBLP website.

Figure 8a (8b) shows the part of the *NBA* (*DBLP*) schema that is relevant to the queries in our experiments. An asterisk indicates that the corresponding node can have an arbitrary number of siblings of the same type. For example, an `NBA` node can have multiple child nodes of type `league`. Table 1 lists the main statistics about each dataset.

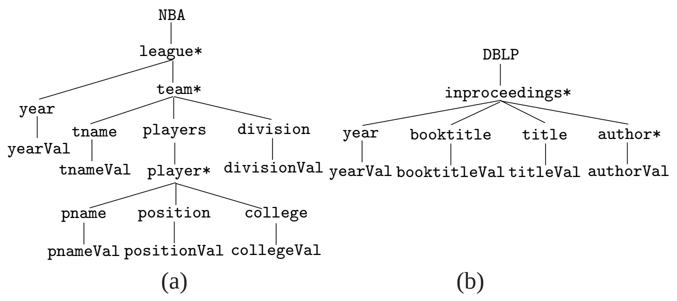


Figure 8: Schemas of *NBA* and *DBLP* relevant to our queries

	NBA	*DBLP*
num. of nodes	135,940	17,501,788
num. of keywords	223,500	48,191,004
num. of distinct keywords	8,302	2,893,195

Table 1: Dataset statistics

We will first explore the characteristics of NK search, and then assess the usefulness of the NK operator in XPath evaluation and GST computation, respectively. Our experiments were performed on a computer that was running Linux, and equipped with an Intel DUO CPU at 3.0Ghz and 4 GB of memory. All the data were memory resident.

Performance of NK search. In our indexing scheme, named *TVP*, we followed the convention mentioned in Section 1.1 to associate nodes with keywords. Specifically, every node had its type as a keyword. Furthermore, if u is a value node, each word w in the value of u was taken as another keyword of u, after w was prefixed with the type of u (see Remark 1 of Section 4.1). The name of a person/team was always regarded as a single word. For instance, if u has type `pnameVal` and a value `Kobe Bryant`, then it carries a (prefixed) keyword `pnameVal:Kobe-Bryant`. Table 2 shows the space consumption of *TVP* and the raw dataset, while Table 3 gives the construction time of *TVP*.

The next experiment compares the query efficiency of *TVP* and the *BFT* algorithm described in Section 3.1. For this purpose, we report their performance on the (realistic) queries in Figure 9, each of which can be handled by a single NK operator. Queries denoted with a first letter N (D) were designed for *NBA* (*DBLP*). For each

	NBA	*DBLP*
TVP	3.8	972
raw dataset	2.4	807

Table 2: Space (mega bytes)

NBA	*DBLP*
$<$ 1 second	$<$ 2.5 minutes

Table 3: Construction cost of *TVP*

$\boldsymbol{NQ1}$: Find the team of Shaquille O'Neal in 2000.
q = the `pnameVal` *node with value Shaquille-O'Neal in the league 2000;* w = `tnameVal`

$\boldsymbol{NQ2}$: Were Shaquiile O'Neal and Kobe Briant in the same team in year 2000?
q = same as $NQ1$*;* w = `pnameVal:Kobe-Bryant`*; distance (of a positive answer) = 6*

$\boldsymbol{NQ3}$: Same as $NQ2$ but in 2002.
q = same as $NQ1$ *but in 2002;* w *and distance as in* $NQ2$

$\boldsymbol{DQ1}$: Find the conference name of "Holistic Twig Joins: Optimal XML Pattern Matching".
q = the `ptitleVal` *node of the paper;* w = `booktitleVal`

$\boldsymbol{DQ2}$: Is Nicolas Bruno an author of the paper in $DQ1$?
q = same as $DQ1$*;* w = `authorVal:Nicolas-Bruno`*; distance = 4*

$\boldsymbol{DQ3}$: Same as $DQ2$ but with respect to Jim Gray.
q and distance same as $DQ2$*;* w = `authorVal:Jim-Gray`

Figure 9: Query set for examining NK efficiency

query, Figure 9 clarifies the node q and keyword w of the corresponding NK operator. For a boolean query, the figure also points out the distance (to q) that the nearest w-neighbor should have in order to return a positive answer. For instance, the answer for $NQ2$ is "yes" if and only if the retrieved neighbor has distance 6 to q.

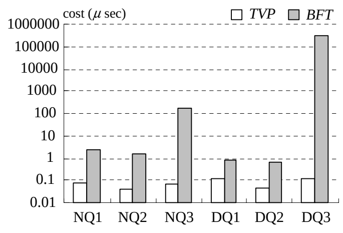


Figure 10: Cost of NK search

Figure 10 illustrates the cost of *TVP* and *BFT* for each query. Note that the vertical axis is in logarithmic scale, and has the unit of μ-second (= 10^{-6} second). *BFT* is especially slow for the boolean queries $NQ3$ and $DQ3$ with answers "no". This is expected because, for these queries, the retrieved neighbor has a large distance to the query node.

XPath evaluation. We proceed to assess the efficiency of the NK operator in answering XPath queries. Our technique integrated the proposed access method *TVP* with the `xpath` algorithm developed in Section 4.1. As discussed in Section 5, the existing algorithms for holistic twig joins can be classified into the *sequential* and *indexed* categories, with the solutions in the latter category significantly faster than those of the former. Therefore, we chose to compare *TVP* against the state-of-the-art indexed algorithm *TSGeneric*$^+$ [21].

Figures 11a and 11b explain the meaning and twig patterns of the queries examined, respectively. The box in each pattern signifies the anchor node chosen by `xpath`. Observe that $NQ7$ and $DQ7$ *violate* the anchor condition (i.e., Condition 2) in Section 4.2. Nevertheless, they are still reducible to NK search and, in fact, can be answered directly by the `xpath` algorithm *with no modification*.

For fairness, our implementation of *TSGeneric*$^+$ utilizes an inverted index to filter all the nodes that do not satisfy the keyword conditions. For example, for $NQ4$, only the `collegeVal` nodes

$NQ4$: Find the names of all players from Duke University.

$NQ5$: Find the names of all centers from the west division.

$NQ6$: Find the names of all players that came from Boston College but were in a team of the west division in year 1999.

$NQ7$: Find the teams where Hakeem Olajuwon and Charles Barkley were teammates.

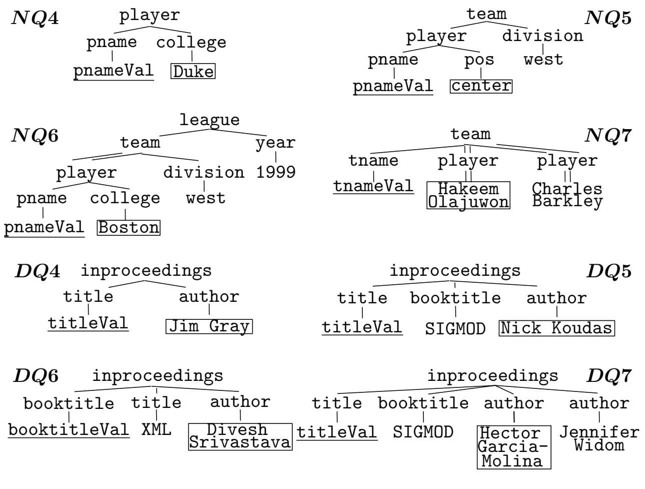
$DQ4$: Find the titles of all papers by Jim Gray.

$DQ5$: Find the titles of the SIGMOD papers by Nick Koudas.

$DQ6$: Find the conferences of the papers by Divesh Srivastava that contained "XML" in the titles.

$DQ7$: Find the titles of the SIGMOD papers co-authored by Hector Garcia-Molina and Jennifer Widom.

(a) Query description



(b) Twig patterns

Figure 11: Query set for examining XPath efficiency

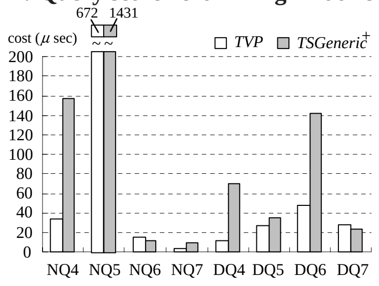


Figure 12: Cost of XPath queries

whose values are equal to `Duke` are considered, as opposed to all the `collegeVal` nodes (as proposed in [21]). This optimization reduces the average overhead of the original *TSGeneric*$^+$ by at least an order of magnitude. Figure 12 demonstrates the execution time of *TVP* and *TSGeneric*$^+$ for all queries. *TVP* lost narrowly in two out of the eight queries, but was at least twice faster in four queries.

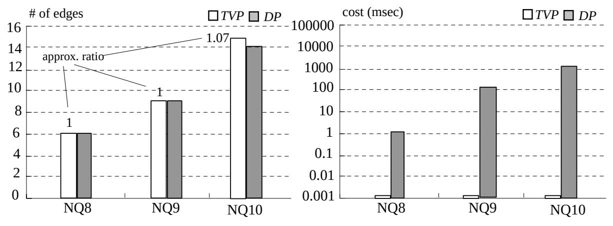
GST computation. The final set of experiments investigate the efficiency and effectiveness of the NK operator in keyword search. Our approach combined the *TVP* indexing scheme with the `approxGST` algorithm in Section 4.2. We compared the approach against the dynamic programming algorithm of [12] (as reviewed in Section 5), hereafter denoted by *DP*. Recall that *DP* returns the *exact* GST. We evaluated the two methods only with *NBA*, as the excessive memory requirements of *DP* rendered it infeasible to run experiments with *DBLP* on our hardware. As the XML schema is irrelevant to keyword search, the *TVP* built here does not require the keyword prefixing explained earlier. In other words, each element node (as before) is associated with its type, whereas each

$NQ8$: {`Shaquille-O'Neal`, `Anfernee-Hardaway`}

$NQ9$: {`Shaquille-O'Neal`, `Anfernee-Hardaway`, `Orlando-Magic`}

$NQ10$: {`Shaquille-O'Neal`, `Anfernee-Hardaway`, `Orlando-Magic`, `2000`}

Figure 13: Query set for keyword search evaluation



(a) Number of edges (b) CPU time

Figure 14: Quality and efficiency of GST computation

value node is simply associated with the words in its value. Figure 13 illustrates the queries executed.

Figure 14a depicts, for each query, the number of edges in the trees output by *TVP* and *DP*, respectively. It also indicates the approximation ratio of *TVP*. Notice that *TVP* returns the exact GST for $NQ8$ and $NQ9$, whereas its result for $NQ10$ has one more edge than the optimal solution. This shows that, in practice, the actual approximation ratio of *TVP* is much better than predicted by theory (Lemma 6). Figure 14b plots the CPU time of the two methods. *TVP* outperforms *DP* by two to five orders of magnitude. Moreover, unlike *DP* whose cost surges with the number of query words, the overhead of *TVP* is only slightly affected.

To better capture the usefulness of *TVP* in keyword search, Figure 15 presents the trees returned by *TVP* in the previous experiment. The result of $NQ8$ essentially states that `Shaquille-O'Neal` and `Anfernee-Hardaway` played for the same team at least once in their career. The tree of $NQ9$ implies that the above players were once teammates in `Orlando-Magic`. Finally, the $NQ10$ result indicates that `Shaquille-O'Neal` and `Anfernee-Hardaway` belonged to different teams in `2000`, neither of which was `Orlando-Magic`.

We conclude our experiments by comparing *TVP* and *DP* in searching for the top-k GSTs of $NQ9$ with $k > 1$. *TVP* now modifies `approxGST` as discussed in Remark 3. Note that *both TVP* and *DP* are approximate, namely, *DP* guarantees returning the top-1 GST, but such optimality is not ensured for $k > 1$. Figure 16a plots the result quality (in the number of tree edges) as a function of k. The two algorithms output trees with identical sizes when

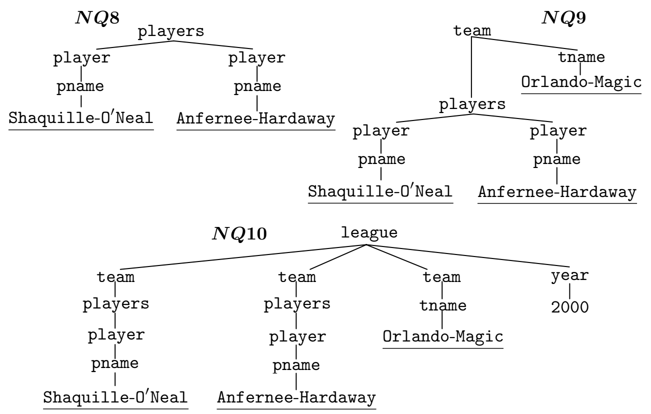
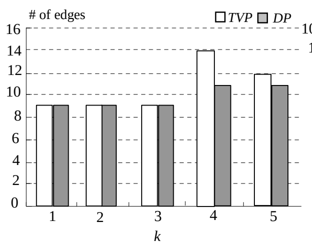
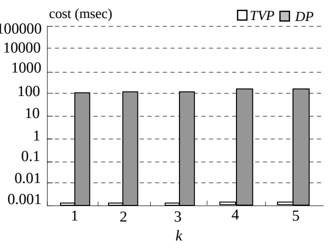


Figure 15: MCTs returned by *TVP* for the queries of Figure 13



(a) Number of edges



(b) CPU time

Figure 16: Quality and efficiency of top-k GST computation

$k \in [1, 3]$, whereas *DP* offers slightly smaller trees for $k \in [4, 5]$. Figure 16b gives their CPU cost. The overhead for both methods gradually elevates with k. *TVP* constantly outperforms *DP* by more than four orders of magnitude.

7. CONCLUSIONS

This paper proposed the problem of NK search on XML documents. Given a node q and a keyword w, an NK query returns the node in the XML tree that has the shortest distance to q, among all nodes associated with w. We solved the problem with a novel technique called tree Voronoi partition that gives rise to an indexing scheme with rigorous worst-case performance guarantees. Specifically, our scheme consumes linear space, and answers every NK query in time logarithmic to how many nodes carry the query keyword. We also demonstrated, both theoretically and experimentally, the usefulness of the NK operator in supporting several important tasks in XML databases. In particular, our technique results in (i) a new methodology for solving a wide class of XPath queries that is both asymptotically and practically efficient, and (ii) a fast algorithm for finding an approximate GST with bounded quality.

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